Dynamic Incentive Strategy for Voluntary Demand Response based on TDP Scheme

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***Abstract*—The enhanced real-time metering and communication capabilities from smart meters and their associated advanced metering infrastructure make it possible for utility company to extend demand response (DR) to small customers through time-dependent pricing (TDP). Considering the economic reason and infrastructure cost, the utility company has to design an incentive scheme to attract the traditional flat pricing (FP) users to be engaged in the TDP scheme. In this process, the utility company may share its revenue from the TDP scheme to those TDP users. It is found, with properly analyzing the energy procurement cost and user elasticity, a dynamic incentive strategy can be considered in dual-tariffs system when flat pricing (FP) and TDP pricing are co-existed. This dynamic incentive strategy gives appropriate stimulus to the users who are involved into the TDP program, and guarantee the utility company's profit at the same time.**

I. Introduction

In traditional electricity market, end users seldom participate into the market activities, where they simply accept a flat price of electricity. With the introduction of enhanced real-time metering and communication capabilities from smart meters and their associated advanced metering infrastructure, utility company will have the ability to extend the demand response (DR) to small customers through time-dependent pricing (TDP). The users can adjust their behaviors according to real time prices, and the utility company can utilize the demand side information to negotiate in the wholesale market.

Traditional demand side management (DSM) programs are mainly targeted at large industrial or commercial users through direct load control (DLC) [1]. There is limited application to extend this program to residential users. With the increasing demands of balancing and reserve power due to penetration of renewable energy sources, there is an increasing interest in extending DR program to residential users through price based DSM programs, or Demand Response (DR) [2], [3]. DR programs encourage customers to voluntarily reduce or reschedule their consumption during peak periods through time-dependent electricity tariffs [4], [5], [6], [7], [8]. The advantage of this program includes several aspects. First, the total electricity costs and price volatility in wholesale market [9] can be reduced by proper *load reducing* and *load shifting*; second, the customers can benefit by participating TDP program to save the cost; third, the utility company may have better negotiating power in wholesale market. In addition, DSM programs also improve the reliability of electricity network through load reduction, thus reducing the need for distribution and transmission infrastructure reinforcements and upgrades. Long term benefits of DSM programs are environmental and sociological as well, as improved energy efficiency through DSM will eventually reduce the energy usage and green house gas emission.

The obstacles to TDP programs include cost, concerns over uncertainty of TDP, and privacy issue. The cost issue mainly invloves the infrastructure renovation. This becomes less important with the development of enhanced real-time metering and communication capabilities from smart meters and their associated advanced metering infrastructure. In addition, the uncertainty of TDP can be solved by transmitting TDP prices to electricity users sufficient ahead of time (e.g., day-ahead) so that they could have time to plan their activities based on the price information provided in advance. The privacy concern can also be mitigated since the price signals can be calculated based on information on aggregated loads rather than scheduling from individual home users [10]. As the above mentioned obstacles to TDP programs can be well addressed by appropriate mechanisms to overcome from both system operators and regulators, the pricing schemes in TDP programs become more and more important. The problem of optimal day-ahead pricing problem for retail market was studied in [11] based on cost minimization for utility, and [12] based on social welfare maximization. Heterogeneous consumption behaviors of different loads were considered at device level in those works using utility modeling for different loads. In [13], different load shedding/shifting behaviors from individual appliance are modeled using a statistical time-utility function and the optimal retail price is then calculated based on maximization of expected social welfare.

To motivate residential users to participate into the TDP program, utility company may have to share part of its profit obtained from TDP program to customers. In addition, the elastic demand of those participated TDP users may further reduce the profits of the utility companies obtained from wholesale market. This makes the profit of the utility company uncertain when TDP program is employed. The above mentioned schemes cannot guarantee the positive profit of utility company under TDP program. In this paper, the profit of the utility company is discussed. A dual-tariffs system that comprises of both conventional regulated flat tariff, and a voluntary TDP scheme is discussed. The flat tariff provides regulated flat prices to users. On top of it, electricity users further have the flexibility to opt for TDP to enjoy additional economic

