Adv. Polar Upper Atmos. Res., **20**, 38–45, 2006

Research note

Anti-diamagnetic (paramagnetic) properties of slow mode waves in anisotropic plasma pressures

Osuke Saka

Department of Physics, Kurume National College of Technology, Kurume 830-8555
E-mail: saka.o@pep.ne.jp

(Received October 12, 2005; Accepted January 4, 2006)

Abstract: In this report, we examined diamagnetic and anti-diamagnetic (paramagnetic) properties of magnetosonic modes in anisotropic fluid pressure condition by incorporating double adiabatic equations of state. In the context of liner perturbations, it is found that there appear two wave modes corresponding to a slow phase velocity and a fast phase velocity as is resembled to those in isotropic plasmas. For the fast phase velocity mode, pressure perturbations and field perturbations exhibited paramagnetic relations, similarly to isotropic plasmas. For the slow phase velocity mode, paramagnetic properties appear, unlike the isotropic case, at higher plasma beta part of lower pressure anisotropy (perpendicular/parallel) region.

key words: anisotropic plasmas, magnetosonic waves, diamagnetic and anti-diamagnetic properties

1. Introduction

In the midnight magnetosphere, slow mode waves were often observed as common disturbances of the magnetic field lines (*e.g.* Nakamizo and Iijima, 2003). Meanwhile, earthward fast plasma flows that are thought to carry a major substorm energy flux toward the earth are often observed in the magnetotail region (Nakamura *et al*., 2001). In addition, the earthward fast plasma flows carry a dipolarization front (Nakamura *et al*., 2002). The dipolarization front can be regarded as a tangential discontinuity as the pressure balance is maintained and no plasma flows is found across the front (Sergeev *et al*., 1996). Therefore, it is reasonable to interpret a tangential discontinuity as a large amplitude slow mode wave. We expect that slow mode and/or tangential discontinuity are common disturbances in the magnetotail region. On the other hand, low and high energy plasmas have been reported to behave differently in the midnight magnetosphere. During increased emission of auroral in ground optical data, electron fluxes in 0.05–5 keV range dropped while those in 30–37 keV enhanced at Geos-2 satellite conjugate to the ground observation (Shepherd *et al*., 1980). The electron flux modulations were observed by ATS-6 at Pi2 frequency range in association with ground Pi2 onset. The modulations at the geosynchronous altitudes were out of phase below and above 5 keV (Arnoldy, 1986). The high energy flux increase and simultaneous pressure decreases in plasma sheet are well-known signatures during substorm expansion. The decrease is attributed to a reduction of plasma particles in lower energy ranges from plasma sheet flux tube (Lyons *et al*., 2003).

In this report, we examine first order perturbations of magnetosonic waves in anisotropic plasmas based on compressive-mode analysis introduced by Kadomtsev (1976). The magnetosonic waves can carry divergence of the cross-field plasma displacement $\xi_\perp$. The plasma displacement accompanies a change of filed magnitude and plasma density. We show that those two quantities vary in parallel or oppositely in more complex way in slow mode perturbations. We suggest that a different behavior of plasmas in different energy ranges that was often observed in the magnetotail can be attributed to diamagnetic nature inherent in the plasmas therein.

2. MHD equations in anisotropic plasmas

We examine field-aligned flow acceleration by assuming anisotropic plasma pressure condition ($P_{0,1\perp} \neq P_{0,1z}$). Here, $\perp$ and z denote "perpendicular" and "parallel" to the field lines. The suffix "0" and "1" indicate the background and perturbed quantities.

We start with the field-aligned acceleration of the plasma flow by the following equation (*e.g.* Parks, 1991).

$$\rho \frac{d\mathbf{V}_z}{\mathrm{d}t} = -\nabla_z P_z - (P_z - P_\perp)\,\mathbf{b}\,(\nabla \cdot \mathbf{b}). \tag{1}$$

