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A Linear Programming Approach for Balancing Flexible Rotors

The feasibility of a new flexible rotor balancing method is demonstrated. Discrete "effective" unbalance components which produce the same observed response as the actual rotor unbalance are identified, and subsequently removed, using linear programming. In addition to satisfying rotor runout observations, the unbalance may be identified such that it is potentially harmful to response at a speed above the level at which the shaft can safely run without balancing. The potentially harmful response corresponds to the linear programming objective function, while the observations become constraints. In addition, further constraints can be included to assure that the size of the calculated balance weights are within practical limits. The versatility of the new approach is demonstrated with example problems using a rotor model for which the response is obtained with a computer code.

Introduction

The deflection of a rotating shaft can be influenced by a broad range of factors, but one of the difficulties most frequently encountered is the existence of forced steady-state response due to unbalance. A rotor suffers from unbalance if its mass axis does not coincide with its axis of rotation. Smooth operation of a rotor demands the reduction of unbalance forces, and this is accomplished at present by a process of balancing in which small correction masses are distributed along the length of the shaft (or material is removed) so that the unbalance response is reduced to acceptable levels over the range of operating speeds. The balancing process requires the formulation of systematic methods for selecting and positioning suitable correction masses, and this paper presents a new method, based on linear programming concepts.

The balancing process involves determining the balance mass magnitudes, the angular locations in planes of rotation, and the axial locations of the planes of rotation in which the balance masses are applied. A comprehensive review of balancing methods for flexible shafts is provided in reference [1].[2] The various balancing techniques which have been developed may be broadly grouped into two essentially different but not contradictory schools of thought: modal balancing and influence coefficient approaches. Since both approaches are discussed in detail in reference [1], no in-depth consideration of them will be given here. However, it is useful to outline the influence coefficient method since the proposed approach belongs to that category. Influence coefficient techniques (see, for example, references [2, 3]) have developed from suggestions initially offered by Goodman [4].

Although the inherent unbalance of a rotating shaft is, in general, distributed continuously along its length, the initial step of the influence coefficient approach involves assuming that this continuous distribution may be "lumped" into a finite number of discrete unbalance elements. The net unbalance for the p-th axial element is therefore regarded as being resolved into a single equivalent unbalance moment vector, $\bar{U}_p$ which is given by [1]

$$\bar{U}_p = \rho_p A_p \int_0^{l_p} \bar{a}(z)\, dz \qquad (1)$$

In equation (1) ρ_p is the weight density of the rotor material in the p-th element, of which there are n such elements. A_p is the cross-sectional area, and $\bar{a}(z)$ is the mass eccentricity vector function. As indicated, the integration is performed from one side of the element ($z = 0$) to the other ($z = l_p$). The distribution of unbalance $\bar{a}(z)$ is, of course, unknown and the integration is not actually performed. Instead, the integral equation merely indicates that the distributed residual unbalance may be represented in the form of n discrete unbalance moments $\bar{U}_p$, one for each of the n elements into which the rotor is divided. These unbalance moments are then related to vibration amplitudes measured at various speeds and at various locations along the axis of the shaft by the matrix equation

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[2] Numbers in brackets designate References at end of paper.

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$$\{w\} = [A]\{U\} \quad (2)$$

where $\{w\}$ is a displacement vector with m elements, $[A]$ is an m by n matrix of influence coefficients, and $\{U\}$ is a vector containing the n unbalance moments, U_p. Because of the two-dimensional nature of the deflections and unbalance moments, the elements of each of the matrices in the preceding equation are complex numbers. It is convenient to express these complex elements in terms of their real two-dimensional (x,y) components. If displacements are measured at υ locations along the rotor, and at N speeds, then $m = \upsilon N$. The elements of $[A]$, denoted by a_{jp}, represent the linear coefficients which give the effect on deflection of element j of the unbalance moment at location p. The balancing procedure involves solution of the foregoing equation for $U_1, U_2, \ldots, U_n$ (or, more precisely $U_{1x}, U_{2x}, \ldots, U_{nx}, U_{1y}, U_{2y}, \ldots, U_{ny}$), which are the components of residual unbalance in the n balancing planes. The process requires measurement of amplitudes and phase angles of rotor vibration, calculation of the influence coefficients a_{jp}, and finally inversion, or, perhaps pseudoinversion, of the $[A]$ matrix so that the solution for the U_p quantities may be obtained, i.e.,