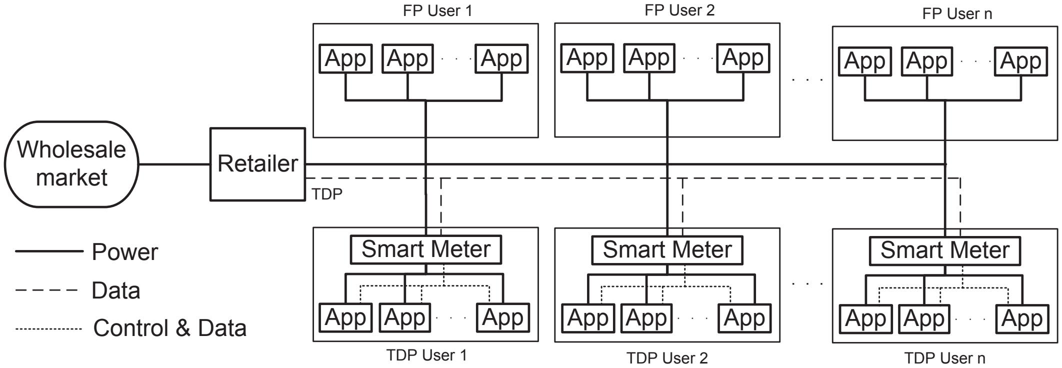

Fig. 1. Block diagram of a simplified smart grid system composed of a utility company implementing dual-tariffs scheme. The time-dependent electricity price signals are announced to TDP users by utility company via digital communication network enabled by smart grid.

benefits. We focus on time-ahead TDP where the computing of price signals is formulated as a cost minimization problem considering both the price response behaviors from electricity users, and energy procurement costs. The resulting TDP prices are transmitted to electricity users sufficient ahead of time (e.g., day-ahead) so that they could have time to plan their activities based on the price information in advance. It is shown that the proposed DR scheme is effective in terms of inducing the desired consumption behaviors from electricity users towards more efficient grid operation. In addition, the positive profit of the utility company is guaranteed.

The rest of this paper is organized as follows. Section II introduces the system model as well as models for price-responsive energy consumption and energy procurement costs. The dynamic incentive day-ahead TDP under the dual-tariffs scheme is formulated in Section III. Numerical examples of the proposed pricing scheme are given in Section IV and finally, the paper is concluded in Section V.

II. System Model

In this paper, a dual-pricing system is considered where a traditional FP scheme and a voluntary TDP scheme (Fig. 1) are co-existed. For FP users, which are also called price-taking customers, the electricity price is regulated under the natural monopoly doctrine using either price regulation or rate-of-return regulation. The FP users can be guaranteed with the certainty of the electricity price without over-spending. For TDP users, which are also called price-responsive users, day-ahead price is announced considering the demand elasticity. TDP customers could have sufficient knowledge about future electricity prices in order to adjust their usage accordingly. Under this dual-tariffs scheme, users have the flexibility to choose either FP scheme so that they can enjoy assured electricity tariff, or TDP after upgrading to smart grid to enjoy its potential economic benefits.

A. Utility company behavior

In power supply system, electricity market may have a number of generating firms, which owns different number of units. According to the marginal cost of each unit and its corresponding operational constraints, the utility cost can be modeled as [14]

$$C_t = \sum_{l=0}^{L-1} (u^{l,t} N^l + \sum_{b=0}^{B-1} MC^{l,b} P_{S_g}^{l,b,t} + S^{l,t}) \tag{1}$$

where, L is the number of generating units, B is the number of segment in the generator's offer curve, $u^{l,t}$ is the on/off status of the generating unit l at period t, N^l is the no-load cost of generating unit l, $MC^{l,b}$ is the marginal production cost of generating unit l on segment b of its piecewise linear cost curve, $P_{S_g}^{l,b,t}$ is the output of generating unit l on segment b of its piecewise linear cost curve during period t, and $S^{l,t}$ is the star-up cost of generating unit l at period t.

B. Customer behaviors

Let's assume that the original customer demand d_{to} is available for study at the beginning. When users start to adopt the TDP, the actual demand can be written as:

$$d_t = \bar{d}_t + \tilde{d}_t, \forall t \in \Gamma, \tag{2}$$

where $\bar{d}_t$ is the aggregated demand from FP users, which may be written as $\bar{d}_t = (1-\alpha)d_{to}$ where α is the ratio defined as the percentage of TDP customers, $\tilde{d}_t$ is the aggregated demand from TDP users, and Γ is the set of all time slots. Here, the time period of study is divided into T time slots, where $T \triangleq |\Gamma|$.

