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**A DECISION SUPPORT FORMULATION FOR DESIGN TEAMS: A STUDY IN PREFERENCE
AGGREGATION AND HANDLING UNEQUAL GROUP MEMBERS**

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ABSTRACT

Supporting the decision of a group in engineering design is a challenging and complicated problem when issues like consensus, consistency, conflict, and compromise must be taken into account. In this paper, we present two developments extending the Group Hypothetical Equivalents and Inequivalents Method (Group-HEIM) and making it applicable to new classes of group decision problems. The first extension focuses on handling forms of value functions other than the traditional L_1 -norm. The second extension focuses on updating the formulation to place unequal importance on the preferences of the group members. Typically, there are some group members whose experience, education, and/or knowledge makes their input more important. The formulation presented in this paper allows team leaders to emphasize the input from certain group members. Illustration and validation of the developments are presented using a vehicle selection problem. Data from twelve engineering design teams is used to demonstrate the application of the method.

1. INTRODUCTION

Technology has changed consumer buying behavior dramatically. The choices for products are increasing while product life cycles are decreasing [1]. Product design companies have been under pressure to speed up their operations to get to the market faster with relevant products and services. The market environment is also becoming more and more complex with the underlying design technologies and networks of customers, suppliers, designers, and channels to distribute products. For this reason, many decision support tools have been developed by researchers for product design decisions, including a number focused on multiattribute selection decisions.

Nevertheless, most companies are reluctant to use these tools as most of the decision tools are either user-unfriendly or require a certain level of technical knowledge to understand and to use them. Only those decision tools that are easy to use are adapted by the product design companies even though those tools may have limitations or theoretical flaws. For example, the Analytical Hierarchy Process (AHP) method has been proven to have some theoretical flaws [2], but it is still widely used in industry because of its flexibility, ease of use and ease of implementation. Therefore, when developing new decision tools, the tools have to be user-friendly, easy to implement, and must help speed up the design cycle.

It is with this incentive that the Hypothetical Equivalents and Inequivalents Method (HEIM) was developed to provide a theoretically sound approach for determining the attribute weights in multiattribute selection problems. In addition it was developed to be easy to use, only requiring the elicitation of preferences over a series of hypothetical product design alternatives [3]. It has since been expanded to handle alternatives with uncertain attributes [4], and in [5], a group formulation was developed for HEIM, in the form of Group Hypothetical Equivalents and Inequivalents (Group-HEIM). Group-HEIM recognizes that in common preferences among group members can rarely be guaranteed, unless individual freedom is greatly limited [6]. Group-HEIM instead allows individuals to freely express preferences over a number of Hypothetical Alternatives (HA) and then explores the level of conflict or differences from the aggregated group preferences. In addition, Group-HEIM reveals the source of the conflict in the group preferences, along with the most preferred group solution. With the information, the group can determine if they may decide to go back and focus on the conflicting preferences and attempt to get consensus on them. While the group formulation answered some important research questions with

respect to handling inconsistencies among groups, applying it to some actual product design group decisions revealed some areas of improvement and further development, which is the focus of this paper.

The primary contributions of this paper are in the areas of investigating the role of different value function formulations on the resulting decisions, and developing an approach for considering one group member's opinions as more important than another. Some decision methods have been applied to group decision making by structuring their preferences in some way [6]. For example, Keeney uses a cardinal utility to demonstrate that transitivity can be guaranteed by aggregating individual ratings for each alternative [7]. In addition, the Analytical Hierarchy Process has also been used to aggregate preferences in a group using a pair-wise approach [8-9]. However, a significant assumption for each of these methods is that the decision makers in the group are assumed to be equally important. That is, their information is handled equally without any preference given to one group member over another.

While this is certainly a democratic approach to handling ensuring equal treatment of opinions, in [10] and in a number of pedagogical studies on unequal engineering teams, (e.g., [11]), it is noted that the contributions, experience, and knowledge of every team member is rarely equal. One team member may have more experience making the types of decisions the group is currently tasked with and therefore his/her opinion should be given more importance in the decision. Experienced engineers who have designed similar systems for many years perhaps should be given more credence than new engineers with only a little experience at a company.

To accommodate this natural and appropriate bias in a group's decision making would not only be beneficial in the team's outcomes but also creates a need for formal decision support tools to handle unequal group members. In [12], preferences from unequal group members are integrated using relative weights. While weights can be used to emphasize certain group members, determining exact weights for group members is subject to the same challenges and limitations as determining exact weights for attributes or objectives [13-15]. We present a new group formulation that accounts for group member differences implicitly in the decision formulation itself and investigate how the solution is affected in their favor. In addition we investigate the role of different value functions on the group formulation. While the basic group formulation was presented in [5], in this paper, we present important developments that increase the applicability of the approach to broader classes of design problems. In the next section, the primary developments of this paper are put into the context of the entire method.

2. G-HEIM: A FORMULATION FOR GROUP DECISION MAKING

In this section, we present a detailed explanation of Group-HEIM with emphasis on the new developments that are the focus of this paper. First, an overview of the basic mechanics of both Group-HEIM and HEIM is presented.

2.1 Basic Premise

In HEIM for individual decision makers and in Group-HEIM for groups, the decision maker(s) do not have to specify precise attribute weights individually or as a group, easing the

burden of the decision process and eliminating a typically challenging task in multiattribute decision making. The attributes weights are found through setting up and comparing a set of hypothetical alternatives (HA). The "equivalents" part of the method allows decision makers to make statements like "hypothetical alternatives A^1 and A^2 are equivalent in value to me." On the other hand, the "inequivalents" part of the method allows decision makers to make statements like "I prefer hypothetical alternative A^1 over A^2 ". Therefore, when a preference is stated, by either equivalence or inequivalence, a constraint is formulated.

The equality constraints are developed based on the stated preference of "I prefer alternatives A^1 and A^2 equally." In other words, the values of these alternatives are equal, resulting in Eq. (1).

$$V(A^1) = V(A^2) \text{ or } V(A^1) - V(A^2) = 0 \quad (1)$$

The value of an alternative (alternative A^j in this case) is given in Eq. (2).

$$V(A^j) = \sum_{i=1}^n w_i r_i^j \quad (2)$$

Equation (2) is a L_1 -norm additive value function where r_i^j is the normalized rating of alternative A^j on attribute i . For instance, for a set of vehicle alternatives whose attributes include miles-per-gallon (MPG), the MPG rating for one of the vehicles would simply be the vehicle's MPG value, normalized between 0 and 1 using the highest and lowest MPG values of all the candidate vehicles. While the L_1 -norm form of an alternative's value is shown here for simplicity, other forms of the value function are investigated in later sections of this paper.