$$\{U\} = [A]^{-1}\{w\} \quad (3)$$

If the number of unbalance moments U_p identified is equal to the number of response observations, i.e., $m = n$, then $[A]$ is a square matrix and $[A]^{-1}$ of equation (3) is inverted in the usual sense. We choose to call this "direct" inversion. However, for the case where the number of observations exceeds the number of unbalance moments to be identified, i.e., $m > n$, $[A]^{-1}$ requires a pseudoinversion operation, which is usually referred to as a least square fitting procedure. The linear programming formulation presented here requires that $m < n$.

In spite of the considerable acceptance of the influence coefficient method of flexible rotor balancing, Lund and Tonneson [3] *note that it has met with only qualified approval by many*. There are several shortcomings, not the least of which is the inability to limit the magnitude of the unbalance moments identified to those that can practically be removed or added. For example, on thin-walled rotors the addition or removal of too large a balance weight may introduce undesirable stress concentrations.

In considering the feasibility of developing an improved approach, one's attention is directed to the fact that the success of present methods has indicated that it is not necessary to balance a rotor exactly, but rather to select and position a set of discrete balance weights such that shaft vibration due to inherent distributed unbalance is held below a certain limit. It is well known that for a given unbalanced rotor-support system, a number of different sets of balance weights may be selected which produce rotor behavior satisfying the specified response criterion of equation (2). It is natural, therefore, to ask which of these several different sets of balance weights might produce a type of *optimum* rotor behavior; that is, which set would satisfy the response criterion *and* produce rotor behavior which is in some way better than the behavior resulting from the addition of each of the other sets of correction weights. The balancing problem is formulated here as an optimization problem, in particular, in terms of linear programming concepts. This makes it possible to identify, for subsequent removal, the unbalance distribution which satisfies a set of observations of the behavior of the unbalanced shaft and which *also* is potentially harmful to response at a location or speed which is not or cannot be actually observed. This additional requirement on the solution cannot be imposed using any of the balancing techniques currently employed. Another advantage of the linear programming technique is that it is possible to specify limits on the magnitudes of the calculated balance weights.

Formulation

The general linear programming problem consists of determining $(U_1, U_2, \ldots, U_n)$ such that

$$[C]\{U\} \quad (4)$$

is minimized (or maximized) subject to

$$\{U\} \geq \{0\} \quad (5)$$

$$[A]\{U\} = \{w\} \quad (6)$$

where

$[c]$ $= (c_1, c_2, \ldots, c_n)$ row vector
$\{U\}$ $= (U_1, U_2, \ldots, U_n)$ column vector
$[A]$ $= (a_{ij})_{m \times n}$ matrix; m rows, n columns
$\{w\}$ $= (w_1, w_2, \ldots, w_m)$ column vector
$\{0\}$ $=$ n-dimensional null column vector
n $> m$

Large scale computer software routines for solving this problem are readily available.

The conversion of the balancing problem to the linear programming formulation is straight forward. Suppose the number of observations is less than the number of unbalance moments sought, i.e., $m < n$, then equation (2) becomes constraint (6). Let

$$w_s = [c]\{U\} \quad (7)$$

represent the response of the rotor at a particular speed and axial location, where $[c]$ is a row vector of influence coefficients. Equation (7) can be taken to represent the objective function (4). If w_s is chosen at a particular axial location (e.g., at a bearing, where large deflection would produce large forces) and/or at a particular speed (e.g., at a high speed that cannot be successfully negotiated with the unbalanced rotor), then maximization of w_s leads to the identification and subsequent removal of a potentially harmful (e.g., large forces at high speeds) unbalance distribution.

The control of the size of the unbalance moments is introduced through constraints of the type

$$U_p^L \leq U_p \leq U_p^U, \; p = 1, 2, \ldots, n \quad (8)$$

where U_p^L, U_p^U are lower and upper bonds that are chosen to be imposed on U_p. In terms of linear programming, this inequality constraint is converted to the equality form of equation (6), so that equation (6) is increased in dimension. In practice, with most major linear programming software systems this conversion is fully automatic and presents no problem to the user.