Generally, customers tend to consume more of a good or service when its price decreases and less when its price increases. This information, which reflects the proportion of the demand over the changing of the prices, is defined as elasticity factor. In electricity market, demand of electricity shows not only self-elasticity where the demand changes is inversely proportional to the prevailing electricity prices, but also cross-time elasticity where demand at one time slot may also depend on the prices prevailing at other time slots [15], [16]. In mathematics, the self-elasticity factor (ε_{tt}) and the cross-time elasticity factor ($\varepsilon_{t\tau}$) are defined as

$$\varepsilon_{tt} = \frac{\Delta d_t / d_{to}}{\Delta p_t / p_{to}}, \tag{3}$$

$$\varepsilon_{t\tau} = \frac{\Delta d_t / d_{to}}{\Delta p_\tau / p_{\tau o}}. \tag{4}$$

Here, the cross-time elasticity factor ($\varepsilon_{t\tau}$) concerns the change in demand of time slot t with respect to price changes in time slot τ, and d_{to} and p_{to} are, respectively, demand and price values at the equilibrium points where elasticity is measured. Considering both self and cross-time elasticity, the change in electricity demand from the prices imposed to the consumers can be written in vector format as follows:

$$\Delta \mathbf{d} = \mathbf{E} \Delta \mathbf{p}, \tag{5}$$

where $\mathbf{d} \triangleq (d_1, \ldots, d_T)^T$, $\mathbf{p} \triangleq (p_1, \ldots, p_T)^T$, and $\mathbf{E}$ is the elasticity matrix of which the diagonal elements represent self elasticities and the off-diagonal elements correspond to the cross-time elasticities. Here both prices and consumptions are assumed to be normalized to equilibrium points (d_{to}, p_{to}). In practice, the values of both self and cross-time elasticity factors have to be determined through data analysis on customers' response to price signals. Privacy concerns can be avoided here if the data analysis is performed only on aggregated demands, or on a subset of voluntary customers who are willing to share their consumption information to improve the efficiency of power grid.

III. Dynamic Incentive TDP Scheme under Dual-tariff Scheme

Based on the system model provided in Sec. II, the problem of calculating the optimal TDP price signals under the dual-tariffs DR program which provides dynamic incentive for TDP customers is discussed in this part. The objective in this dual-tariffs system is to maximize the profit, or minimize the cost of the utility company by introducing TDP scheme. Considering revenue from FP and TDP users, the cost to utility company can be expressed as follows:

$$\mathcal{C} = \sum_{t=1}^{T} C_t - \bar{p}\mathbf{1}\bar{\mathbf{d}} - \tilde{\mathbf{p}}^T \tilde{\mathbf{d}}, \tag{6}$$

where C_t is the energy costs as defined in (1), $\bar{\mathbf{d}} \triangleq (\bar{d}_1, \ldots, \bar{d}_T)^T$ the vector of aggregated demands from FP users, $\tilde{\mathbf{d}} \triangleq (\tilde{d}_1, \ldots, \tilde{d}_T)^T$ the vector of aggregated demands from TDP users, $\bar{p}$ the flat price for FP users, and $\tilde{\mathbf{p}} \triangleq (\tilde{p}_1, \ldots, \tilde{p}_T)^T$ the TDP price vector. $\bar{p}\mathbf{1}\bar{\mathbf{d}} + \tilde{\mathbf{p}}^T\tilde{\mathbf{d}}$ constitutes the revenue of utility company. Considering the demand elasticity, $\tilde{\mathbf{d}}$ is a function of TDP price vector given as:

$$\tilde{\mathbf{d}} = \alpha \left[\mathbf{d}_o + \mathbf{E}(\tilde{\mathbf{p}} - \mathbf{1}^T \bar{p}) \right], \tag{7}$$

where $\mathbf{d}_o \triangleq (d_{1o}, \ldots, d_{To})^T$ is the vector of original demand if all users are under FP scheme. Here it is assumed that demand elasticity factors are measured at the flat price, and have been properly scaled according to equilibrium points $(d_{to}, \bar{p}), \forall t \in \Gamma$. The aggregated demand from FP users is simply given by $\bar{\mathbf{d}} = (1-\alpha)\mathbf{d}_o$.

