

*Dynamics of Continuous, Discrete and Impulsive Systems*  
*Series B: Applications & Algorithms* **19** (2012) 377-403  
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## RISK-INDUCED BACKWARD BIFURCATION IN *HSV-2* TRANSMISSION DYNAMICS

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**Abstract.** A risk-structured, two-sex, model for the transmission dynamics of herpes simplex virus type 2 (HSV-2) in a population is designed and qualitatively analysed. It is shown that adding risk structure (i.e., the risk of transmitting or acquiring HSV-2 infection) to an HSV-2 transmission model causes the phenomenon of backward bifurcation when the associated reproduction threshold is less than unity. This dynamical feature, which has non-trivial consequence on the persistence or elimination of the disease (when the reproduction threshold is less than unity), can be removed if the susceptible male and female sub populations are not stratified according to the risk of acquiring HSV-2 infection.

**Keywords.** HSV-2; low- and high-risk groups; equilibria; stability; backward bifurcation.

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Received April 2010; revised October 2011.

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