For the linear programming formulation applied to a rotor, the vectors $\{U\}$, $\{w\}$ contain the components of the unbalance moments and observations. Thus if there are n balance planes and m observations, $\{U\}$, $\{w\}$ are $2n$ and $2m$ dimensional column vectors and $[A]$ is $2m \times 2n$. Since the unbalance moment components may be negative, a change in variables is required to satisfy equation (5). This adjustment is automatically taken into account in major linear programming systems.

For a given rotor-support system, the influence coefficients a_{ij} may be generated either experimentally or theoretically. If the objective function is taken at a speed or location that cannot be dealt

Nomenclature

$\bar{a}$ = distributed unbalance function
a_{ij} = influence coefficient
$[A]$ = matrix of influence coefficients
A_p = cross-sectional area of pth axial element of rotor
l_p = length of pth axial element of rotor
m = number of deflection observations
n = number of identified unbalance values
U_p = unbalance moment of pth axial element
w = rotor deflection
x,y,z = mutually orthogonal coordinates
ρ_p = weight density of rotor material in pth axial element

with experimentally then it is necessary to compute the influence coefficients for the objective function theoretically. This would normally be accomplished using a rotor response computer program. Usually it is advisable to verify the accuracy of the computer model utilized in the program at those speeds that can be observed experimentally.

The linear porgramming formulation includes the maximization of the objective function, equation (7). It would be desirable, however, to maximize the magnitude of the response at a given speed and location, as given by

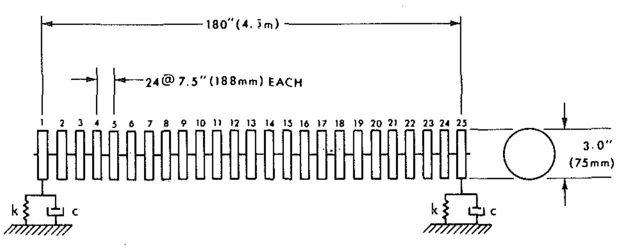
$$|w_s| = \sqrt{w_{sx}^2 + w_{sy}^2} \tag{9}$$

Thus $|w_s|$ is a nonlinear function of the response components, and therefore of the unbalance components. Use of equation (9) as an objective function would cast the problem into a quadratic programming form, which is not as desirable as linear programming from the standpoint of available adequate computer software. We choose, therefore, to maximize the lingar combination $|w_{sx} + w_{sy}|$ of the x and y components of objective function response. It should be noted that the set of unbalance components thus identified might not, in all cases, correspond to the set which produces maximum objective function response, although the response will be large. Thus, instead of identifying unbalance which satisfies all observations and is potentially *most* harmful to objective function response, we obtain the unbalance distribution which is simply potentially harmful. It will be seen that this formulation, involving maximizing the linear combination of the objective function response components instead of the actual magnitude of the response, and thus admitting the possibility of a linear programming solution, produces excellent results. In fact, several sets of unbalance moments have been computed using linear and quadratic programming with virtually identical results.

Example Problem

As an example of the application of the linear programming formulation, consider the rotor of Fig. 1. A hypothetical situation is proposed whereby it is supposed that the rotor is to operate at 6000 rpm. In order to reach operating speed, the rotor must negotiate three flexural criticals. These critical speeds, along with their associated characteristic mode shapes, are given in Fig. 2.

An initial arbitrary unbalance distribution is assumed. Our goal is to calculate balance weights such that response due to this initial unbalance is reduced to the extent that the assumed operating speed may be reached without deflections exceeding some allowable maximum value. In order to approximate the general case of a distributed initial unbalance, the rotor mass has been "lumped" into 25 equally spaced elements. Initial two-dimensional (x and y components) arbitrarily selected values of unbalance are introduced at each of the 25 mass stations, as given in Table 1. Typical response values due to this initial unbalance are shown in Fig. 3. These and subsequent response data were calculated using a rotor response computer program described in reference [5] and modified by E. J. Gunter of the University of Virginia. It should be mentioned that the response plots shown in Fig. 3 are only approximate, since the response due to the initial unbalance is three di-



k = 5000 lb/in (875 kN/m)
c = 3.0 lb-sec/in (0.525 kN-sec/m)
E = 10.4 (10^6)psi (72.8 GN/m^2)
I = 0.96 in^4 (39.96 cm^4)