The inequality constraints are developed based on the stated preference of "I prefer A^1 over A^2 ." In other words, the value of alternative A^1 is more than alternative A^2 , as shown in the following equations:

$$\begin{aligned} V(A^1) &> V(A^2) \\ V(A^1) - V(A^2) &> 0 \\ V(A^1) - V(A^2) + \delta &\geq 0 \end{aligned} \quad (3)$$

where δ is a small positive number to ensure inequality. Group-HEIM is similar to the multiattribute approach described in [16] because it uses stated equality preferences from the decision maker based on hypothetical alternatives. However, Group-HEIM is different because it accommodates inequality preference statements and is easily scalable to problems with many attributes because it avoids having to address preferential independence or reduction of dimensionality when there are three or more attributes. In the following section, the steps of Group-HEIM are detailed, illustrating how these preference statements are used to find the attribute weights for multiattribute group decisions.

2.2 Group-HEIM Outline

There are six steps in Group-HEIM to process and aggregate group preferences.

Step 1 Identify the Attributes that are relevant and important in the decision problem. Group-HEIM is not able to identify the absence of an important attribute. If an unimportant attribute is included in the process, Group-HEIM will indicate the attribute's limited role with a low weighting factor. Some techniques such as conjoint analysis, factor analysis and value-focused thinking [17-19] can be used to identify the key attributes.

Step 2 Determine the Strength of Preference (SOP) for each individual on each attribute. These strength of preferences (other notations in the literature include single attribute utility functions) reflect the decision makers' true preferences on a certain attribute. A nonlinear SOP representation is suggested to better reflect the decision maker true preferences. Some ways to assess these SOPs are lottery, mid-level splitting and indifferent point methods [20-22].

Step 3 Set up Hypothetical Alternatives and Elicit Preference Structure for each Group Member. In this step, the information used to construct the equality and inequality constraints is elicited from each group member. Hypothetical alternatives (HA) are a key ingredient in Group-HEIM. It is critical that when evaluating two hypothetical alternatives, one is not dominated by another. That is, one alternative should not be better across all attributes compared to another alternative. There should be some balance of attributes that forces the decision maker to process and make a tradeoff when choosing one of the hypothetical alternatives.

In order to develop an appropriate set of HA's, the attribute space is sampled in a structured and balanced manner, using Design of Experiments (DOE) [23]. In most applications of DOE, the input factors are design or control variables. However, in this application of DOE, the input factors are the product attributes relevant to the decision. The attributes dictate the performance of the product and do not reflect a specific design configuration, size, material, or any other typical design variable. They only reflect the performance of a hypothetical design alternative. In Figure 1, two different DOE examples used to sample the performance space are shown. The attributes, f_1 and f_2 , are two conflicting attributes in a typical 2-D performance space, where the objective is to maximize both attributes (e.g., miles-per-gallon and horsepower). The example on the left is a Full Factorial Design while a Central Composite Design is shown on the right.

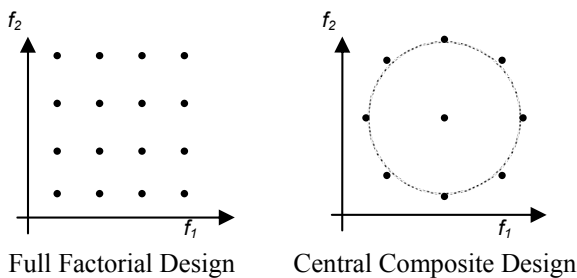


Figure 1 Examples of Sampling the Performance Space

In Group-HEIM, two of these candidate combinations of attributes (which represent one alternative each) are presented to a decision maker. Obviously some of the comparisons will not elicit any valuable information. For instance, in the Full Factorial Design, the point in the upper right (most preferred

region) is certainly preferred to the points in the lower left. This information is not helpful in determining attribute importances. However, other sets of two points will reveal important tradeoff information.

Once the HA's are set up (sampled) appropriately in the performance space, the decision makers are then asked to compare pairs of HA's in order to set up the preference constraints for Step 5. The main purpose of the comparison is to identify the most preferred location in the performance space so that the relative attribute weights can then be solved for. However, some comparisons may not lend any useful or non-redundant information. Figure 2 is presented to further explain the types of HA comparisons that can be made.

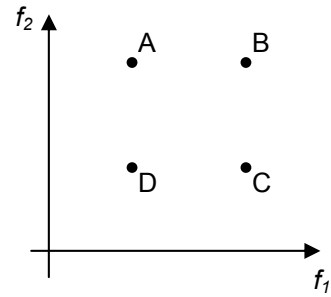


Figure 2 Typical 2-D Performance Space

In Figure 2, assume A, B, C and D are four sampled hypothetical alternatives from the performance space with two conflicting objectives, f_1 and f_2 , that are to be maximized. Therefore, we need to compare the alternatives that will provide useful information. If we compare D with B, no useful information will be obtained since B dominates D by having higher values for both f_1 and f_2 . However, if we compare C with A, then the decision maker has to decide which objective to sacrifice in order to get a higher value on the other objective. This comparison, as a result, provides useful information for the optimization formulation to solve for attribute importances in Step 5.

In Fig. 3, the useful and non-useful projections of comparison are shown. Figure 3a) shows the useful direction of comparison, which includes the comparison of alternatives A and C in Figure 2. Figure 3b) shows the direction where no useful information will be generated, as any two alternatives along one of these directions will include one dominating alternative. In other words, when we compare two HA's, we must make sure that one of the alternatives is not *equal to or worse than* the other alternative with respect to every attribute. The comparison of alternatives B and D in Figure 2 is in this category and is not a useful comparison.

