odic regime of pressure fluctuations could conceivably occur when the wavelength of the resulting ice structure is of the order of the tube length.

Acknowledgment

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Solidification in Two-Phase Flow

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Nomenclature

a = crust thermal diffusivity
A = heat flux parameter
B = latent heat parameter
c = crust heat capacity
h = convective heat transfer coefficient
j = superficial gas velocity
k = crust thermal conductivity
L = latent heat of fusion
q = convective heat flux
T_c = methanol coolant temperature (−70°C)
T_ℓ = liquid temperature
T_{MP} = fusion temperature
T_W = wall temperature
t = time
α = void fraction
δ = crust thickness
δ_{ss} = steady-state crust thickness
δ_w = wall thickness (0.238 cm)
τ = dimensionless time
Δ = dimensionless crust thickness
θ = dimensionless wall temperature

1 Introduction

If, in the process of solidification, the liquid phase contains a gas in solution, the gas will be rejected at the phase interface due to the difference in solubility of the gas in the liquid and solid. The evolution of the gas may result in the entrapment of gas bubbles in the solid phase by the advancing solidification front [1]. The resulting porosity alters the properties of the solid [2]. If gas is present in the body of the liquid as a discontinuous phase, it is not clear at present what effect the two-phase mixture will have on the solidification process. The problem of predicting solidification rates in a two-phase mixture arises in safety studies for the fast breeder reactor [3] and in the design of air bubblers as ice control devices for cold-water ports and harbors [4, 5].

A study of the transient solidification of a Wood's metal-nitrogen gas mixture in a cold tube was performed by Greene, et al. [6, 7]. The authors postulated the entrapment of nitrogen gas bubbles within the solid phase and concluded from their experimental results that the rate of solidification may be several times faster for the two-phase case than for the corresponding single phase case.

This paper presents the results of an experiment designed to measure directly the growth of an ice layer (crust) in a water-nitrogen gas mixture.

2 Experiment

A planar test section on which ice crusts were grown was vertically suspended in a pool of water contained within a Lucite bubble column of square cross-section (5.08 cm by 5.08 cm). Nitrogen gas bubbles were formed at a perforated plate located at the bottom of the column. The pressure in the column was atmospheric. A schematic of the apparatus is shown in Fig. 1. The test section was constructed from a copper block 7.62 cm in length and 0.635 cm thick. A serpentine coolant channel was milled into one face of the block. The freezing surface was 0.238 cm thick. Thermocouples (±1°C) were used to measure the test section temperature and were located in the freeze plate 0.2 cm from the freezing surface. The entire test section except for the freeze surface was cast in epoxy material to insulate the edges and back of the test section. The coolant supply line, connecting the coolant reservoir with the test section, was insulated with rubber tubing. A lateral-traversing thermocouple probe (not shown in Fig. 1), similar to that developed by Savino and Siegel [8], was used to measure the instantaneous ice crust thickness as a function of time.

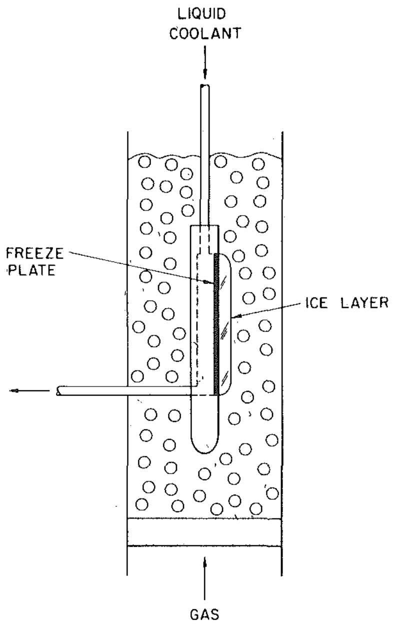

Fig. 1 Schematic diagram of the test section showing an ice layer growing on the freeze plate in an ice-water mixture

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[3] Void fraction was calculated as the difference between the height of the expanded pool and the initial height of the pool with zero gas flow divided by the expanded pool height.

