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COHESIVE FINITE ELEMENT BASED MODELING OF DAMAGE IN COMPOSITE MATERIALS

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ABSTRACT

Damage in composite laminates affects its overall viscoelastic response. Constitutive equations have been developed for composite laminates considering a fixed damage state. A complete description, however, requires suitable damage evolution laws. This paper is focused on studying damage evolution in viscoelastic laminates using a cohesive finite element approach. A two dimensional, four nodded finite element is developed incorporating a rate-independent traction-displacement cohesive law. This element is used in conjunction with plane strain bulk elements behaving in a linear viscoelastic manner to simulate crack evolution between two existing transverse cracks in symmetric cross-ply laminates. The effects of loading strain-rate, ply constraint and initial crack density are studied. This study shows expected trends in the behavior and indicates the suitability of cohesive zone modeling to study damage evolution in viscoelastic composite materials.
Keywords: damage evolution, viscoelasticity, composite materials, cohesive finite element.

INTRODUCTION

Composite materials are prone to damage when they are subjected to quasi-static, fatigue or dynamic loads. Damage is defined as any irreversible change in the material brought about by multitude of distributed microstructural rearrangements such as microcracks and microvoids. When a composite lamina is subjected to tension-tension fatigue loading in the fiber direction, different damage mechanisms that develop depend on the level of applied strain. If the applied strain is high enough, catastrophic fiber breakage takes place. If the strain is lower, progressive damage beginning with matrix cracking is followed by debonding between fibers and matrix at the tips of these cracks. If the applied strain is below a critical value, damage either does not occur or existing damage does not grow. These damage mechanisms are discussed in Talreja [1].

Damage mechanisms observed in a multi-directional laminate, however, depend on the laminate configuration. Damage in the form of intra-laminar cracking occurs first in the off-axis layers. These cracks quickly grow to span the entire layer thickness as well as laminate width. These cracks are more or less uniformly spaced. With applied cyclic loading, their number increases and then reaches a saturation level. This level has been termed as Characteristic Damage State (CDS) by Reifsnider [2]. After reaching a saturation crack density, deterioration in the material occurs in the form of internal delaminations at the tips of these cracks (Jamison [3]). These delaminations may grow with further cyclic loading. The final failure of the laminate occurs by fiber breakage in the 0° plies. A typical pattern of damage that emerges in laminates is shown schematically in Fig. 1.

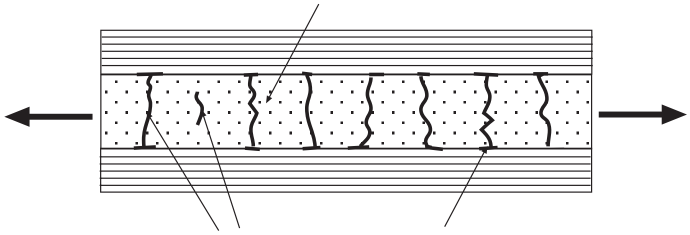

Figure 1. A Typical Pattern of Damage in Laminated Composites

The effects of damage on the elastic behavior of laminated composite materials are extensively studied. Hashin [4], Varna and Berglund [5,6], McCartney [7] and Gudmundson and Zang [8] have presented micromechanics based approaches to determine the effective elastic properties of laminates at a fixed level of matrix cracks. Elastic behavior of laminates with internal delaminations in conjunction with matrix cracks has been investigated by Akshantala and Talreja [9], Fan and Zhang [10], Kumar and Talreja [11] and Kashtalyan and Soutis [12]. An alternative to the micromechanics approach is a continuum damage mechanics approach. Talreja [13,14] and Allen et al. [15,16] have used continuum damage mechanics to obtain the constitutive equations for damaged composite materials undergoing small deformations.