Considering the benefit of TDP, utility companies are more willing to promote TDP scheme. However, to encourage more FP consumers to adopt TDP, utility company should, preferably, offer monetary incentive to TDP customers in electricity bill with respect to what they would have been charged under the regulated flat price. Traditionally, this incentive is given in forms of discount in electricity bill, which also serves as an important restriction on TDP to avoid an excessive increase of them due to low elasticity of electricity demand from customer. This incentive scheme is specified as:

$$\frac{\tilde{\mathbf{p}}^T \tilde{\mathbf{d}}}{\mathbf{1}\tilde{\mathbf{d}}} \leq (1-\gamma)\bar{p}, \tag{8}$$

where γ is the discount rate. When high discount rate is given, users under TDP scheme may receive more monetary incentive from the utility company. However, this incentive scheme considers only the benefit on the customer side. The profit of utility company is not taken into consideration. In some cases, the profit of utility company cannot be guaranteed under certain discount.

In the proposed system, a dynamic incentive scheme is introduced which considers the net benefit under TDP program from both utility company and electricity customers, which is calculated as:

$$(\mathcal{C}_o - \mathcal{C}) + (\bar{p}\mathbf{1}\tilde{\mathbf{d}} - \tilde{\mathbf{p}}^T \tilde{\mathbf{d}}) \tag{9}$$

where $\mathcal{C}_o$ is the utility company cost without TDP program implemented, which is defined as

$$\mathcal{C}_o = \sum_{t=1}^{T} \mathcal{C}_t - \bar{p}\mathbf{1}\mathbf{d_o}, \tag{10}$$

Here, $\bar{p}\mathbf{1}\tilde{\mathbf{d}} - \tilde{\mathbf{p}}^T\tilde{\mathbf{d}}$ is the the benefit for the customers under the proposed TDP scheme. The utility company and TDP clients can share the net benefit with pre-defined distribution. For example, when less users are involved into TDP program, the utility company can give more shares to TDP users to attract non-TDP users.

In such a way, the formulation of the optimization problem can be defined as

$$\min_{\tilde{\mathbf{p}}} \mathcal{C}, \tag{11}$$

subject to

$$\underline{p}_t \leq \tilde{p}_t \leq \overline{p}_t, t = 1, ..., T; \tag{12}$$
$$\tilde{d}_t \geq \tilde{d}_{t,\min}, t = 1, ..., T; \tag{13}$$
$$\tilde{\mathbf{p}}^T \tilde{\mathbf{d}} < \bar{p}\mathbf{1}\tilde{\mathbf{d}}; \tag{14}$$
$$(\mathcal{C}_o - \mathcal{C}) = \beta \cdot (\bar{p}\mathbf{1}\tilde{\mathbf{d}} - \tilde{\mathbf{p}}^T\tilde{\mathbf{d}}), \tag{15}$$

where $\underline{p}_t$ and $\overline{p}_t$ are the lower and upper price bounds necessary to avoid extraneous results from the optimization process, β is the distribution factor which may affects the benefit partition between the utility company and TDP customers, and (13) are operating restrictions that specify the minimum demand for each time slot from critical loads.

This optimization problem is a typical nonlinear programming (NLP) [17] problem. In practical applications, this is simplified as quadratic cost functions. Considering constant

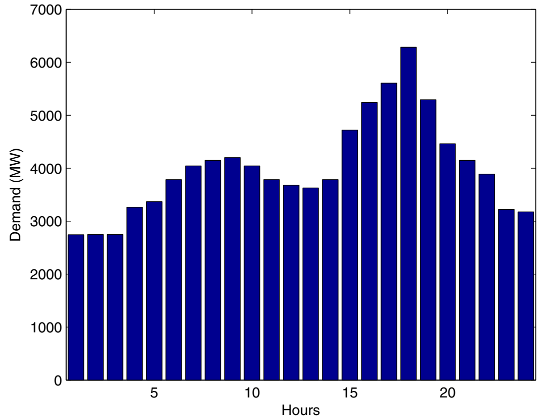

Fig. 2. Original aggregated load from electricity users.

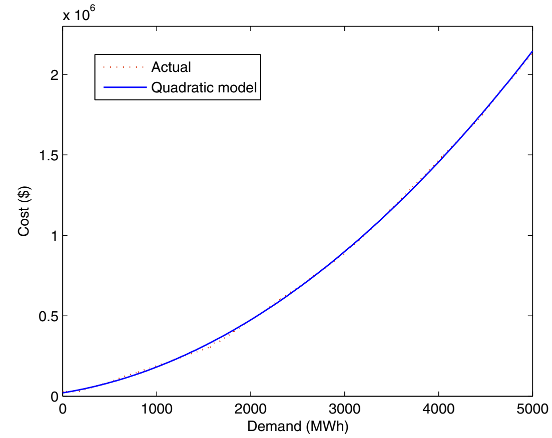

Fig. 3. 10-Unit system cost function.

demand elasticity factors, The above optimization problem becomes a quadratically constrained quadratic programming (QCQP) problem [18]. It is possible to solve this problem efficiently using approximation algorithms such as sequential quadratic programming [19], or semi-definite programming (SDP) on a semi-definite relaxation of Eq. (11) [20].