$I_p = \begin{cases} 0.726 \text{ lb-in}^2 \ (212.41 \text{ kg}\cdot\text{mm}^2) \text{ for 1 and 25} \\ 1.453 \text{ lb-in}^2 \ (425.11 \text{ kg}\cdot\text{mm}^2) \text{ for 2 thru 24} \end{cases}$

DISK MASS $= \begin{cases} 0.345 \text{ lb } (0.156 \text{ kg}) \text{ for 1 and 25} \\ 0.690 \text{ lb } (0.313 \text{ kg}) \text{ for 2 thru 24} \end{cases}$

Fig. 1 Model of uniform shaft on two supports

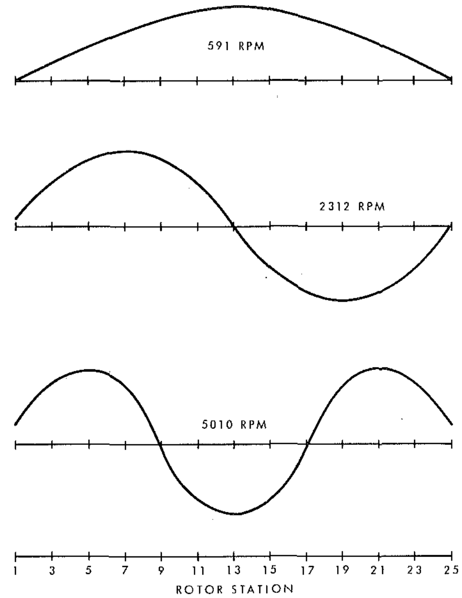


Fig. 2 Critical speeds and mode shapes for rotor of Fig. 1

Table 1 Arbitrary initial unbalance distribution for rotor of Fig. 1

Rotor Station	x-component oz-in.	(mN·m)	y-component oz-in.	(mN·m)
1	0.56	(4.0)	0.74	(5.2)
2	0.84	(5.9)	0.37	(2.6)
3	0.47	(3.3)	0.74	(5.2)
4	0.74	(5.2)	0.46	(3.3)
5	−0.20	(−1.4)	0.28	(2.0)
6	0.46	(3.3)	−0.59	(−4.2)
7	0.37	(2.6)	−0.44	(−3.1)
8	−0.58	(−4.1)	−0.25	(−1.8)
9	−0.14	(−0.99)	0.50	(3.5)
10	0.58	(4.1)	0.26	(1.8)
11	0.24	(1.7)	0.46	(3.3)
12	−0.42	(−3.0)	−0.29	(−2.1)
13	0.46	(3.3)	−0.44	(−3.1)
14	0.10	(0.8)	−0.29	(−2.1)
15	0.68	(4.8)	0.27	(1.9)
16	0.27	(1.9)	0.41	(2.9)
17	0.66	(4.7)	−0.52	(−3.7)
18	−0.23	(−1.6)	0.36	(2.5)
19	−0.52	(−3.7)	0.09	(0.6)
20	0.49	(3.5)	0.31	(2.2)
21	0.42	(3.0)	−0.77	(−5.4)
22	−0.18	(−1.3)	0.69	(4.9)
23	−0.36	(−2.5)	−0.96	(−6.8)
24	0.18	(1.3)	0.25	(1.8)
25	−0.30	(−2.1)	−0.50	(−3.5)

Table 2 Comparison of unbalanced and balanced response of rotor of Fig. 1, seven observations, eight planes

Rotor Station	Speed rpm	Initial Unbalanced Response mils	(mm)	Balanced Response mils	(μm)	Per Cent Reduction
13	600	654	(16.4)	5.37	(134)	99.2
7	2400	109	(2.73)	2.52	(63.0)	97.6
19	2400	91.6	(2.29)	2.28	(57.0)	97.4
5	4800	56.1	(1.40)	1.14	(28.5)	98.0
11	4800	75.5	(1.89)	0.33	(8.3)	99.5
12	4800	82.4	(2.06)	0.56	(14)	99.1
21	4800	52.0	(1.30)	1.08	(27.0)	98.1
16	6000	23.9	(.598)	0.29	(7.3)[1]	98.8

[1] Objective function, no observation

mensional. However, in the vicinities of the critical speeds, deviation from a planar response curve is small, so that little accuracy is lost in a two-dimensional plot.