In many service conditions, composite materials are likely to be subjected to high temperature and moisture in addition to the usual mechanical loads. Such conditions induce time dependent viscoelastic response of the constituents and hence a coupling between damage and viscoelasticity. While damage in elastic composite materials is extensively studied, there is relatively little research in understanding damage in viscoelastic composite materials. Most work in this area is limited to particulate composites (cf. Schapery [17] and references therein). For laminated composite materials, Zocher et al. [18] and Kumar and Talreja [19] have applied micromechanics approach to obtain the overall effective properties at a fixed level of damage. Recently Kumar and Talreja [20] have proposed an internal variables based continuum damage model for studying the linear viscoelastic behavior of laminated composites. Damage constitutive relationships for viscoelastic laminates, as discussed in the foregoing, are developed for a fixed damage state. These must be supplemented with suitable damage evolution laws.

In the present paper, we investigate the evolution of damage in the form of matrix cracking in linear viscoelastic composite laminates using cohesive finite element method. The paper is organized as follows: first we qualitatively describe the mechanism of damage evolution in laminates. The cohesive finite element approach is described next followed by its application to study the evolution of matrix cracks in linear viscoelastic cross-ply laminate. The paper ends with the presentation of simulation results and conclusions.

DAMAGE EVOLUTION IN LAMINATED COMPOSITE MATERIALS

Damage evolution in elastic composite laminates occurs by nucleation, growth and subsequent multiplication of transverse cracks in the off-axes plies. Wang and Crossman [21] performed finite element analysis of a cross-ply laminate and showed, using energy arguments, that a transverse crack of width equal to about 85% of the 90° plies thickness is formed instantaneously. This is followed by a stable growth until the crack is arrested by the stiff 0° plies. Dvorak and Laws [22] considered growth of a transverse crack along the thickness direction as well as along the width direction. The crack growth along the width direction of the laminate is referred to as a tunneling crack. While growth along the thickness and the width direction is possible before a full transverse crack develops, most experimental observations show that the process is almost instantaneous. Thus the mechanism of damage evolution is "popping-in" of fully developed transverse cracks. No cracks are seen before a certain threshold is reached. Beyond this threshold, the crack density (inverse of crack spacing) increases monotonically until it reaches a saturation level. The threshold stress and the saturation level are functions of ply thickness and laminate configuration. A schematic of this evolution is shown in Fig. 2.

Nairn [23] and Liu and Nairn [24] have analyzed instantaneous formation of a new transverse crack between two existing cracks using the strain energy release rate based criterion. The evolution of crack density in cross-ply laminates is predicted for various configurations and a good agreement with the experimental data is shown.

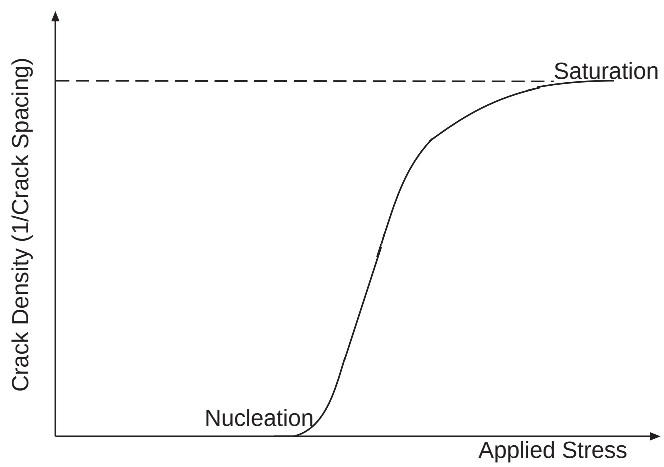

Figure 2. A Schematic of Crack Density Evolution in Composite Laminates

Studies on damage evolution in viscoelastically deforming laminated composites are scarce. Moore and Dillard [25] observed time-dependent evolution of matrix cracks in Kevlar/epoxy and graphite/epoxy cross-ply laminates at room temperature. They observed significant effects of loading rate on the crack density evolution. Raghavan and Meshii [26] conducted creep and quasi-static tests on AS4/3501-6 cross-ply laminates at room temperature. Again, a significant effect of strain rate on the nucleation and multiplication of transverse cracks was observed. Akshantala and Brinson [27] have reported a damage evolution model for viscoelastic cross-ply laminates. This model is based on extending Hashin's elastic analysis [4] of a cracked cross-ply laminate to the viscoelastic domain. Assuming maximum stress failure criterion, they studied instantaneous formation of a transverse crack midway between two existing cracks. They showed loading rate dependency on the crack formation.