IV. Case Study and Results Analysis

In this section, dynamic incentive TDP scheme described in Sec. III is implemented, and simulation results are given to show the load shifting/shedding effect of the proposed scheme in dual-tariffs system. Here, we assume the 24-hour initial demand from electricity users when a flat price of \$430/MWh is applied (see Fig. 2). We further allow TDP rates to fluctuate between a lower bound of \$129/MWh and a higher bound of \$860/MWh over the time period as long as they, collectively, satisfy the constraint on expected rebate to the TDP users. The self and cross-time elasticity factors of electricity users are specified as follows:

$$\varepsilon_{t\tau} = \begin{cases} -0.5, & mod(\tau - t, 24) = 0 \\ \frac{1}{6}, & mod(\tau - t, 24) = 1 \\ \frac{1}{30}, & mod(\tau - t, 24) = 2, 3, 4, 22, 23 \\ \frac{1}{60}, & mod(\tau - t, 24) = 5, 6, 7, 19, 20, 21 \\ \frac{1}{150}, & mod(\tau - t, 24) = 8, 9, 10, 16, 17, 18 \\ \frac{1}{300}, & mod(\tau - t, 24) = 11, 12, 15 \\ 0, & mod(\tau - t, 24) = 13, 14 \end{cases} \quad (16)$$

Obviously, the elasticity factor satisfies $\mathbf{1} \cdot \mathbf{E} \prec \mathbf{0}$, which indicates the existence of load shedding.

First, we may study the utility cost function given in (1). In real application, the calculation of this cost function requires high computational cost. It is hard to do it in real time. In our simulation, quadratic model is used to approximate this cost function. Let's use the numerical data in [21] as example. In [21] Appendix C.1, a 10-Unit system was given. Based on this system, the corresponding cost function is generated as shown in Fig. 3. This cost function can be approximated by quadratic function $\hat{C}_t = 21152 + 94.368d + 0.0661d^2$, where d is power demand in MWh, which is also shown in the same figure. The coefficient of determination (R^2) is used to evaluate this approximation result. The coefficient of determination (R^2) is defined as

$$R^2 = 1 - \frac{\sum_d (C_t - \hat{C}_t)}{\sum_d (C_t - \bar{C})} \quad (17)$$

where C_t is the actual cost data and $\bar{C}$ is its expectation. With this approximation, the value of R^2 is 0.9998 which means the quadratic function approximation is highly accurate. It can be well understand when more generator units are involved, the final results may be better approximated as a quadratic function.

Based on the above simulation, quadratic function may be used as the energy procurement cost function in the following simulation. The energy procurement cost adopts the quadratic funtion $21152 + 94.368d + 0.0661d^2$. An equal benefit distribution scheme between the utility company and the TDP customers is considered, which means $\beta = 1$. When less users are involved in TDP program, these TDP users can get high benefit from TDP program. When more users participates into the TDP program, more users share the same quota of benefit. Less benefit will be awarded to individual customers.

As can be seen from Fig. 4, when less users participate into TDP scheme ($\alpha = 0.2$), the demand is in fact inelastic since the majority of the electricity users is still under flat tariff, resulting less load shifting from peak hours. The peak load is as high as 5988MW at $T = 18$. When more users are involved into TDP scheme ($\alpha = 0.7$), more loads are shifted from peak hours to non-peak hours, and peak load at $T = 18$ becomes 5100MW. Clearly, the reduced peak demand and improved load profile when more users participate in TDP scheme will help to improve the reliability of the power grid, and save the energy procurement costs of utility company. This is evident from Fig. 5, which shows that the revenue to utility company has in fact, increased when more users participate in TDP

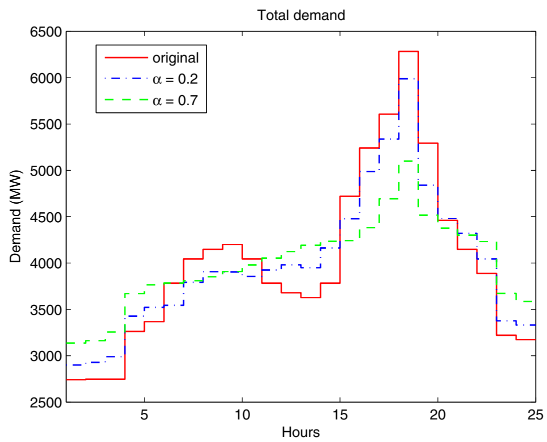

Fig. 4. Electricity demand from users under dual-tariffs system with dynamic incentive $\beta = 1$.