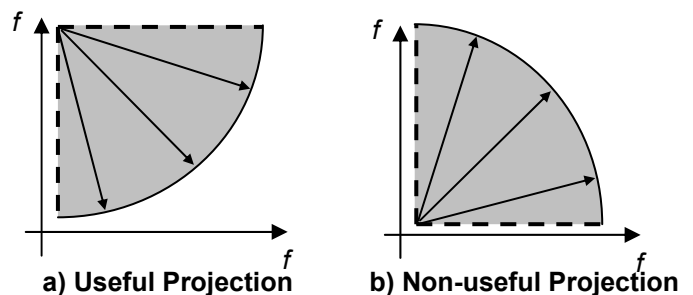


Figure 3 Possible Alternative Comparisons

To improve the efficiency of preference elicitation, the total number of hypothetical alternatives should be kept minimal while maximizing the amount of useful information. In this work, we use a D-Optimal experimental design to both effectively sample the performance space while minimizing the number of hypothetical alternatives and maximizing the value of the information gained from each alternative comparison. The tradeoffs expressed by decision makers can then be used to identify the most preferred location in the performance space and to solve for the attribute importances. To accomplish this, a way to measure the overall value of the alternatives (real and hypothetical) must be chosen.

Step 4 Aggregate Individuals' Preference Structures as an Optimization Problem. The stated preferences (both equivalent and inequivalent) of HA's provided by each decision maker are aggregated into a single optimization problem formulation. In HEIM (for single decision makers), Eqs. (1) and (3) are used as the constraints. However, if the constraints from each group member preference in Step 3 were placed together into one optimization problem, there would most likely be no feasible solution. This is because it is not likely that all members of the group would agree on all their assessments. For example, if one group member preferred alternative A^1 over alternative A^2 and another preferred A^2 over alternative A^1 , this would result in two constraints of the form given in Eq. (3). However, these constraints would be the reverse of each other and would prevent a feasible solution from being found.

In fact, Arrow's Theorem demonstrates that consistency among a group can not be guaranteed [28]. In practice, it is rare for every member of a product development group to have exactly the same preferences as well. Some common group decision methods have effectively aggregated group preferences to avoid the limitation by limiting the freedom of individuals in the group [6-9,29-30]. Group-HEIM instead allows individuals to freely express preferences over a number of HA's and then explores the level of conflict or inconsistency in the aggregated group preferences. The Group-HEIM approach for group decision making does not propose a way to circumvent Arrow's Theorem. Instead, Group-HEIM acknowledges it and uses *compromise variables* to identify and minimize the conflict in the group.

Based on the least-distance approximation method [31], Group-HEIM extends the single decision maker formulation in HEIM by adding variables into the constraints in Eq. (1) and Eq. (3). These variables, called *compromise variables*, are used to identify the conflicts in preferences among group members. The basic formulation is shown in Eq. (4).

$$\text{Minimize} \quad \sum_{\{>\}} (x_{jk})^p + \sum_{\{\sim\}} |z_{st}|^p \quad (4)$$

$$\text{Subject to} \quad V(A^j) - V(A^k) + x_{jk} \geq \delta$$

For all inequality preferences

$$V(A^s) - V(A^t) + z_{st} = 0$$

For all equality preferences

$$\sum w_i = 1$$

$$\text{Side constraints:} \quad w_i \geq 0, \quad x_{jk} \geq 0$$

where i is the number of attributes, p is an integer, δ is an arbitrarily small constant, r_i^j is the rating of alternative A^j on

attribute i , $\{>\}$ is the set of inequality preferences, $\{\sim\}$ is the set of equality preferences, x_{jk} is the compromise variable for inequality preference of alternatives A^j and A^k , z_{st} is the compromise variable for the equality preference of alternatives A^s and A^t , and $V(A^j)$ is the value of alternative A^j . These compromise variables are both calculated and minimized, since they appear in the objective function in Eq. (4).

Different value function formulations have been applied to decision based design [16,20,24,25]; nevertheless, we can expect different outcomes based on the different value assessments since the process used to make the decision influences the outcome 97% of the time when there are at least six alternatives [26]. In addition, some misconceived limitations to utility analysis in the context of engineering design could also be expected [27]. In previous implementations of Group-HEIM, a L_1 -norm additive value function is used. However, there may be instances where a multiplicative form or another norm is more appropriate and effective. One of the contributions of this paper is to investigate the effect of different value functions on the decision outcome in the group formulation.

An L_1 -norm additive value function, or Weighted-Sum Method, has been used in previous work to find the solution in multi-dimensional space [5]. Although the basic formulation in Eq. (2) has shown to be effective in solving convex problems [13], there is no guarantee that the set of product alternatives in a design selection problem form a convex space. As has been well documented in [13-15], if the nondominated set of alternatives in the performance space is not convex, there is no guarantee that all the nondominated alternatives can be found using an L_1 -norm. In Fig. 4a), the nondominated set of discrete alternatives appear to be convex, while the set in Fig. 4b) is not convex. The L_1 -norm would not be able to identify alternatives C or D in Figure 4b). Therefore, in this paper, we investigate the effect of different value function formulations on the final selected alternative.

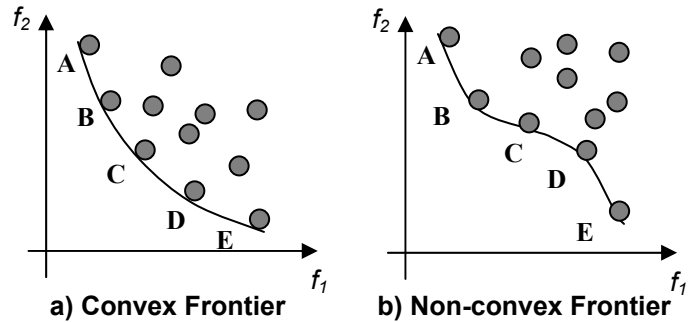


Figure 4 Possible Product Alternatives

The purpose of the objective function in Eq. (4) is to minimize the compromise variables, or the amount of conflict in the group's preferences. The compromise variables, x_{jk} and z_{st} , in the least-distance approximation are utilized to ensure that equality and inequality preferences are satisfied. For instance, in the case of a conflicting preference, the compromise variable will be nonzero to ensure the inequality preference is always greater than zero. Also observe that the

objective function in Eq. (4) is a convex space; thus, the constraint of sum of the weights is equal to 1 is needed to avoid the trivial solutions.