COHESIVE FINITE ELEMENT APPROACH

The concept of cohesive zone models derives from the work of Dugdale [28] and Barenblatt [29] who considered a zone of non-vanishing tractions at the crack tip in order to alleviate stress singularities. In materials, locally high stresses result in a narrow and highly damaged zone in front of the crack tip. The material in this zone progressively deteriorates until it is completely separated. The mechanisms of damage within this zone are dependent on the material. For example, in brittle materials, atomic plane separation is the dominant mechanism; in ductile materials, void nucleation and coalescence occur in this zone; and in polymers, crazing (yielding), entanglement pull-out and chain fracture are the predominant damage modes. Cohesive zone is a highly heterogeneous region and the resulting stresses and strains within this region are very complex. In cohesive zone or cohesive surface modeling approach, the damage zone is replaced by two planes that are coincident in the initial, undamaged state. The complex nature of the failure processes within the cohesive zone is replaced, at the continuum level, by a traction-displacement relationship between these planes. The relationship is such that the traction between the two surfaces increases with increasing displacement, reaches a maximum and then decreases to zero. The parameters characterizing this relationship are the peak traction, critical opening displacement and fracture energy (area under the curve). These parameters are usually empirical fitting parameters though some attempts to derive them through micromechanical considerations have been pursued (Allen and Searcy [30]) A representative traction-displacement curve is shown in Fig. 3.

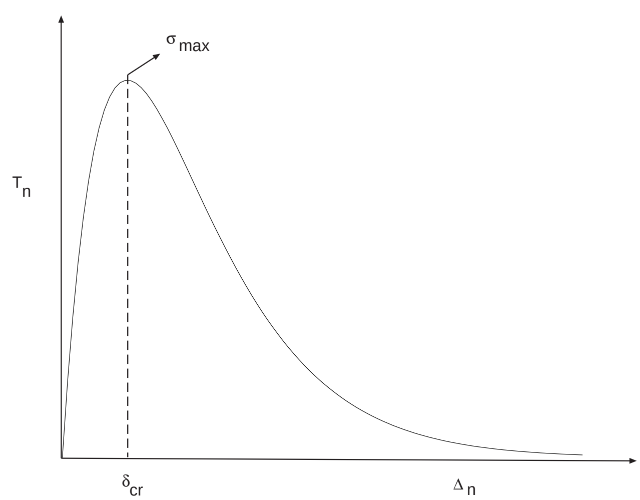

Figure 3. Traction-Displacement Cohesive Law

Various traction-displacement cohesive laws are available in the literature. Most common relations are bilinear and exponential laws. Fortunately, it was shown by Tvergaard and Hutchinson [31] that the overall mechanical response is usually insensitive to the shape of the adopted cohesive law. Most cohesive laws are rate-independent. However, some researchers are beginning to adopt rate-dependent cohesive laws (see Allen and Searcy [30] and Rahulkumar et al. [32], for example).

The use of cohesive surface concept into computational mechanics framework has been pioneered by Needleman [33]. Since then, this approach has been extensively used to simulate void nucleation (Needleman [33], Xu and Needleman [34]), interface decohesion (Needleman [35,36]), dynamic fracture and damage (Xu and Needleman [37]), Lo and Allen, [38], Needleman [39], Camacho and Ortiz [40], Pandolfi et al. [41], Zhai and Zhou [42], Zavattieri et al. [43]), fatigue crack growth (de-Andres et al. [44], and viscoelastic fracture (Allen and Searcy [30], Rahulkumar et al. [45]).

Cohesive zone based computational modeling is compatible with conventional finite elements. One of the advantages of this approach is that no fracture and node separation criteria are needed - crack nucleation and growth are a natural outcome of the formulation. Cohesive surfaces can be embedded between every pair of elements in the mesh if arbitrary fracture path is to be simulated. However, if the failure plane is known a priori, cohesive surfaces may be embedded only in that plane. When cohesive elements are embedded between every pair of elements in a finite element mesh, there could be a significant overall compliance enhancement due to these elements. In such a case, size of the bulk elements must be chosen such that the artificial compliance increase is minimal (Zhai [46]). It should also be noted that cohesive zone modeling introduces a characteristic length scale in the model related to the size of damage zone at the macroscopic crack tip.