Here, $\mathbf{b}$ is unit vector of the magnetic field $\mathbf{B}$, ρ is mass density, $\mathbf{V}_z$ is field-aligned velocity. The second term of the right-hand side of the equation can also be written as $\frac{(P_z - P_\perp)}{B} \nabla_z B$, because $\mathbf{b}\,(\nabla \cdot \mathbf{b}) = -\frac{\mathbf{B}}{B^2} \nabla_z B$. Equations of motion perpendicular to the field lines in anisotropic plasma can be given by the equation (*e.g.* Parks, 1991)

$$\rho \frac{\mathrm{d}\mathrm{V}_\perp}{\mathrm{d}t} = \mathbf{j} \times \mathbf{B} - \nabla_\perp P_\perp - (P_z - P_\perp)\ (\mathbf{b} \cdot \nabla)\mathbf{b}. \tag{2}$$

Here, $\mathbf{j}$ is a current vector.
The field equation is given by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}). \tag{3}$$

In addition, plasma motions normal and along the field lines are assumed to be decoupled. This condition leads to double adiabatic equations of state given by (*e.g.* Parks, 1991)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P_\perp}{\rho B}\right) = 0, \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P_z B^2}{\rho^3}\right) = 0. \tag{5}$$

We will derive first order perturbations from eqs. (1) through (5) assuming uniform background field geometries and uniform distribution of background plasma density, pressure. The pressure is however anisotropic. The first order perturbations are expressed as,

$$\rho_0\frac{\partial \mathbf{V}_{1z}}{\partial t}=-\nabla_z P_{1z}+\frac{(P_{0z}-P_{0\perp})}{B_0}\nabla_z B_1, \tag{1'}$$

$$\rho_0\frac{\partial \mathbf{V}_{1\perp}}{\partial t}=\mathbf{j}_1\times\mathbf{B}_0-\nabla_\perp P_{1\perp}-(P_{0z}-P_{0\perp})(\mathbf{b}_0\cdot\nabla)\mathbf{b}_{1\perp}, \tag{2'}$$

$$\frac{\partial \mathbf{B}_1}{\partial t}=\nabla\times(\mathbf{V}_1\times\mathbf{B}_0), \tag{3'}$$

$$P_{1\perp}=\frac{P_{0\perp}}{\rho_0}\rho_1+\frac{P_{0\perp}}{B_0}B_1, \tag{4'}$$

$$P_{1z}=-2\frac{P_{0z}}{B_0}B_1+3\frac{P_{0z}}{\rho_0}\rho_1. \tag{5'}$$

Note that the suffix "0" and "1" indicate the background and perturbed quantities. We assume no background flow, $\mathbf{V}_0$ and no background currents, $\mathbf{J}_0$. The first term of right-hand side of eq. (2′) may be expressed as (Kadomtsev, 1976),

$$-\frac{1}{\mu_0}\nabla_\perp(\mathbf{B}_0\cdot\mathbf{B}_1)+\frac{B_0^2}{\mu_0}(\mathbf{b}_0\cdot\nabla)\mathbf{b}_{1\perp}.$$

By introducing the plasma displacement $\boldsymbol{\xi}$, first order quantities of field vector and field magnitudes can be written as (Kadomtsev, 1976),

$$\mathbf{B}_1=-\mathbf{B}_0\,\mathrm{div}\boldsymbol{\xi}_\perp+B_0\frac{\partial\boldsymbol{\xi}_\perp}{\partial z}, \tag{6}$$

$$B_1=-B_0\,\mathrm{div}\boldsymbol{\xi}_\perp. \tag{7}$$

The eq. (7) can be obtained by substituting the eq. (6) into the following relation.

$$B_1=\sqrt{(\mathbf{B}_0+\mathbf{B}_1)^2}-\sqrt{(\mathbf{B}_0)^2}\approx\frac{\mathbf{B}_0\cdot\mathbf{B}_1}{B_0}.$$

The density perturbations can be readily obtained from the continuity equation as,

$$\rho_1=-\rho_0\,\mathrm{div}\boldsymbol{\xi}. \tag{8}$$

Substituting the above expressions into eqs. (1′) to (5′), the first order equations of motion along and perpendicular to the field lines in (1′) and in (2′) can be written as,

$$\frac{\partial^2\xi_z}{\partial t^2}-3C_z^2\frac{\partial^2\xi_z}{\partial z^2}-C_\perp^2\frac{\partial}{\partial z}\mathrm{div}\boldsymbol{\xi}_\perp=0, \tag{9}$$

$$\frac{\partial^2\boldsymbol{\xi}_\perp}{\partial t^2}-(C_A^2-C_z^2+C_\perp^2)\frac{\partial^2\boldsymbol{\xi}_\perp}{\partial z^2}-C_\perp^2\nabla_\perp(\mathrm{div}\boldsymbol{\xi}+\mathrm{div}\boldsymbol{\xi}_\perp)-C_A^2\nabla_\perp(\mathrm{div}\boldsymbol{\xi}_\perp)=0. \tag{10}$$