In general terms, the objective is to minimize the level of inconsistency in the set of group preference constraints. Eq. (4) provides a unique and single solution even when conflict occurs in the preference structures. If all the preferences are consistent and can all be satisfied, all the compromise variables will be equal to zero. If a set of preferences is conflicting, the corresponding compromise variables in Eq. (4) will be non-zero.

Step 5 Solve for the Attribute Weights. Depending on the value of p shown in Eq. (4), various optimization methods can be applied to solve the formulation. For example, if $p = 1$, then the formulation is a linear problem and can be solved using linear programming. In this work, we use $p = 2$, and the optimization solver in Excel (based on the GRG method).

Step 6 Evaluate the Solution and Make Decision. Once the attribute weights are known, the chosen value function from Step 4 can be used to calculate the overall value of the actual alternatives. The decision is then made based on the alternative with the highest value.

Another advantage of Group-HEIM is that it allows a group to take a number of actions based on this initial decision. First, Group-HEIM has the ability to identify conflicting preferences or inconsistency among the group members. The group has the option to go back and focus on these preferences and attempt to get consensus on them. Previous work in [5] studied the value of performing this action.

Second, a group leader can decide to discount or elevate the opinion of certain members of the group based on their experience, their education, or their contribution to the team thus far. Being able to place more emphasis or importance on certain group members' opinions is not only necessary when some group members may have valuable experience and reliable insight on a certain product line, but may also help speed up a design process and improve the probability of success of a product [1]. We accomplish the emphasis of certain group members directly in the optimization formulation of Eq. (4) by adding constraints that limit the compromise of a certain group member to be less than another.

Graphically, the effect of these new constraints is shown in Figure 5. In the figure, the basic idea of Group-HEIM is illustrated, where three constraints representing three group members are shown in the attribute weight space. There is no feasible combination of w_1 and w_2 (the hatched sides of the constraints represent infeasibility). The objective of Group-HEIM is to minimize the amount of conflict between group members by identifying a compromise solution even when the preferences among the group members are conflicting. In Figure 5, this compromise solution is shown in the center where the x_1 , x_2 , and x_3 variables are the compromise variables in Eq. (4). Since all the group members are equal, this solution would be centered, and would not favor any one group member. However, by introducing the additional constraints in Eq. (5), a team leader can shift the solution to favor the contribution of one group member over another. For instance, assume group member 1 (represented by constraint g_1) is a senior member of the team with many years of experience in product design with the company. If the team leader wants to consider the opinion

of group member 1 more than the other members, the solution in Figure 5 will tend to move towards g_1 , indicating that the group member is compromising *less than* the other group members.

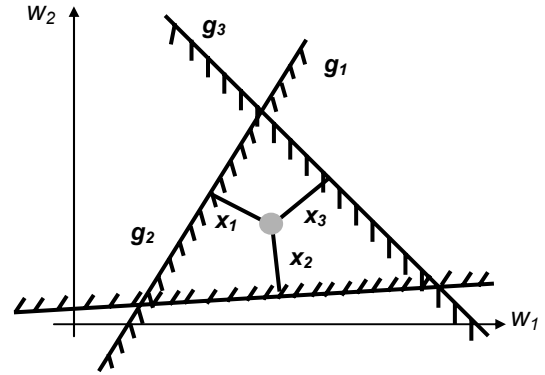


Figure 5 Graphical Representation of Group-HEIM

With the basic incentive behind the approach shown in Figure 5, additional constraints can be added to Eq. (4) to account for unequal importances among group members. For instance, if a group leader determines that designer n is more important than designer m (e.g., by using the number of years of experience with the company, or any other reasonable metric), the formulation is updated as shown in Eq. (5). In Eq. (5), the additional constraint that the sum of all compromise variables for designer n , $\sum x_{jk}|_n$, is less than the sum of all compromise variables for designer m , $\sum x_{jk}|_m$, which implies that designer n is more important than designer m . In other words, the solution should compromise the preferences of designer n *less than* those of designer m .

$$\text{Minimize } \sum_{\{j\}} (x_{jk})^p + \sum_{\{s\}} |z_{st}|^p \quad (5)$$

$$\text{Subject to } V(A^j) - V(A^k) + x_{jk} \geq \delta$$

For all inequality preferences

$$V(A^s) - V(A^t) + z_{st} = 0$$

For all equality preferences

$$\sum x_{jk}|_m - \sum x_{jk}|_n \geq \delta$$

For all unequal group members

$$\sum w_i = 1$$

$$\text{Side constraints: } w_i \geq 0, x_{jk} \geq 0$$

Note that only different between Eq. (5) and Eq. (4) is the added constraint for all unequal group members. Explanation of the other terms in Eq. (5) is given in Eq. (4).

With the given understanding of the basics of the Group-HEIM process, including the new developments emphasized in this paper, in the next section we exercise the approach. We focus on studying the results from the two primary developments of this paper: investigating the effects of different value function formulations, and being able to weight the preferences of each group member differently.

3 EXERCISING THE APPROACH

In this section, a case study is used to exercise the group formulation in the context of the two primary developments of this paper. A brief overview of the case study is given first, followed by a study of each area.

The case study is the vehicle case study, first reported in [5]. It consists of eight concept cars in the small sedan vehicle segment being evaluated using five attributes. The eight alternatives and their attribute levels are shown in Table 1. This problem is simplistic and is not realistic in terms of how car manufacturers select vehicle designs. It is rather meant to illustrate the contributions of this work.