In this research, we developed a 2D cohesive finite element with the traction-displacement law as given by Xu and Needleman [37]. This law is derived from a potential Φ, which for 2D problems is given by

$$\Phi(\Delta_n, \Delta_t) = \Gamma_o \left[1 - \left(1 + \frac{\Delta_n}{\delta_{cr}} \right) \exp\left(-\frac{\Delta_n}{\delta_{cr}} \right) \exp\left(-\frac{\Delta_t^2}{\delta_{cr}^2} \right) \right] \quad (1)$$

In this equation, the fracture energy per unit area Γ_o and the critical opening displacement δ_{cr} (corresponding to the maximum traction) are assumed to be same in both normal and tangential directions. The fracture energy is related to the maximum stress and the critical opening displacement through

$$\Gamma_o = e\,\sigma_{\max}\delta_{cr} \quad (2)$$

Normal and tangential tractions are obtained from this potential by differentiating it with respect to the corresponding opening displacements, as

$$T_n = \frac{\partial \Phi}{\partial \Delta_n} = \Gamma_o \frac{\Delta_n}{\delta_{cr}^2} \exp\left(-\frac{\Delta_n}{\delta_{cr}} \right) \exp\left(-\frac{\Delta_t^2}{\delta_{cr}^2} \right) \quad (3)$$

and

$$T_t = \frac{\partial \Phi}{\partial \Delta_t} = \frac{2\Gamma_o \Delta_t}{\delta_{cr}^2} \left(1 + \frac{\Delta_n}{\delta_{cr}} \right) \exp\left(-\frac{\Delta_n}{\delta_{cr}} \right) \exp\left(-\frac{\Delta_t^2}{\delta_{cr}^2} \right) \quad (4)$$

A plot showing the variation of normal traction as a function of normal opening displacement, when there is no tangential displacement, is shown in Fig. 3. These tractions are used to calculate the element tangent stiffness matrix.

The two dimensional, four noded cohesive element is implemented as a user element (UEL) in the ABAQUS finite element analysis program (HKS, Inc. [47]). The details of the finite element formulation are described in Kumar [48]. An implicit integration formulation is used for the analysis which involves incremental-iterative solution of the nonlinear equations using full Newton-Raphson algorithm. A number of single element simulations were conducted to verify the element formulation. The element was also verified for a quasi-static crack growth in an elastic double cantilever beam specimen similar to that used by Rahulkumar et al. [45].

EVOLUTION OF TRANSVERSE CRACKS IN LINEAR VISCOELASTIC CROSS-PLY LAMINATES

Damage evolution in viscoelastic laminated materials is not fully understood. Experimental observations by Moore and Dillard [25] and Raghavan and Meshii [26] suggest that transverse cracks form instantaneously when the laminate is subjected to a constant loading rate at room temperature. However, when the material is subjected to high temperature or other environmental condition, there may be a significant time spent in crack growth before a full transverse crack is formed. Thus the mechanisms of damage evolution in laminated materials are likely to have nucleation, growth and multiplication components. Further, these mechanisms need not occur sequentially. For example, crack multiplication may occur even before the existing cracks have fully grown in the thickness and the width directions. The relative importance of these mechanisms is dependent on the material, applied loading and environmental conditions. The cohesive zone based computational micromechanics analysis appears to be an attractive framework to address nucleation, growth and multiplication of damage as well as to study the effects of existing damage, ply constraints and loading rate on the evolution characteristics. A comprehensive parametric study must be conducted before a reliable phenomenological damage evolution law can be established.

The cohesive finite element formulation described in the previous section is used to study the evolution of transverse cracks in linear viscoelastic cross-ply laminates. Even though crack growth is possible in the thickness and the width directions of the laminate, in the present work, we restrict our study to growth in the thickness direction only as a simplified first analysis. For this purpose, a two-dimensional analysis is performed. A schematic of the configuration analyzed is shown in Fig. 4. This figure shows an edge view of a symmetric cross-ply laminate with two fully developed cracks in the 90° plies separated by a distance of *2L*. We are interested in studying nucleation and growth of a new crack in between these cracks.