To take a divergence of the eqs. (9) and (10), we have the following expressions.

$$\frac{\partial^2}{\partial t^2}\mathrm{div}\xi_z - 3C_z^2\frac{\partial^2}{\partial z^2}\mathrm{div}\xi_z - C_\perp^2\frac{\partial^2}{\partial z^2}\mathrm{div}\boldsymbol{\xi}_\perp = 0, \tag{11}$$

$$\frac{\partial^2}{\partial t^2}\mathrm{div}\boldsymbol{\xi}_\perp - (C_A^2 - C_z^2 + C_\perp^2)\frac{\partial^2}{\partial z^2}\mathrm{div}\boldsymbol{\xi}_\perp - (C_A^2 + C_\perp^2)\Delta_\perp\mathrm{div}\boldsymbol{\xi}_\perp - C_\perp^2\Delta\mathrm{div}\boldsymbol{\xi} = 0. \tag{12}$$

Here, $C_\perp^2 = \dfrac{P_{0\perp}}{\rho_0}$, $C_z^2 = \dfrac{P_{0z}}{\rho_0}$, and $C_A^2 = \dfrac{B_0^2}{\mu_0\rho_0}$. μ_0 is magnetic permeability in vacuum. We assume that the first order quantities vary as,

$$e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}.$$

Here, ω and $\mathbf{k}$ are angular frequency and wave vector of the perturbations. Then, eqs. (11) and (12) can be expressed as follows.

$$(\omega^2 - 3C_z^2k_z^2)k_z\xi_z - C_\perp^2k_z^2\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp = 0, \tag{13}$$

$$C_\perp^2k^2k_z\xi_z - (\omega^2 - C_A^2k^2 + (C_z^2 - C_\perp^2)k_z^2 - C_\perp^2k_\perp^2 - C_\perp^2k^2)\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp = 0. \tag{14}$$

Equations (13) and (14) can be written as,

$$\begin{pmatrix} (\omega^2 - 3C_z^2k_z^2), & (-C_\perp^2k_z^2) \\ (C_\perp^2k^2), & (-(\omega^2 - C_A^2k^2 + (C_z^2 - C_\perp^2)k_z^2 - C_\perp^2k_\perp^2 - C_\perp^2k^2)) \end{pmatrix}\begin{pmatrix} k_z\xi_z \\ \mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp \end{pmatrix} = 0. \tag{15}$$

The term, $k_z\xi_z$ and $\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp$ can be obtained, if

$$\begin{vmatrix} (\omega^2 - 3C_z^2k_z^2), & (-C_\perp^2k_z^2) \\ (C_\perp^2k^2), & (-(\omega^2 - C_A^2k^2 + (C_z^2 - C_\perp^2)k_z^2 - C_\perp^2k_\perp^2 - C_\perp^2k^2)) \end{vmatrix} = 0. \tag{16}$$

Then two eigen-values are,

$$\frac{\omega^2}{k^2} = \frac{1}{2}\left[C_A^2 + 2C_\perp^2 + 2C_z^2\frac{k_z^2}{k^2} \pm\sqrt{4\frac{k_z^2}{k^2}\left\{C_\perp^2(C_\perp^2 - 6C_z^2) + 3C_z^2\left(C_z^2\frac{k_z^2}{k^2} - C_A^2\right)\right\} + \left(C_A^2 + 2C_\perp^2 + 2C_z^2\frac{k_z^2}{k^2}\right)^2}\,\right]. \tag{17}$$