To establish a particular problem, we must choose the speeds and axial locations at which observations of unbalanced rotor response are to be taken, as well as the number and axial locations of the balance weights to be applied. In making these selections, some experience with the actual behavior and balancing of rotating shafts can be extremely valuable. Accordingly, we choose observations corresponding to speeds at or near resonance values and at axial locations for observations and balancing weights which correspond to some of the antinodes of the respective modes.

For a specific balancing problem, assume the deflections at 4800 rpm shown in Fig. 3 are the maximum allowable at that speed. Since the rotor is to operate at 6000 rpm, balancing is required. Suppose the rotor can be run safely at speeds up through 4800 rpm and displacement observations are made. In terms of the linear programming formulation, we wish to identify (and subsequently remove) the unbalance which satisfies observations through 4800 rpm *and* is potentially harmful at 6000 rpm, a speed at which the rotor cannot be safely run prior to balancing.

At 6000 rpm, response will involve a combination of third and fourth mode effects. Since the calculated balance weights will reduce the response at 4800 rpm (because observations are taken there), a speed close to the third critical, the third mode component at 6000 rpm will also be reduced. We therefore focus our attention on the fourth mode component and select the axial location for the objective function at an antinode of the fourth mode, station 16 in Fig. 1.

We choose to identify eight balance weights to be located at stations 5, 7, 11, 12, 13, 16, 19, 21 such that observations at the seven stations (and speeds) 13 (600 rpm), 7 (2400 rpm), 19 (2400 rpm), 5 (4800 rpm), 11 (4800 rpm), 12 (4800 rpm), and 21 (4800 rpm) are satisfied. After the observation responses are computed, it is assumed that the initial unbalance distribution is unknown and that only these response observations are available for use in calculating the balance weights.

The first attempt by the authors to obtain a solution, using a standard linear programming code, resulted in an unbounded objective function. Physically the explanation for this phenomenon is that it is possible to obtain a set of unbalance moments (U_1, U_2, $\ldots$, U_8) which satisfy the observations (equation (6)), but which can produce unbounded response at station 16 at 6000 rpm. Such a set of unbalance moments would consist of very large magnitudes of components, U_x and U_y, but the orientations of the unbalance vectors are such that the observations are still satisfied. For example, very large components might be calculated for stations 11, 12, and 13, but the signs of these components could be such that the resultant unbalance moment for all three stations is fairly small. Such a combination could produce an unbounded objective function, while still satisfying equation (6).

In order to overcome this mathematical difficulty, it is necessary to place constraints on the maximum size of the unbalance components. This requirement leads to an additional set of equations of the form of equations (8). As mentioned in the discussion of the linear programming formulation, this additional set of constraints is actually quite appropriate since it may be desirable in practice to obtain balance weights of minimum size. However, it was observed that there is a lower bound on $|U_p|$ such that attempts to calculate balance weight components smaller than this lower bound result in no feasible solution, i.e., it is not possible to satisfy the observations, equations (6). Thus the procedure consists of determining balance weights for progressively smaller values of the bounds on U_p, until the smallest possible values have been identified, and then using those weights to improve the response of the rotor.

Using this procedure the balance weights were identified and then added. Representative resulting responses are shown in Table 2. It is seen that a significant reduction in response was obtained at station 16 at 6000 rpm (the objective function), although no observation at station 16 was used to calculate the balance weights. Examination of other response values indicates reduced response at 6000 rpm at all stations, although observations were assumed to be available up to only 4800 rpm.

Additional Results

The effectiveness of the linear programming approach was compared with direct inversion balancing. For the speed range up to 4800 rpm the two approaches are for the most part of equal quality. As shown in Table 3, however, the differences become more significant at station 16 at 6000 rpm (the linear programming objective function). Here the balance weights obtained using the linear programming scheme lead to a substantially better response than those obtained by the direct inversion technique.

Additional differences between the results obtained using the two approaches become evident when the calculated balance weight distributions are compared. Such a comparison is given in Table 4. We note that the balance weights calculated using the direct inversion technique for application at stations 11, 12, and 13 tend to be rather large in magnitude, but their directions (or signs) are such that the net resultant for all three stations is small. This phenomenon has been encountered in practice when the direct inversion technique has been applied to the balancing of actual rotors. In some cases the balance weight magnitudes calculated have been so large that application was impossible. The large magnitudes still satisfy the required observations, however, since the angular orientations of the balance weights in adjacent planes differ by approximately 180 deg.