The thermocouple probe was constructed by mounting a 30 gauge chromel-alumel thermocouple within a 13 cm long, 0.32 cm o.d. tube. The bare-bead junction, extending 1.0 cm beyond the tube, was trimmed to minimize the response time, flattened slightly, and supported by epoxy. The lateral displacement of the probe from the freezing surface was measured using a micrometer (±0.005 cm). During an experiment, the probe was initially located at a distance from the plate equal to the selected displacement increment used in subsequent measurements. The probe temperature was recorded on a strip-chart recorder. When the ice crust contacted the probe, as determined by watching the strip-chart recorder, the probe was moved laterally away from the crust a distance equal to the displacement increment. Thus, the strip-chart recording provided a means of determining the times of successive equal increments of crust thickness. A more detailed description of this procedure including representative tracings from the experiments is given in [9]. The water pool temperature was measured with a mercury thermometer (±0.1°C).

Two water pool temperatures were studied; in one case the water was at 0°C (saturated), in the other case the water pool was maintained at 14°C (superheated). The experiments covered a range of nitrogen gas volume (void) fractions from 0 to 90 percent. Void fractions[3] from 0 to 65 percent were obtained by simply bubbling nitrogen gas through the perforated plate at superficial velocities ranging from 0 to 12 cm/s into ordinary tap water. Foams having void fractions between 80 and 90 percent were produced by adding a small quantity of Kodak Photo-Flow solution to the pool in the ratio of about 2 drops to 150 ml of water. The ice growth transient was initiated by suddenly allowing the coolant (methyl alcohol at −70°C) to flow into the test section channel. The plate temperature- and crust thickness-time histories were recorded during the experiment. Over the course of an experiment with saturated water (~1 min.), the freeze plate temperature decreased to approximately −65°C and the resulting instantaneous crust thickness was approximately 0.65 cm. In the experiments with superheated water, the crust thickness grew more slowly in comparison to the experiments with saturated water due to the convective heat transport into the crust from the two-phase mixture. In these latter experiments, the crust thickness achieved a steady-state value after approximately three min.

3 Results and Discussion

The measured (dimensionless) crust thickness as a function of dimensionless time is shown in Fig. 2 for water maintained at 0°C. The data flags in this figure represent the standard deviation of a total of 18 runs in which the void fraction was varied from 0.0 to 90 percent. The coolant flowrate was fixed in the experiment so that the freeze plate experienced approximately the same cooling transient during each run. The freeze plate temperature-time curve can be found in reference [9]. For a given crust thickness, the measured times and void fractions for the 18 runs were subjected to a linear regression analysis. It was found that the slope of the regression line was not statistically different from zero. Thus, it can be concluded that, in these experiments, the void fraction does not determine the time for a given crust thickness to be achieved. The small variation in the measurements can be attributed to the small variations in the coolant flow rate from run to run. In addition, the physical characteristics of the ice crusts retrieved from the bubble column after the experiments did not change over the entire range of void fractions investigated. Thus, solidification of a two-phase structure (solid plus gas), as postulated in references [6] and [7] for the freezing of gas-liquid flows in a tube, was not indicated by the post-test observations.

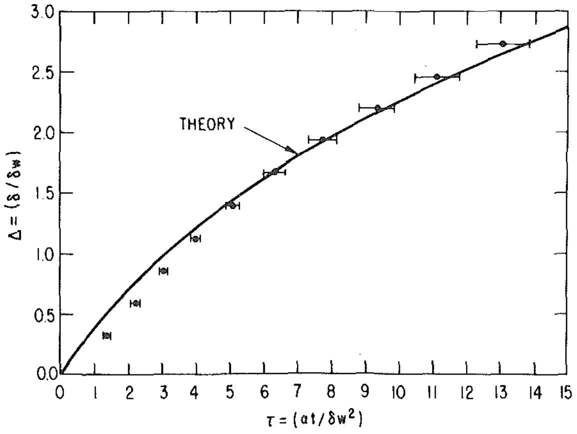

Fig. 2 Dimensionless crust thickness versus dimensionless time for ice growing in a two-phase water-nitrogen gas mixture maintained at 0°C. The data flags represent 18 runs covering the void fraction range 0–90 percent