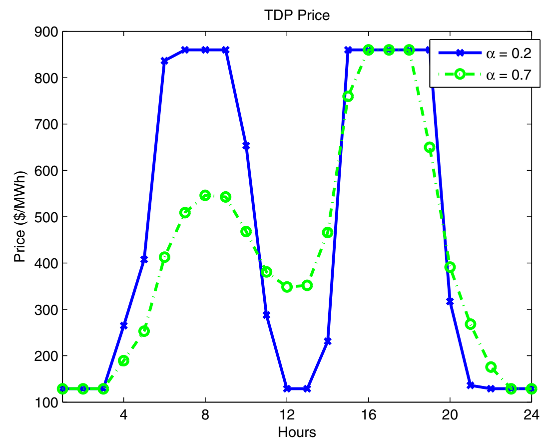

Fig. 6. TDP prices ($\beta = 1$).

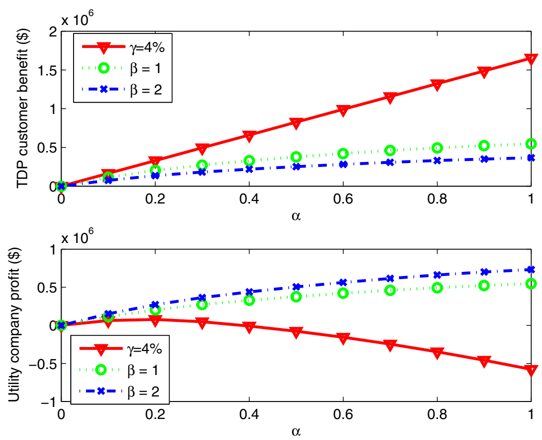

Fig. 5. Comparison of TDP customer and utility company benefits under different incentive schemes.

scheme even when TDP users share the same benefit as the utility company ($\beta = 1$).

However, if a fixed discount rate is given to the TDP customers, the utility company's profit cannot be guaranteed. This can be observed in Fig. 5. When $\gamma = 4\%$ is considered, which means only 4% discount rate is given to the TDP customers, the utility company's profit may drop when more customers participate into the TDP program. In this case, the customer's overall benefit increased constantly. However, the utility company's profit becomes negative. Under the same simulation condition, dynamic incentive scheme guarantees the utility company's profit under different benefit distribution schemes ($\beta = 1$ and $\beta = 2$). This motivates the utility company to promote the TDP program to potential clients.

The TDP price signals obtained from the optimization problem with dynamic incentive scheme $\beta = 1$ are illustrated in Fig. 6. It can be seen in both cases ($\alpha = 0.2$ and $\alpha = 0.7$), two price peaks are presented which correspond to the peak load period as shown in Fig. 2. In particular, when the level of TDP user participation is low ($\alpha = 0.2$), more dramatic changing TDP is required in order to encourage load shifting from peak hours to non-peak hours. In this case, the TDP price signals closely resemble a simple Time-of-Use (ToU) rate design that charges end users a higher rate during peak hours. The dynamic of TDP is reduced when more users participate in the TDP scheme, and the elasticity of electricity demand is improved ($\alpha = 0.7$).

V. Conclusion

In this paper, a dynamic incentive scheme for dual-tariffs based DR program is investigated. Since conventional pricing scheme and voluntary TDP scheme are co-existed, dynamic incentive scheme is used to attract the FP customers to convert to TDP program, and guarantee the utility company's profit. The TDP price signal actually provides a control signal to guide the power consumption towards more efficient operation of power grid, and the dynamic incentive schemes distribute the benefit from TDP program to the utility company and TDP customers. As the profit of the utility company can be guaranteed by this dynamic incentive scheme, the utility company is well motivated to promote the TDP program to clients. The proposed DR scheme can well reduce the variation of the load and increase the total benefit.

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