Unlike the previous use of the study in [5], in this work, we exercise the developments using twelve actual design groups consisting of four to six mechanical and aerospace senior engineering students each. Each group is given the same alternative and attribute information (Table 1), and they follow the steps described in Section 2.2. An automated MS-Excel interface is used by the individuals in the groups to determine their strength of preference functions using lottery questions. The Excel algorithm then creates hypothetical alternatives based on the responses, elicits the individual preferences over the HAs, and solves for the attribute weights using the Excel Solver. We use the group responses to study the effect of different value aggregation functions and the effect of imposing unequal importances to the group member preferences.

| Alternative | Attributes and Relative Weights | | | | |
|-------------|---------------------------------|----------------|----------------|----------------|----------------|
| | w ₁ | w ₂ | w ₃ | w ₄ | w ₅ |
| Car #1 | 2.0 | 145 | 36 | \$12,906 | 8.6 |
| Car #2 | 1.7 | 127 | 38 | \$13,470 | 10.5 |
| Car #3 | 2.2 | 140 | 33 | \$11,995 | 9.9 |
| Car #4 | 1.8 | 130 | 40 | \$13,065 | 9.5 |
| Car #5 | 2.0 | 132 | 36 | \$12,917 | 10.0 |
| Car #6 | 2.0 | 130 | 31 | \$13,315 | 10.4 |
| Car #7 | 2.2 | 140 | 33 | \$13,884 | 7.9 |
| Car #8 | 2.0 | 135 | 33 | \$12,781 | 9.8 |

Table 1 Attribute Data for Vehicle Alternatives

3.1 Value Function Study

The simplest value function aggregation is the L₁-norm representation, as shown in Eq. (2). In the previous implementations of HEIM and Group-HEIM, this form was used. However, in this paper we investigate the impact of using other aggregation functions, including the L₂-norm shown in Eq. (6) [32], and the aggregation function based on the Method of Imprecision, shown in Eq. (7) [33].

$$V(A^j) = \sqrt{\sum_{i=1}^n (w_i r_i^j)^2} \quad (6)$$

$$P_s(\alpha_1, \alpha_2; \omega_1, \omega_2) = \left(\frac{\omega_1 \alpha_1^s + \omega_2 \alpha_2^s}{\omega_1 + \omega_2} \right)^{\frac{1}{s}} \quad (7)$$

In Eq. (7), P is the aggregation operators, α_1, α_2 are individual preference values (or single attribute utility values) to be aggregated, ω_1, ω_2 are the attribute weights corresponding with the preference values, and s can be interpreted as a measure of level of compensation, or trade-off strategy. Higher values of s indicate a greater willingness to allow high preference for one criterion to compensate for lower values of

another. As reported in [34], the aggregation function of Eq. (7) satisfies a set of axioms that an aggregation function, appropriate for rational design decision making, must obey.

The resulting weights for the 12 design groups, their chosen vehicle, and the overall value of the winning vehicle are shown in Table 2 for the L₁-norm value function. Note that the L₁-norm representation is used to find the value in the final column, as well as the values in the Group-HEIM formulation given in Eq. (4) in order solve for the attribute weights.

| Group | Attribute Weights | | | | | Car # | Value |
|-------|-------------------|----------------|----------------|----------------|----------------|-------|--------|
| | w ₁ | w ₂ | w ₃ | w ₄ | w ₅ | | |
| 1 | 0.083 | 0.147 | 0.228 | 0.403 | 0.138 | 3 | 0.6752 |
| 2 | 0.157 | 0.178 | 0.299 | 0.331 | 0.036 | 3 | 0.6905 |
| 3 | 0.046 | 0.146 | 0.268 | 0.337 | 0.204 | 3 | 0.5940 |
| 4 | 0.156 | 0.187 | 0.308 | 0.258 | 0.091 | 3 | 0.6385 |
| 5 | 0.083 | 0.147 | 0.228 | 0.403 | 0.138 | 3 | 0.6752 |
| 6 | 0.083 | 0.147 | 0.228 | 0.403 | 0.138 | 3 | 0.6752 |
| 7 | 0.101 | 0.166 | 0.277 | 0.299 | 0.157 | 3 | 0.6175 |
| 8 | 0.036 | 0.157 | 0.319 | 0.238 | 0.251 | 1 | 0.5962 |
| 9 | 0.108 | 0.151 | 0.272 | 0.434 | 0.035 | 3 | 0.7196 |
| 10 | 0.084 | 0.132 | 0.198 | 0.458 | 0.127 | 3 | 0.7111 |
| 11 | 0.083 | 0.147 | 0.228 | 0.403 | 0.138 | 3 | 0.6752 |
| 12 | 0.237 | 0.164 | 0.326 | 0.225 | 0.048 | 3 | 0.6640 |

Table 2 Relative Attribute Weights and Selected Vehicle - L₁ Norm

Table 2 shows that the most preferred car for 11 of the 12 groups was vehicle #3. The weights in bold for each group show the most important attribute. Nine groups indicated vehicle price as the most important attribute. Four groups have the exact same relative attribute weights, which indicates that the group members have the same or very close to the same preference structures between the groups.

To compare with the results from Table 2, the results when the L₂-norm is used are shown in Table 3. The L₂-norm is a common approach to finding nondominated solutions in nonconvex spaces. The exact same preference structures for all group members were used. The values in Eq. (4) and the last column in Table 3 were calculated using Eq. (6). The attribute weight values for each group have changed, as would be expected, since the constraints in Eq. (4) were altered slightly with the new value function. Eight groups now indicate that vehicle price is the most important attribute. The most preferred vehicle has stayed the same for every group except for Group 3, which switched from vehicle #3 to #1.

| Group | Attribute Weights | | | | | Car # | Value |
|-------|-------------------|----------------|----------------|----------------|----------------|-------|--------|
| | w ₁ | w ₂ | w ₃ | w ₄ | w ₅ | | |
| 1 | 0.178 | 0.191 | 0.199 | 0.249 | 0.184 | 3 | 0.6513 |
| 2 | 0.194 | 0.194 | 0.220 | 0.235 | 0.157 | 3 | 0.6538 |
| 3 | 0.141 | 0.189 | 0.229 | 0.233 | 0.208 | 1 | 0.6093 |
| 4 | 0.189 | 0.192 | 0.227 | 0.217 | 0.176 | 3 | 0.6353 |
| 5 | 0.178 | 0.191 | 0.199 | 0.249 | 0.184 | 3 | 0.6513 |
| 6 | 0.178 | 0.191 | 0.199 | 0.249 | 0.184 | 3 | 0.6513 |
| 7 | 0.183 | 0.199 | 0.214 | 0.213 | 0.191 | 3 | 0.6315 |
| 8 | 0.127 | 0.184 | 0.245 | 0.235 | 0.209 | 1 | 0.6063 |
| 9 | 0.141 | 0.154 | 0.255 | 0.294 | 0.157 | 3 | 0.6384 |
| 10 | 0.133 | 0.151 | 0.172 | 0.389 | 0.155 | 3 | 0.7052 |
| 11 | 0.178 | 0.191 | 0.199 | 0.249 | 0.184 | 3 | 0.6513 |
| 12 | 0.205 | 0.191 | 0.232 | 0.210 | 0.161 | 3 | 0.6421 |