The dark line in Fig. 2 is based on calculations of ice growth into saturated water in the absence of voids. The theory is based on solving the one-dimensional heat conduction equation in the crust using an accurate collocation technique [10]. The resulting governing equation is

$$\frac{d\Delta}{d\tau} = -\left(\frac{1}{\Delta} + \frac{1}{2}\frac{A}{B}\right) + \left\{\left(\frac{1}{\Delta} - \frac{1}{2}\frac{A}{B}\right)^2 + \frac{2}{B}\frac{\theta}{\Delta^2}\right\}^{1/2} \tag{1}$$

where

$$\theta(\tau) = \frac{T_W(\tau) - T_{MP}}{T_c - T_{MP}} \quad \text{(dimensionless plate temperature)} \tag{2}$$

$$\Delta(\tau) = \frac{\delta(\tau)}{\delta_w} \quad \text{(dimensionless crust thickness)} \tag{3}$$

$$\tau = \frac{at}{\delta_w{}^2} \quad \text{(dimensionless time)} \tag{4}$$

$$A = \frac{q\delta_w}{k(T_{MP} - T_c)} \quad \text{(dimensionless convective heat flux)} \tag{5}$$

$$B = \frac{L}{c(T_{MP} - T_c)} \quad \text{(dimensionless latent heat)} \tag{6}$$

For the case of a saturated liquid, A is set equal to zero. To solve equation (1) the measured wall temperature-time variation was used. The theory is seen to agree favorably with the data. All this strongly implies that the ice growth rate is the same in all runs studied, and is thus independent of the nitrogen gas void fraction, at least up to 90 percent. At early times, the crust thickness is overpredicted by the model probably due to the formation of a metastable, subcooled water layer adjacent to the freezing surface. For low gas void fractions, it was observed that the formation of an ice crust did not occur until the plate temperature decreased several degrees below 0°C and then a crust "flashed" on the surface. Presumably, this phenomenon also occurred at higher void fractions but the presence of the bubbles obscured visual observation.

In the experiments conducted with superheated water at 14°C, a steady-state ice crust was achieved when the conduction heat flux through the crust equaled the convective heat flux to the crust from the two-phase mixture at $\tau \rightarrow \infty$. The measured local crust thickness at the center of the freeze plate as a function of time is shown in Fig. 3 with A as a parameter. It is apparent that the steady-state crust thickness is a function of the superficial gas velocity j, varying from 0.724 cm for $j = 0.1$ cm/s to 0.152 cm for $j = 12.0$ cm/s. Since the steady-state crust thickness, δ_{ss}, is related to the convective heat transfer coefficient by

$$q = h(T_\ell - T_{MP}) = \frac{k(T_{MP} - T_W)}{\delta_{ss}}, \tag{7}$$

it can be concluded that the convective heat transfer coefficient is increased when the superficial gas velocity is increased. This result is in agreement with studies concerned with the enhancement of convective heat transfer in liquids by gas injection [11, 12]. Values of the convective heat transfer coefficient can be calculated from equations (5) and (7) and the values of A given in Fig. 3.

The dark lines in Fig. 3 represent the predictions obtained with equation (1) wherein the convective heat-transfer coefficient h (or dimensionless parameter A) was evaluated using equation (7) together with the measured values of δ_{ss}. This method of determining steady-state heat transfer coefficients from ice crust thickness measurements is well established [13]. Experimental studies involving