It is known from analytical micromechanics solution that maximum axial stress in the 90° plies occurs midway between the existing cracks. Thus a new transverse crack can nucleate and grow in a plane located here and lying parallel to the existing crack planes. Hence, cohesive surfaces are embedded only in this plane.

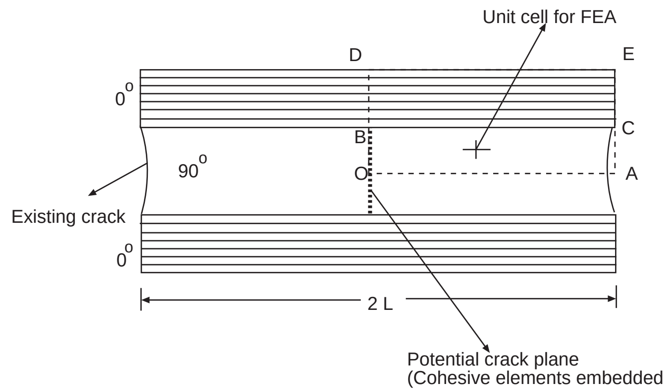

Figure 4. A Schematic of the Configuration Analyzed

Growth of such a crack is studied under applied displacement in the longitudinal direction. The bulk material is considered to be linear viscoelastic. The cohesive law, however, is assumed to be rate-independent. As discussed earlier, cohesive law represents the failure processes occurring within the cohesive zone. In the present case, cohesive surfaces are embedded in the 90° plies. The failure process before a complete macro-crack develops in a 90° ply consists of debonding of fibers and matrix followed by growth and coalescence of debonds. Now if the fiber volume fraction is high, as is usual in unidirectional laminae, narrow matrix ligaments between the fibers will be highly constrained and will behave in a brittle manner. Thus the growth of these debonds through the matrix will be sudden. This suggests that the failure process may be considered as rate independent even when the bulk material is behaving in a rate-dependent manner, especially in the case of high fiber volume fraction. The rate dependency of the failure process is also likely to be dependent on the environmental conditions such as high temperature and moisture. A rate dependent cohesive law, if needed, can be easily incorporated by extension into the cohesive finite element framework presented earlier.

A typical finite element mesh used in the analysis is shown in Fig. 5. Note that only a quarter of the unit cell (shown in Fig. 4) is considered for the finite element simulation due to the symmetry. In Fig. 5, *OACB* represents 90° layers and *BCED* represents 0° layer. *OA* is the laminate mid-plane and hence a plane of symmetry. Plane *OD* is also a plane of symmetry. *AC* is the traction free surface representing the existing crack. Length *OA* depends on the initial crack density. Cohesive elements are embedded only in the 90° plies, i.e., along the plane *OB*. The laminate is loaded by applying displacement *u* on the plane *CE*.

cohesive law are chosen as $\gamma_o = 22$ J/m^2 and $\sigma_{\max} = 12.65$ MPa. The critical opening displacement, δ_{cr}, using Eq. (2) is obtained as $0.64\ \mu m$. These parameters are of an order of magnitude expected to be typical for a transverse ply. However, they are not true material properties. These parameters must be adjusted by comparing the simulation results with experimental data. However, due to the lack of suitable experimental data, this is not done in the present study. Our aim is to merely evaluate the suitability of the cohesive modeling approach by observing the trends in the simulation results. Crack growth is based on the criterion that a point fails when the normal opening displacement Δ_n is greater than or equal to five times the critical opening displacement δ_{cr}, i.e., when $\Delta_n \geq 5\delta_{cr}$. Similar criterion has been adopted by other researchers in conjunction with crack growth in elastic materials.