This difficulty may be eliminated using the linear programming balancing technique, since constraints on the maximum size of the balance weight components may be included in the problem formulation. For the case presented in Table 4, the maximum size of each component is required to be less than or equal to 3.0 oz-in. (21.2 mN·m). Although it is seen that the balance weights calculated by the linear programming technique tend to follow the same

Table 3 Comparisons of balanced response of rotor of Fig. 1 obtained using different balancing techniques

Rotor Station	Speed rpm	Initial Unbalanced Response mils	(mm)	Balanced Response: Linear Programming 8 obs., 7 planes mils	(mm)	Direct Inversion 7 obs., 7 planes mils	(μm)
13	600	654	(16.4)	5.37	(134)	5.37	(134)
7	2400	109	(2.73)	2.52	(63.0)	0.45	(11)
19	2400	91.6	(2.29)	2.28	(57.0)	0.16	(4.0)
5	4800	56.1	(1.40)	1.14	(28.5)	0.87	(22)
11	4800	75.5	(1.89)	0.33	(8.3)	0.72	(18)
12	4800	82.4	(2.06)	0.56	(14)	0.58	(14)
21	4800	52.0	(1.30	1.08	(27.0)	0.63	(16)
16	6000	23.9	(.598)	0.29[1]	(7.3)	5.90[2]	(148)

[1] no observation, objective function
[2] no observation, no balance plane

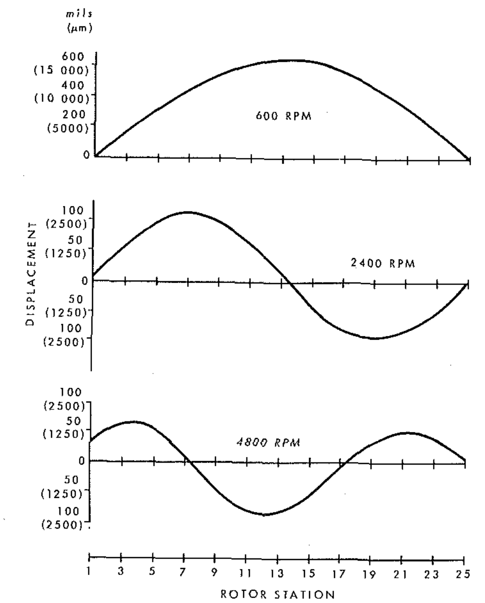


Fig. 3 Response of rotor of Fig. 1 due to initial unbalance

pattern for stations 11, 12, and 13 (large magnitudes with opposite signs), their magnitudes are considerably less than those obtained by simple direct inversion.

A better understanding of the response of the balanced rotor may be gained by considering the balanced responses given in Fig. 4, obtained by the addition of the balance weights calculated with linear programming on the basis of seven observations and eight balance planes. We note that, especially at the higher speeds, the greatest reductions in response are obtained at those stations where observations are taken, namely, station 13 at 600 rpm, stations 7 and 19 at 2400 rpm, and stations 5, 11, 12, and 21 at 4800 rpm. For the remainder of the rotor, especially the ends (where the bearings are located), the reductions in response are not as significant. It seems reasonable to postulate, therefore, that balance weights calculated on the basis of satisfying additional observa-