Table 3 Relative Attribute Weights and Selected Vehicle - L₂ Norm

Investigating this a bit further, in Table 4, the difference in values for vehicles #3 and #1 are shown for Group 3 using both norms. When the L_1 -norm additive model is used, the value difference between the first and second ranked car is 5.3%. However, when the L_2 -norm is used, the difference is only 0.02%. Therefore, by comparing the differences in the values, vehicle #3 could still be considered as the most preferred car for Group 3 since the difference in the values is so small. This observation, nevertheless, suggests that perhaps the top two or three alternatives should be kept under consideration in the design process until more information can clearly distinguish them.

| Rank | L_1 -norm Model | | L_2 -norm Method | |
|--------|-------------------|--------|--------------------|--------|
| | Vehicle | Value | Vehicle | Value |
| First | #3 | 0.5840 | #1 | 0.6093 |
| Second | #1 | 0.5528 | #3 | 0.6092 |
| | Difference | 5.3% | Difference | 0.02% |

Table 4 The Decision of Group 3 Using Different Value Functions

Next, the aggregation function in Eq. (7) is used as the value representation for each hypothetical and actual alternative. It is argued in [33] that values of $s \leq 0$ are more appropriate for design because they satisfy the annihilation axiom, which says that if the preference for any one attribute of an alternative is zero, then the value of the entire alternative is zero. However, in our studies with over one hundred engineering students, engineers, and engineering managers, very few, if any, of them attribute a value of zero to an alternative that has the poorest performance on one attribute. Therefore, values of $s \leq 0$ that satisfy the annihilation axiom do not seem appropriate for implementations of HEIM and Group-HEIM, but current studies are examining the theoretical foundations of this observation. As a result, a value of $s=5$ is used in this study, which as noted in [33], behaves closer to a max operator, where the attributes that have high fulfillment are given more importance. Table 5 shows the results of using Eq. (7) with $s=5$.

| Group | Attribute Weights | | | | | Car # | Value |
|-------|-------------------|--------------|--------------|--------------|-------|-------|--------|
| | w_1 | w_2 | w_3 | w_4 | w_5 | | |
| 1 | 0.211 | 0.239 | 0.180 | 0.152 | 0.218 | 7 | 0.8620 |
| 2 | 0.156 | 0.132 | 0.236 | 0.378 | 0.098 | 3 | 0.8905 |
| 3 | 0.143 | 0.210 | 0.221 | 0.213 | 0.212 | 3 | 0.8318 |
| 4 | 0.207 | 0.219 | 0.221 | 0.164 | 0.189 | 7 | 0.8480 |
| 5 | 0.211 | 0.239 | 0.180 | 0.152 | 0.218 | 7 | 0.8620 |
| 6 | 0.211 | 0.239 | 0.180 | 0.152 | 0.218 | 7 | 0.8620 |
| 7 | 0.211 | 0.253 | 0.188 | 0.112 | 0.236 | 7 | 0.8696 |
| 8 | 0.000 | 0.001 | 0.330 | 0.657 | 0.012 | 3 | 0.9196 |
| 9 | 0.000 | 0.000 | 0.330 | 0.660 | 0.010 | 3 | 0.9204 |
| 10 | 0.002 | 0.002 | 0.029 | 0.963 | 0.003 | 3 | 0.9931 |
| 11 | 0.211 | 0.239 | 0.180 | 0.152 | 0.218 | 7 | 0.8620 |
| 12 | 0.214 | 0.217 | 0.227 | 0.168 | 0.175 | 7 | 0.8452 |

Table 5 Relative Attribute Weights and Selected Vehicle – Method of Imprecision ($s=5$)

Compared to the results with the L_1 -norm and L_2 -norms, the results in Table 5 demonstrate the effect of the aggregation function of Eq. (7) with $s=5$. In five of the groups, Car 3 is chosen as the most preferred vehicle, which is not that much

unlike the results in Tables 2 and 3. Indeed, Car 3 is the cheapest and has the largest engine, supporting the effect of the max operator. However, the other seven groups choose Car 7, which is not identified by any group in Tables 2 and 3. From Table 1, Car 7 has the largest engine (tied with Car 3) and by a significant margin, the best acceleration. Therefore, the aggregation formulation of Eq. (7) using $s=5$ identifies the alternatives that perform the best on as many of the attributes as possible (Cars 3 and 7 each are the best in 2 of the 5 attributes). As further evidence of the effect of the max operator, the values of the winning vehicles for each group in Table 5 (final column) are much larger than the winning values in Tables 2 and 3. This indicates that the aggregation approach is placing great importance and value on alternatives that are the best in each attribute, increasing their score by multiplying a large attribute weight times a high attribute fulfillment.

The choice of $s=5$ is somewhat arbitrary, other than keeping it positive to avoid satisfying the annihilation condition. In [35], “indifference points” are used to effectively identify the most appropriate compensation strategy, s . However, finding two designs that are exactly of equivalent value to a decision maker can be a challenging and time-consuming task [27], specifically in the context of constructing utility functions. Therefore, as part of the ongoing studies, the concept of using hypothetical inequivalents or “difference points” to determine the value of s will be considered.

3.2 Discussion

For the eleven groups that chose vehicle #3 in Table 2, there were eight different sets of weights. Six of these eight sets had price as the most important attribute, while two had miles-per-gallon as the most important attribute. This consistency in the final selection, even with differences in the attribute weights and importances can be explained by a simple two dimensional representation shown in Figure 6. The black circles represent actual nondominated vehicles and their aggregation is represented by the thick line, approximating the nondominated set of vehicles (this is hypothetical and only meant to explain the results from Table 2).