As shown in Fig. 5, a uniform mesh is used for the analysis. Ten elements are used per ply thickness. The number of elements along the length (axis - 1) depends on the initial crack density and the aspect ratio of the elements. The connection between a bulk element and a cohesive element is shown schematically in Fig. 5. Face *PS* of the cohesive element is connected to the bulk element, whereas face P_1S_1 is supported on "rollers" as required by the symmetry condition. The symmetry condition also requires the *v*-displacement of the nodes forming the two cohesive surfaces to be same. This is applied as a constraint condition. In the initial undeformed state, nodes forming the two faces of the cohesive element have same coordinates. Under the action of applied loading, the bulk elements deform in a time-dependent fashion. This causes the two faces of the cohesive elements to separate in a time-dependent manner as well.

Inclusion of cohesive elements in a finite element mesh leads to many convergence issues. The convergence is dependent on cohesive law parameters and element size. Once the cohesive law parameters are fixed, convergence can be achieved by suitably refining the mesh. However, embedding cohesive elements between every pair of elements can lead to artificial increase in the overall compliance. Hence, the mesh cannot be refined below a threshold element size. In a time-dependent analysis, the time step should also be small to achieve the convergence. For all the cases considered in this study, suitable convergence was obtained for the selected cohesive parameters and meshes.

RESULTS AND DISCUSSION

Nucleation of a transverse crack and its growth in the ply thickness direction is considered for cross-ply laminates deforming in a linear viscoelastic manner. Effects of initial crack spacing (or initial crack density), ply constraint and applied strain rate are considered. Figure 6 shows the plot of normalized crack length (normalized with respect to 90° plies thickness) with time for a $[0/90_2]_S$ cross-ply laminate with an

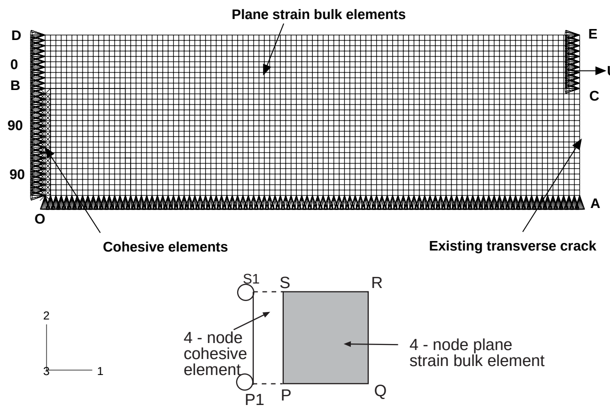

Figure 5. A Typical Finite Element Mesh with Bulk and Cohesive Elements

We consider an IM7/8320 material system. The fibers are assumed to be transversely isotropic and elastic, whereas the matrix is taken as isotropic and linear viscoelastic. The relaxation modulus of the matrix is taken from Zocher et al. [18]. The material properties are listed in Table1.

• Fiber (Transversely Isotropic, Volume fraction = 0.6)
$E_L = 256.76$ GPa
$E_T = 25.51$ GPa
$\nu_{LT} = 0.289$
$\nu_{TT} = 0.380$
$G_{LT} = 22.06$ GPa
$G_{TT} = 9.25$ GPa
• Matrix (Isotropic, Linearly viscoelastic)
$\tilde{E}_m = \dfrac{6.8947 \text{ X } 10^3}{880 + 19.780\,\Gamma(1.33)s^{-0.33}}$ GPa
$\nu_m = 0.3$

Table 1. Fiber and Matrix Properties

Orthotropic lamina properties are derived from fiber and matrix properties using a viscoelastic extension of Composite Cylinder Assemblage (CCA) model. These properties are then used to describe the constitutive behavior of the bulk elements in the finite element mesh via a user material model (UMAT) (see Kumar and Talreja [20] and Kumar [48] for the details). The bulk elements considered in the analysis are four node continuum plane strain elements. The parameters used in the initial crack density of 0.4/mm and loaded with different applied strain rates. It is seen that applied strain rate has an effect on the nucleation time of a crack. However, the rate of crack growth is same for all three cases as the curves can be superimposed by lateral translation. This is a manifestation of the linear viscoelasticity. Time required to nucleate and grow a crack is higher for the laminate loaded at a lower strain rate, as expected.