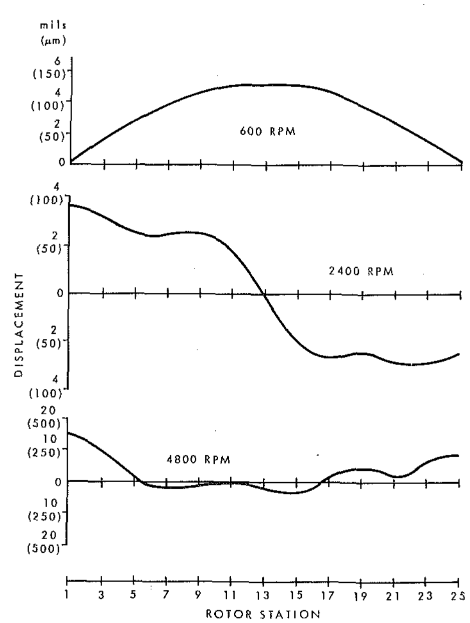


Fig. 4 Response of rotor of Fig. 1 after balance weights are added

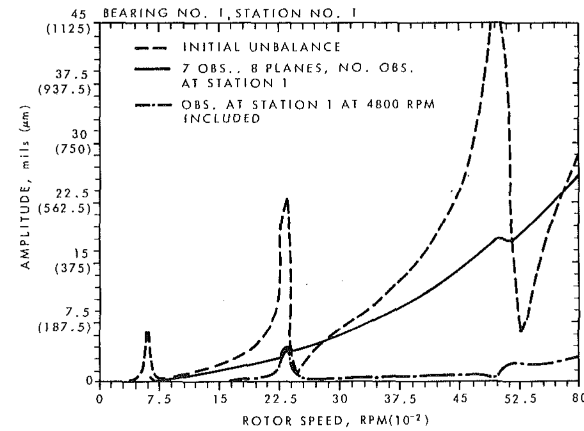


Fig. 5 Effect on balanced response of including an observation at station 1 of rotor of Fig. 1

tions at the bearing would produce smaller balanced response at those points. Fig. 5 indicates that this is true. Shown in Fig. 5 is the response versus speed curve obtained at station 1 when balance weights based on including an observation at station 1 at 4800 rpm are added. Also shown, for purposes of comparison, are response curves obtained due to the initial unbalance distribution and the addition of balance weights previously obtained without an observation at station 1. Note that response at station 1 is drastically reduced by requiring that the observation there be satisfied.

In some cases a reliable theoretical model may not be available to provide influence coefficients for speeds above the safe speeds where observations may be taken. In such cases a linear programming balancing approach still offers certain advantages over techniques presently employed, since it admits the possibility of limiting the size of the balance weights to be calculated. The objective function could be taken to correspond to a station and speed for which observation is possible, and the appropriate influence coefficients determined in the usual fashion. The feasibility of "moving" the objective function into the observable speed range was investigated for the unbalanced rotor considered here, and a solution was readily obtained.

More detailed studies of the linear programming approach are provided in reference [6], where consideration is given to such facets as the effect of varying the numbers of observations. For example, the results indicate that increasing the number of observations results in better balanced response. Furthermore, attaching balance weights calculated on the basis of too few observations can have disastrous effects, producing response greater than that due to the initial unbalance distribution. On the basis of various observation combinations studied, it is shown that significant reductions in response are achieved when observations are taken for each mode under consideration, and even greater reductions are obtained when observations of response at each antinode of each

Table 4 Comparison of balance weight distributions obtained for rotor of

	Linear Programming 7 obs., 8 planes				Direct Inversion 7 obs., 7 planes			
	x		y		x		y	
Rotor Station	oz-in.	(mN·m)	oz-in.	(mN·m)	oz-in.	(mN·m)	oz-in.	(mN·m)
5	-2.11	(-14.90)	-1.33	(-9.39)	-2.03	(-14.3)	-1.49	(-10.5)
7	0.24	(1.70)	0.81	(5.7)	0.22	(1.6)	1.04	(7.35)
11	-1.20	(- 8.48)	-1.50	(-10.6)	-2.45	(-17.3)	-3.44	(-24.3)
12	3.00	(21.2)	2.14	(15.1)	7.23	(51.1)	6.71	(47.4)
13	-1.93	(-13.6)	-0.87	(-4.7)	-6.32	(-44.6)	-3.79	(-26.8)
16	-2.89	(-20.4)	-0.31	(-2.2)	-	-	-	-
19	3.00	(21.2)	-0.03	(-0.2)	-0.17	(- 1.2)	0.46	(3.3)
21	-2.04	(-14.4)	0.41	(2.9)	-0.19	(- 1.3)	-0.04	(- 0.3)

mode are employed.

Summary

The material presented in this paper is the result of an investigation of the feasibility of an apparently new flexible rotor balancing method, based on linear programming techniques. Discrete unbalance components which produce the same observed response as the initial distributed rotor unbalance are identified and subsequently removed. In addition to satisfying the specified observations, the unbalance is identified such that it is potentially harmful to response at a certain speed and axial location. This response corresponds to the linear programming objective function. If the objective function influence coefficients can be calculated theoretically, then it is possible to formulate an objective function corresponding to response which cannot actually be observed. In addition, linear programming techniques make it possible to limit the size of the calculated balance weights.

Acknowledgments

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