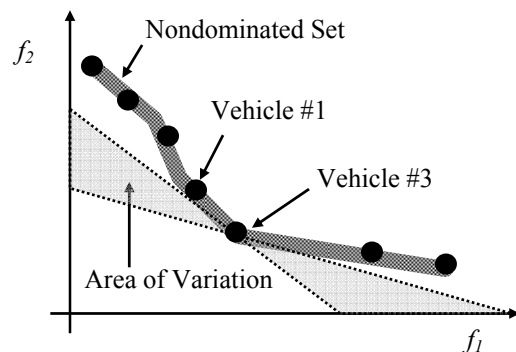


Figure 6 Variation in Weights

It is well known that using an L_1 -norm in two dimensions is equivalent to projecting a line with a given slope (determined by a ratio of the attribute weights) towards the nondominated set until a solution from the set is contacted. This is depicted in this figure for two different lines, representing a range of possible attribute weights. As long as the weights are within this range, the most preferred vehicle will be identified as

vehicle #3, as shown in the figure. Also evident is vehicle #1 which is close to vehicle #3. If the weights were to change slightly, then vehicle #1 may be identified as the most preferred alternative (as indicated by group 3). Therefore, the groups can have different sets of weights and even different most important attributes, but yet still identify the same alternative.

The L_2 -norm method, in contrast with the L_1 -norm employs a circle centered at the Utopia point (hypothetical best point) to project until it contacts the nondominated set. Similar to the L_1 -norm case where a range of weights can translate to the same solution, a range of weights for the L_2 -norm can also translate to the same identified solution. In Figure 7, the same set of possible solutions as in Figure 6 is shown. However, now the geometric interpretation of the L_2 -norm is shown. In the left-most plot, the f_1 axis is scaled by a factor n . In this case, vehicle #3 is identified as the preferred solution since the circle intersects this point first. In the right-most plot, the f_1 axis is now scaled by a factor m . In this case, vehicle #3 is also identified as the preferred solution. This range of scaling factors from n to m indicates the range of relative weights for f_1 and f_2 that result in vehicle #3 as the preferred solution. This explains why 10 of the 12 groups in Table 3 even though seven different sets of weights are used by the groups.

In Figure 8, an illustration is given for the identified vehicle by Group 3 using the L_2 -norm. While vehicle #1 was identified as the preferred vehicle, as shown in Table 3, vehicle #3 is a very close second. In Figure 8, the f_1 axis is scaled by a factor k , which results in the circle contacting both vehicles at the same time, which is what the numerical results indicate. In this case, other considerations must be used to select the best alternative.

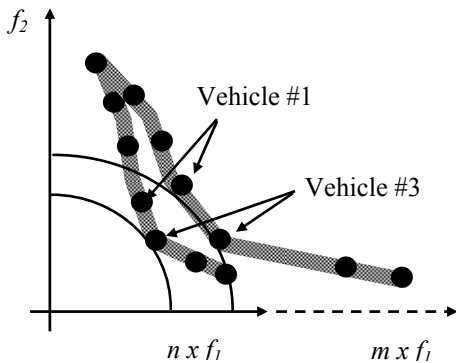


Figure 7 Rescaling the Pareto Set with L_2 Norm Method

The results shown thus far have treated each group member equally. However, rarely does a design team contain group members with equal experience, education, knowledge, and contribution. This may make one group member's insight more valuable or more reliable than another. In the next section, we present an updated formulation for Group-HEIM that takes into account the difference in group members and allows a team leader to effectively "weight" certain group members whose opinions may be more valuable or reliable because of their experience with the product in question.

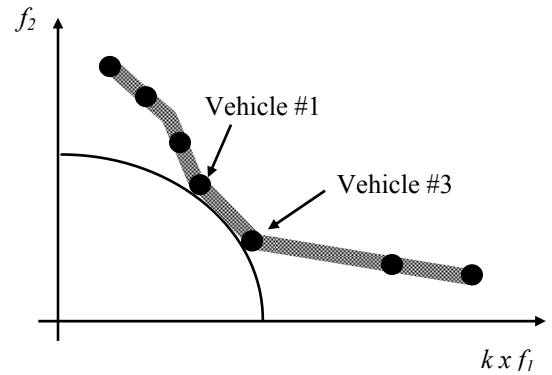


Figure 8 Group 3 Illustrations

3.3. Relative Importance of Group Members

The same vehicle design example, as described in Section 3.1 is used to demonstrate the Group-HEIM formulation for unequal group members. To demonstrate the application of Eq. (5), only group 1 is used to illustrate the approach. There were 4 members of group 1, and Table 6 shows the preference structures for each group member across the sets of hypothetical alternatives. There were 4 sets of 3 hypothetical alternatives generated using a D-Optimal design. The set of hypothetical alternatives for designer #1 are shown in Table 7. The other group members do not necessarily have the same hypothetical alternatives, since their strength of preferences assessments (Step 2) are most likely different from designer #1. The hypothetical alternatives are developed based on the strength of preference assessments (see [3] for more detail on this development).

| Designer | Preference Structures | | | |
|----------|-----------------------|-----------|-----------|-----------|
| #1 | C > B > A | E > D > F | I > H > G | K > L > J |
| #2 | C > A > B | E > D > F | H > I > G | J > L > K |
| #3 | C > B > A | E > D > F | G > I > H | L > K > J |
| #4 | C > B > A | E > F > D | H > I > G | J > K > L |

Table 6 Preference Structures for the Group Members

| HA | Engine (liters) | Horsepower (hp) | MPG | Price (\$) | Acc. (0-60mph) |
|----|-----------------|-----------------|------|------------|----------------|
| A | 2 | 137.8 | 34.6 | \$13,128 | 10.5 |
| B | 2.2 | 137.8 | 40 | \$13,884 | 10.5 |
| C | 2.2 | 145 | 40 | \$13,128 | 7.9 |
| D | 2 | 127 | 40 | \$13,884 | 7.9 |
| E | 2.2 | 137.8 | 34.6 | \$11,995 | 7.9 |
| F | 2.2 | 127 | 31 | \$13,128 | 10.5 |
| G | 2 | 145 | 31 | \$11,995 | 8.9 |
| H | 1.7 | 137.8 | 40 | \$13,128 | 8.9 |
| I | 1.7 | 127 | 40 | \$11,995 | 10.5 |
| J | 1.7 | 145 | 34.6 | \$13,884 | 10.5 |
| K | 2.2 | 127 | 34.6 | \$13,884 | 8.9 |
| L | 1.7 | 137.8 | 31 | \$13,884 | 7.9 |