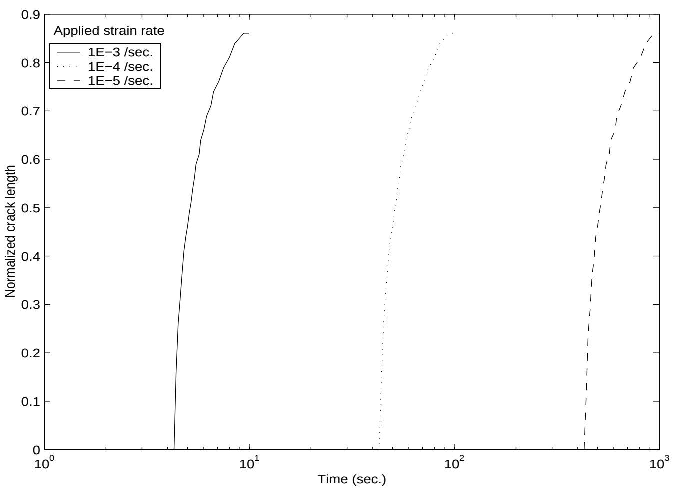

Figure 6. Effects of Loading Strain Rate on the Nucleation and Growth of a Crack in $[0/90_2]_S$ Laminate with Initial Crack Density of 0.4/mm

The stiff 0° plies constrain the deformation behavior and damage evolution in the 90° plies. The constraint effect can be studied by varying the structural stiffness ratio of the two plies. This can be done by varying the thickness of the 90° plies while keeping the thickness of the 0° plies constant. In the present research, we analyzed three configurations, viz., $[0/90]_S$, $[0/90_2]_S$ and $[0/90_3]_S$ laminates. The thickness of the 90° plies is smallest in the first case, and hence it is the most constrained configuration. The $[0/90_3]_S$ laminate is the least constrained configuration. Keeping the material properties, initial crack density (at 0.4/mm) and loading strain rate (1E-3 /sec.) same for the three cases, evolution of a new crack is observed. Figure 7 shows the variation of normalized crack length with time. It is observed that the constraint effect is quite significant on both nucleation and subsequent growth of a crack. The nucleation is significantly delayed in the most constrained case. Further, in such laminates, the final failure (i.e., failure of the 0° plies) may occur even before the crack develops fully.

Finally we consider the effects of initial crack spacing (initial crack density) on nucleation and growth of a new crack. We consider a fixed laminate configuration, $[0/90_2]_S$, and a fixed loading strain rate of 1E-3 /sec. By considering different lengths *2L* of the unit cell (Fig. 4), the initial crack density can be varied. The formation of a crack is then examined using the finite element analysis procedure. Figure 8 shows the crack length as a function of time for different initial crack spacing. It is observed that when the initial crack spacing is smaller (i.e. higher initial crack density), nucleation of a new crack is delayed. This is because as the crack density increases, the axial stress in the 90° plies midway between the two cracks reduces. Hence more stress needs to be applied to form a new crack. This is in agreement with what has been observed for elastic laminates. From Fig. 8, the time required to form a "full" crack (taken here as 85% of 90° plies thickness) can be obtained for each crack density. This would then correspond to the time required to double the crack density. A plot showing the evolution of crack density is shown in Fig. 9. It is implicitly assumed here that a new crack forms only after the previous one has grown fully.

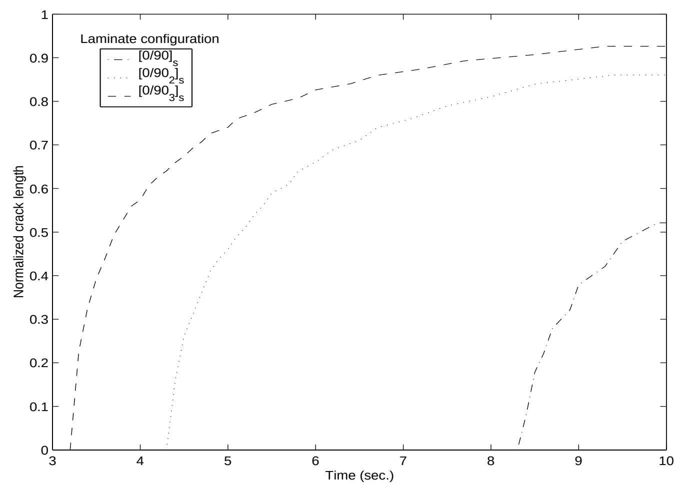

Figure 7. Effects of Ply Constraint on the Nucleation and Growth of a Crack (Initial Crack Density of 0.4/mm; Loading Strain Rate is 1E-3 /sec.)