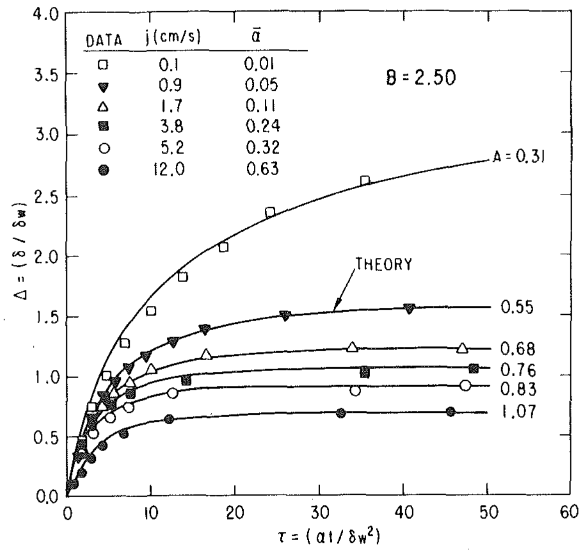

Fig. 3 Dimensionless crust thickness versus dimensionless time for ice growing in a two phase water-nitrogen gas mixture maintained at 14°C, using the superficial gas velocity (or convective heat flux) as a parameter

solidification in pure liquids with forced flow have demonstrated that heat convection is relatively undisturbed by the moving freeze boundary [8, 14]. The good agreement between theory and experiment shown in Fig. 3 is a clear indication that this is also the case for the two-phase solidification experiments reported herein.

4 Conclusions

The following conclusions may be made for the experiments reported herein. For void fractions up to 90 percent, the presence of a discontinuous gas phase in a saturated flowing liquid does not affect the freezing of the liquid on a vertical surface. The crust surface remains smooth and the void in the two-phase mixture is not trapped in the crust in contradistinction to the result of the evolution of a gas resulting from differences in gas solubility in the liquid and solid. For experiments with void fractions from 0 to 65 percent, the two phase flow regimes varied from bubbly flow to churn-turbulent flow and the resulting vigorous agitation at the crust surface presumably prevented the bubbles in the discontinuous phase from attaching themselves to the solid-liquid interface and being incorporated into the solid. However, in the experiments involving foams, it was visually observed that the two phase structure was relatively stagnant. Also, it was found that the crust growth rate and the smooth crust surface was the same as for the experiments at lower void fractions. Thus, it can be concluded that the two-phase fluid mechanics of the relatively stagnant foam structure; that is, water moving in the thin lamallae between the bubbles toward the crust and bubbles moving away from the crust is not important and that conduction in the crust remained the controlling phenomenon. The effect of gas bubbling on the freezing of a flowing, superheated liquid on a vertical surface is to enhance the convective heat transfer from the liquid to the crust. The crust surface remains smooth in this case with no evidence of entrapment of the void. In both cases, the crust growth behavior can be modeled by ignoring the presence of voids (except for the void flux-induced enhancement of the convective heat flux).

Acknowledgment

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Heat Transfer to Generalized Couette Flow of a Non-Newtonian Fluid in Annuli with Moving Inner Cylinder

S. H. Lin[1] and D. M. Hsieh[1]

Introduction

In a previous work, Lin [1] investigated the heat transfer to a generalized non-Newtonian Couette flow between parallel plates. That problem is of practical significance because of its potential applications in the processing of a number of industrially important non-Newtonian fluids, such as polymer melts and solutions and liquid foods. It is noted that the problem of heat transfer to a plane non-Newtonian Couette flow occurs in plate coating with polymers [2]. A corresponding problem of comparable practical importance is the wire or tube coating. The polymer flow in the latter coating problem takes place between two concentric cylinders in which the outer cylinder is motionless while the inner cylinder moves in the flow direction. This is exemplified by a process in which the wire surface is to be coated with a layer of polymer.

In spite of the practical significance of this coating problem, it has received very little attention in the past. Literature in connection with this coating heat transfer problem is virtually not available. Studies of the heat transfer characteristics of this problem can lead to better understanding of the process performance which in turn can facilitate process equipment design. The present study is an attempt to investigate this problem.

Velocity Distribution of Flow

Like the plane non-Newtonian Couette flow, the present flow

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