Those two eigen-values correspond to fast phase velocity (+) and slow phase velocity (−) mode, referred hereafter as fast and slow mode (see Fig. 1). For the slow mode, we may have unstable conditions as the eq. (17) becomes negative at a particular condition. For nearly parallel propagation, this condition takes place when perpendicular pressure decreases to the value below 40% of parallel pressure. Contrarily, when the perpendicular pressure exceeds the parallel pressure by the factor more than 6, the perturbation becomes unstable again. Those instabilities could be classified as garden-hose (or fire-hose) instability and mirror instability (*e.g.* Stix, 1962), respectively. In this study, however, we examined the characteristics of the plasma perturbations by not

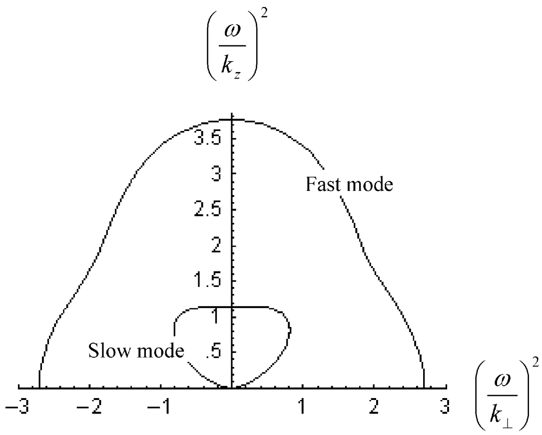

Fig. 1. Polar-plot of $(\omega/\mathbf{k})^2$ (ω is angular frequency, $\mathbf{k}$ is wave vector) for fast and slow modes. Pressure anisotropy (α) and plasma beta (β) is assumed to be 0.91 and 6.0, respectively. Note that slow mode becomes unstable when pressure anisotropy was below 0.3 and above 6.7 (see text).

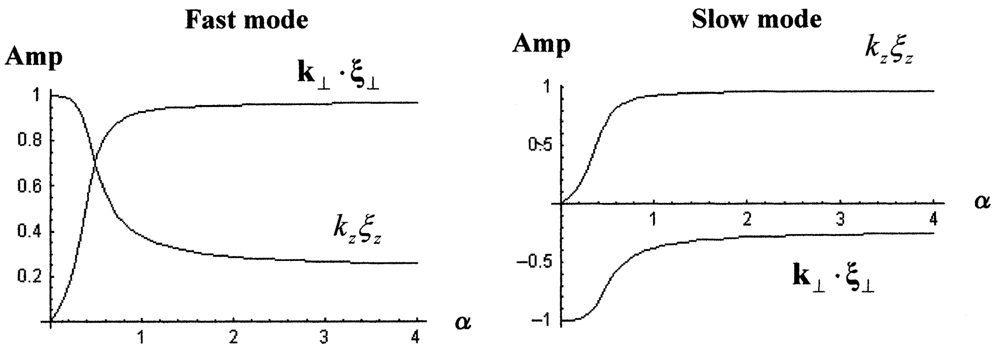

Fig. 2. Plot of $\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp$ and $k_z\xi_z$ for fast mode wave (left-hand panel) and for slow mode (right-hand panel) as a function of pressure anisotropy α. Plasma beta (β) is assumed to be 1.0.

taking account of the stable/unstable conditions.

For respective eigen-values, a set of eigen-vectors, $k_z\xi_z$ and $\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp$ can be given. Instead of showing a lengthy mathematical expression of eigen-vectors, we show in Fig. 2 a plot of those eigen-vectors for the case of nearly parallel propagation ($\theta \approx 5^\circ$) as a function of pressure anisotropy, $\alpha = \dfrac{P_{0\perp}}{P_{0z}}$. The plasma-beta ($\beta$: a ratio of plasma/magnetic pressures) is selected to be 1.0. To calculate the plasma-beta, the plasma pressure P_0 is given by $P_0 = P_{0\perp} + P_{0z}$.

3. Field magnitude and plasma density perturbations

Field magnitude and density perturbations (B_1 and ρ_1) in eqs. (7) and (8) can be written using the eigen-vectors as,

$$B_1 = -iB_0\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp, \tag{7'}$$

$$\rho_1 = -i\rho_0(\mathbf{k}\cdot\boldsymbol{\xi}) = -i\rho_0(\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp + k_z\xi_z). \tag{8'}$$

In Fig. 3, we plot B_1 and ρ_1 as a function of pressure anisotropy. For this case, plasma beta is assumed to be 4.2.