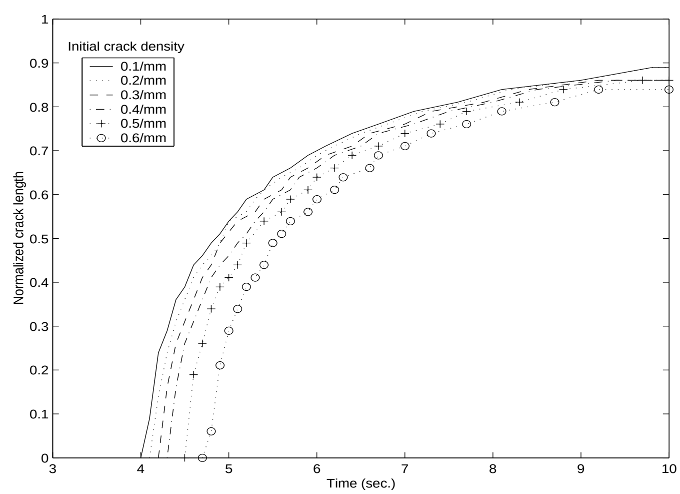

Figure 8. Effects of Initial Crack Density on the Nucleation and Growth of a New Crack in a $[0/90_2]_S$ Laminate Loaded at a Strain Rate of 1E-3 /sec.

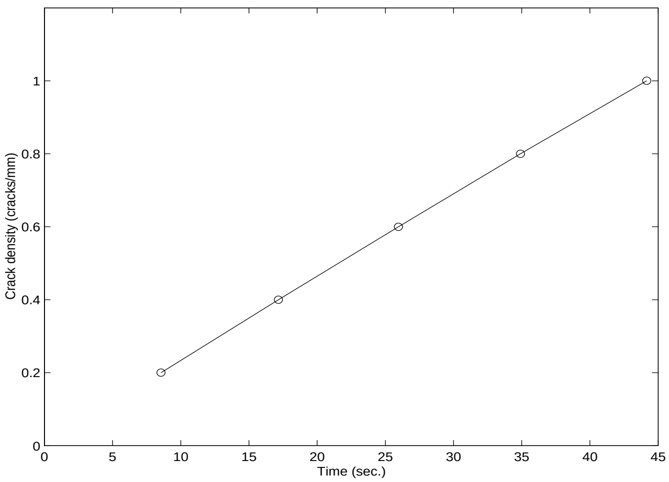

Figure 9. Crack Density Evolution in $[0/90_2]_S$ Laminate Loaded at a Strain rate of 1E-3 /sec.

CONCLUSIONS

A cohesive finite element modeling approach is used to study time-dependent damage evolution in laminated composite materials. Based on the concept of cohesive zone, a two dimensional four noded finite element is developed. The traction-displacement law for this element is taken to be rate-independent based on the potential given by Xu and Needleman [37]. The element is implemented as an user element in the ABAQUS finite element package (HKS, Inc. [47]) and is used in conjunction with the linear viscoelastic user material model developed in [20,48] to simulate crack nucleation and growth in viscoelastic cross-ply laminates. Crack nucleation and growth is considered only in the thickness direction and in between two existing cracks. Effects of loading strain-rate, ply constraints and initial crack density (spacing) are studied. It is observed that these parameters have a significant effect on the nucleation time for a crack. They also affect the subsequent growth of the crack. The results show that cohesive finite element modeling approach is suitable for clarifying damage mechanisms and to study the effects of material and damage parameters on damage evolution in laminated composite materials. However, for realistic cases, a three-dimensional formulation with rate-dependent cohesive law must be adopted. Parametric studies using such a formulation must be used in conjunction with experiments to develop reliable phenomenological damage evolution laws.

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