Table 7 Hypothetical Alternatives for Designer 1, Group 1

Each group member has eight compromise variables corresponding to the eight unique constraints created by the

preferences in Table 6. These are shown in Table 8. We can use these variables to study the effect of making the group member importances unequal.

| Designer | Compromised Variables | | | | | | | |
|----------|-----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| #1 | X _{CB} | X _{BA} | X _{ED} | X _{DF} | X _{IH} | X _{HG} | X _{KL} | X _{LJ} |
| #2 | X _{CA} | X _{AB} | X _{ED} | X _{DF} | X _{HI} | X _{IG} | X _{JL} | X _{LK} |
| #3 | X _{CB} | X _{BA} | X _{ED} | X _{DF} | X _{GI} | X _{IH} | X _{LK} | X _{KJ} |
| #4 | X _{CB} | X _{BA} | X _{EF} | X _{FD} | X _{HI} | X _{IG} | X _{JK} | X _{KL} |

Table 8 Compromised Variables for Each Group Member

In this case, assume the opinion of designer #1 is the most important, the opinion of designer #2 is the second most important, the opinion of designer #3 is the third most important, and the opinion of designer #4 is the least important. In no way are we promoting treating people unequally, but are simply acknowledging that it may be beneficial to consider the technical insight of certain team members (due to experience, education, or other metric) more than others.

The additional constraints necessary to model these importances are shown in Eq. (8) where $\delta = 0.001$ to ensure inequality. The first constraint implies that the sum of the compromise variables (as shown in Table 8) for designer #1 must be *less than* the sum of the compromised variables for designer #2, indicating that the preferences for designer #1 should be adhered to more than the preferences of designer #2. The basic group formulation is updated with these constraints and solved again (using the L_1 -norm).

$$\begin{aligned}
 \sum x_{jk} \Big|_{\text{Designer\#2}} - \sum x_{jk} \Big|_{\text{Designer\#1}} &\geq \delta \\
 \sum x_{jk} \Big|_{\text{Designer\#3}} - \sum x_{jk} \Big|_{\text{Designer\#2}} &\geq \delta \\
 \sum x_{jk} \Big|_{\text{Designer\#4}} - \sum x_{jk} \Big|_{\text{Designer\#3}} &\geq \delta
 \end{aligned} \quad (8)$$

The original set of weights with equal group members (as shown in Table 3) are as follows.

$$\bar{w} = [0.083, 0.147, 0.228, 0.403, 0.138]^T$$

The chosen vehicle for the group is vehicle #3. With the integration of the relative importance constraints from Eq. (8), the new attribute weights are as follows.

$$\bar{w} = [0.090, 0.136, 0.232, 0.398, 0.144]^T$$

The chosen vehicle for the group is again #3. The same vehicle was found with the two different sets of weights for reasons discussed in Sec. 3.2. However, what can be noted is that the solution has shifted in favor of designer #1. One way to validate this is to compare the sum of the compromise variables for the designers from the equal and unequal group member formulations. Table 9 shows the sum of the compromise variables for each group member. The results illustrate that the sum of the compromise variables for designer #1 decreased from the original formulation. In addition, designer #1 now has the smallest amount of compromise among all the designers, followed by designer #2, designer #3, and lastly designer #4. This matches the intent of the constraints formulated in Eq. (8). By placing a priority on the

information from designer #1, Table 9 also clearly shows that the other three designers must compromise their preferences more than the original formulation.

| Designer | Sum of Compromise Values | |
|----------|--------------------------|-----------------------------|
| | Original Solution | Solution w/ Unequal Members |
| #1 | 0.1835 | 0.1578 |
| #2 | 0.1435 | 0.1579 |
| #3 | 0.1009 | 0.1580 |
| #4 | 0.2640 | 0.2758 |

Table 9 Comparison of the Compromise Variables

Another form of validation is to study how the set of attribute weights changes to better accommodate the preferences of designer #1. The weight for horsepower (the second weight) decreases the most indicating that designer #1 may not really place too much emphasis on horsepower. Investigating the hypothetical alternatives in Table 7 and the preference structure in Table 6, the *most preferred* alternatives in the third and fourth sets (I and K, respectively) both have the lowest horsepower available, clearly indicating its lack of importance to designer #1. On the other hand, the weight for engine size (the first weight) increases the most, indicating that designer #1 places seems to emphasize the engine size in their choices. Indeed, three of the four most preferred alternatives (C, E, and K) have the largest engine size of their set. Similar explanations can be offered for the other changes in attribute weights as well. A number of general observations and conclusions are made in the next section.

4 OBSERVATIONS AND CONCLUSIONS

While Group-HEIM was introduced in [5], this paper has extended and improved Group-HEIM in two important areas. First, it develops the basic foundation for using value aggregation approaches other than the common L_1 -norm. Second, it establishes an effective formulation for being able to emphasize the opinions of certain group members.

While the new constraints in Eq. (5) set up a lexicographic priority structure among group members, additional constraints could be used to numerically specify the relative importance of group members, for instance, the ratio of $\sum x_{jk}$ among group members. The approach avoids having to assign weights to group members, which would create a number of implementation and solution challenges.

These important improvements to Group-HEIM help establish its foundations as an easy-to-use, sound approach to supporting decision making when groups of engineers or managers are involved. In related work, a software program is being developed to automate many of the steps in Group-HEIM. As mentioned previously, the twelve experimental groups completed the decision process using an automated MS-Excel interface. A significant advantage is that all the preferences of the group members are captured without having to meet collectively. Therefore, when the group does meet, they can devote their time to discussing the conflicts that have been identified and build consensus. The decision maker's primary responsibility as an individual is to create accurate strength of preference functions (single attribute utility functions) for each attribute and then to state their preferences

over pairs of hypothetical alternatives. Beyond this, everything else can be automatically processed using the underlying mathematical foundation of Group-HEIM.

Current work includes investigating the transitivity of each group member. Group transitivity is rare and can not be guaranteed, but individual transitivity should be established before applying Group-HEIM or any other decision support tool. If decision makers are inconsistent with their preferences, then the validity and accuracy of the resulting decision can not be trusted. Also, current work is focused on studying the appropriate number of constraints necessary in the group formulation. Too many constraints may result in a solution that is largely unusable, while too few constraints may result in an underconstrained problem with no meaningful solution.

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