Figure 3a is for fast mode, while Fig. 3b is for slow mode. Both are assumed to be propagating nearly parallel to the field lines ($\theta \approx 5°$). It is apparent for slow mode that B_1 and ρ_1 varied oppositely above $\alpha = 1.4$, while they changed in parallel below it. Therefore, diamagnetic properties are seen to occur above $\alpha = 1.4$, while the paramag-

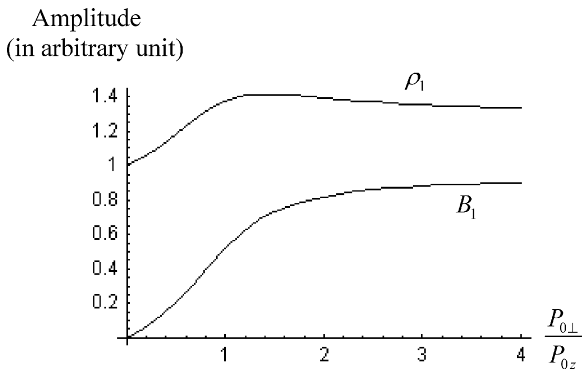

Fig. 3a. Plot of $\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp$ and $\mathbf{k}\cdot\boldsymbol{\xi}$ for fast mode wave as a function of pressure anisotropy α. The terms $\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp$ and $\mathbf{k}\cdot\boldsymbol{\xi}$ are proportional to B_1 and ρ_1, respectively. Plasma beta (β) is assumed to be 4.2.

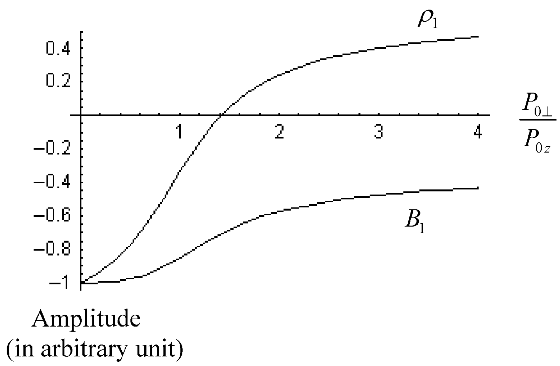

Fig. 3b. Same as Fig. 2a but for slow mode wave.

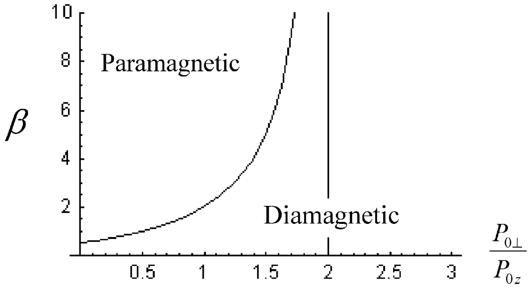

Fig. 4. *Plot of the line in α–β plane that separates diamagnetic and paramagnetic region for nearly parallel propagation ($\theta \approx 5°$). The vertical line at $\alpha = 2$ is an asymptotic line.*

netic properties appeared below it. Such changes can not be seen for fast mode. Only the paramagnetic properties appeared. We demonstrate in Fig. 4 a critical line that separates diamagnetic and paramagnetic regions in α and β plane for a nearly parallel propagation ($\theta \approx 5°$). The asymptotic line at $\alpha = 2$ shifts toward left with increasing angle of the propagation. It is apparent that the paramagnetic properties appeared only at higher plasma beta part in lower pressure anisotropy region. Such a mixture was found to occur only for slow mode waves.

4. Summary and discussion

It was found in anisotropic plasmas that slow mode wave shows mixture of diamagnetic and paramagnetic properties, unlike those in isotropic plasmas. Those wave properties are schematically illustrated in Fig. 5, where waves are assumed to propagate with the angle of $\theta \approx 5°$. It is supposed that plasmas were compressed and decompressed together with the compression and decompression of magnetic field lines. A field-aligned plasma flow modifies those density perturbations. For slow mode, the

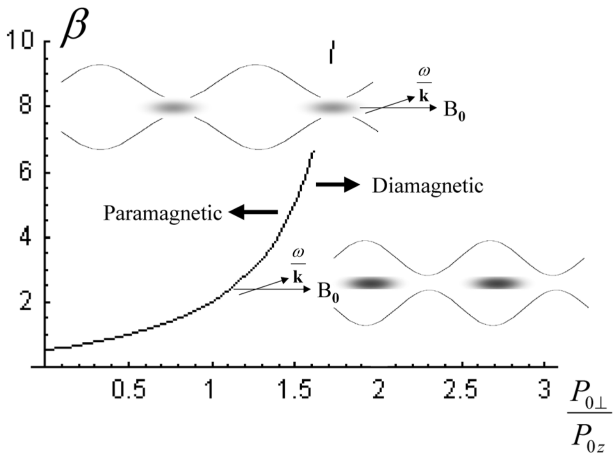

Fig. 5. *Schematic illustration of the field line perturbation (solid lines) and associated plasma density peaks (spots) overlapped in the α–β plane shown in Fig. 4. Wave form is symmetric across the field lines, because of oblique propagation.*

flow along the field lines is toward a weaker field region as it was driven by magnetic mirror force. This may occur in lower beta region below 0.5. In higher beta region above 0.5, however, the density modulation associated with the field line compression and decompression may sustain the paramagnetic perturbations. Nonetheless, diamagnetic perturbations may appear at large pressure anisotropy region, because the magnetic mirror force increased. For fast mode, however, the flow along the field lines is toward a compressed field region as it was driven by parallel pressure force acting on a changing area of the flux tube. Therefore, the paramagnetic nature is unchanged.

We employed double adiabatic equations of state to examine plasma perturbations in anisotropic pressures. Because those equations are valid in the magnetized plasmas, we should be careful in application of the present result to the conditions where the particle orbit is chaotic and definition of gyration around the field lines becomes meaningless.

Acknowledgments

The editor thanks Dr. A. Nakamizo and another referee for their help in evaluating this paper.

References

Arnoldy, R.L. (1986): Fine structure and pitch angle dependence of synchronous orbit electron injections. J. Geophys. Res., **91**, 13411–13421.

Kadomtsev, B.B. (1976): Collective Phenomena in Plasmas (in Japanese). Tokyo, Iwanami Shoten.

Lyons, L.R., Wang, C.-P., Nagai, T., Mukai, T., Saito, Y. and Samson, J.C. (2003): Substorm inner plasma sheet particle reduction. J. Geophys. Res., **108**, doi: 10.1029/2003JA010177.

Nakamizo, A. and Iijima, T. (2003): A new perspective on magnetotail disturbances in terms of inherent diamagnetic processes. J. Geophys. Res., **108**, doi: 10.1029/2002JA009400.

Nakamura, R., Baumjohann, W., Brittnacher, M., Sergeev, V.A., Kubyshkina, M., Mukai, T. and Liou, K. (2001): Flow bursts and auroral activations: Onset timing and foot point location. J. Geophys. Res., **106**, 10777–10789.

Nakamura, R., Baumjohann, W., Kleker, B., Bogdanova, Y., Balogh, A. *et al.* (2002): Motion of the dipolarization front during a flow burst event observed by Cluster. Geophys. Res. Lett., **29** (20), doi: 10.1029/2002GL015763.

Parks, G.K. (1991): Physics of Space Plasmas: An Introduction. Redwood City, Addison-Wesley.

Sergeev, V.A., Angelopoulos, V., Gosling, J.T., Cattel, C.A. and Russel, C.T. (1996): Detection of localized, plasma-depleted flux tubes of bubbles in the midtail plasma sheet. J. Geophys. Res., **101**, 10817–10826.

Shepherd, G.G., Bostron, R., Derblom, H., Falthammar, C.-G., Gendrin, R., Kaila, K., Korth, A., Pedersen, A., Pellinen, R. and Wrenn, G. (1980): Plasma and field signatures of poleward propagating auroral precipitation observed at the foot of the Geos 2 field line. J. Geophys. Res., **85**, 4587–4601.

Stix, T.H. (1962): The Theory of Plasma Waves. New York, McGraw-Hill.