NEURAL COMPUTATION APPROACH FOR THE MAXIMUM-LIKELIHOOD SEQUENCE ESTIMATION OF COMMUNICATIONS SIGNAL

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Abstract: A novel detection approach for signals in digital communications is proposed in this paper by using the neural network with transiently chaos and time-variant gain (NNTCTG) developed by author. The maximum likelihood signal detection problem can be always described as a complex optimization problem with so many local optima that conventional Hopfield-type neural networks cannot be applied. To amend the drawbacks of Hopfield-type networks, the NNTCTG is used to search for globally optimal or near-optimal solutions of the optimization problems with lots of local optima since it has richer and more flexible dynamics over conventional networks only with point attractors. We established a neuro-based detection model for the signal in digital communication and analyzed its working procedure in detail. Two simulation experiments were conducted to illustrate the validity and effectiveness of the proposed approach.

I. INTRODUCTION

It is well known that under the assumption of time-dispersive, time-varying channels and additive Gaussian noise, the maximum-likelihood sequence estimation (MLSE) based signal detection is often adopted. Although this kind of detectors exhibits an optimum error rate performance, it is often impractical to construct due to the computation-intensive complexity [1]. Therefore, an important researching direction is how to design a detector with both good error rate performance and an acceptable computation complexity.

Motivated by the pioneer work of Hopfield and Tank, the collective computational properties and the massively parallel architectures of artificial neural network are extensively utilized in solving for difficult optimization problems in many fields over conventional approaches [2]. The application of the neural network in digital communication systems [3,4] was motivated by its adaptive learning capability and potential of real-time processing.

According to the communication theory, the signal detection problem can be represented as an optimization problem. In particular, the MLSE can be described as a combinatorial minimization of the cost function over all possible sequences which is a large but finite set, of a certain length [3]. Among which we desire to find the one which globally minimizes the cost function involved.

Because of so many local optima in the optimization problem for the signal detection, the main difficulty using conventional Hopfield-like neural network is that the network tends to become trapped in local optima due to its gradient descent dynamics. To avoid getting stuck in local minima, both stochastic simulated annealing (SSA) and various deterministic simulated annealing (DSA) approaches such as hardware annealing and mean field approximate annealing have been proposed [5]. Very recently, [6] and [7] proposed a new artificial neural network with transient chaos and time-variant gain (NNTCTG). Unlike the conventional networks only utilizing gradient descent dynamics, it has richer and far-from equilibrium dynamics with various coexisting attractors, not only of fixed points and periodic points but also of strange attractors. This kind of complicated neuro-dynamics is a promising technique for information processing and optimization. In particular, an intriguing property of chaotic neural network to move chaotically over fractal structure in the phase space may be an efficient heuristic method searching for global optimal or near-optimal solution, avoiding getting stuck at local minima.

On the basis of the chaotic simulated annealing property of NNTCTG, we propose a new detection approach for signals in digital communications and give a concrete signal detector model. This paper is organized as follows. After a concise introduction to NNTCTG is given in section II, we present the NNTCTG-based signal detector in digital communication in detail in section III. In section IV, simulation experiments are performed for the comparison with the existing approaches. Finally, a conclusion is given.

II. CHAOTIC NEURAL NETWORK

It is well known that Hopfield network with continuous-time or asynchronously discrete-time state transitions guarantee convergence to a stable equilibrium solution but suffer from local minimum problems. Since the chaotic neural network is of richer and more flexible neuro-dynamics whose running region is only a fractal structure in the phase space and may be used to efficiently escape from the local minima problem in chaotic manner. Therefore, in order to take advantage of both the chaotic dynamics with convergent dynamics, a neural network with transient chaos and time-variant gain (NNTCTG) is proposed in [6, 7], as defined below:

$$y_i(t+1) = ky_i(t) + \alpha\left(\sum_{j=1, j\neq i}^{n} w_{ij}x_j(t) + I_i\right) - z_i(t)\left(x_i(t) - I_0\right) \tag{1}$$

$$x_i(t) = \left(1 + e^{-y_i(t)(1+\varepsilon_i(t))}\right)^{-1} \tag{2}$$

$$z_i(t+1) = (1-\beta)z_i(t), \quad \varepsilon_i(t+1) = (1-\gamma)\varepsilon_i(t) \tag{3}$$

where x_i and y_i are the output and internal state of *i*th neuron, respectively, w_{ij} is the synaptic weight from neuron *j* to neuron i , I_i is input bias, I_0 is a positive parameter, α is a positive scaling parameter for inputs, k is a damping factor of nerve membrane $(0 \le k \le 1)$, $z_i(t)$ is self-feedback synaptic connection weight or refractory strength, $\varepsilon_i(t)$ is gain parameter of the output function, β and γ $(0 \le \beta, \gamma \le 1)$ are damping factors.

It is clear that NNTCTG in (1)-(3) has related to existing ANN models, such as if $\varepsilon_i(t)=0$ and $z_i(t)=0$, then it is degraded as the famous HNN[2]; if $\varepsilon_i(t)=\varepsilon_i>0$, $z_i(t)=z_i>0$ and $I_0=0$, the NNTCTG is reduced as the chaotic neural network (CNN) [8]; if $\varepsilon_i(t)=\varepsilon_i>0$, it is reduced as TCNN[9]. A switched-capacitor implementation of transiently chaotic neural networks is given in [9].

It can be shown that NNTCTG actually has transiently chaotic dynamics which eventually converges to a stable equilibrium point through successive bifurcations like a route of reversed period-doubling bifurcations, with the temporal evolution of $z_i(t)$ and $\varepsilon_i(t)$ in (3), which harness the chaotic behavior for convergence and the speed of reversed bifurcation since they correspond to the temperature in simulated annealing process in exponential cooling schedule

Generally, the procedure of NNTCTG in solving for general optimization problem can be divided into two phases: chaotic bifurcations phase and gradient convergent phase. In the first phase, a complicated and rich chaotic bifurcations process is created by big values of refractoriness and gain for the network to escape from local minima, whose mechanics can be regarded as a kind of DSA, and called chaotic simulated annealing (CSA). In the second phase, a good initial state at a neighborhood of globally optimal solution is provided for the gradient descent dynamics so that the network can easily reach the global optimal or near-optimal solution of the problem.

In solving optimization problem, one can map the problem onto the network by letting

$$\sum_{j=1}^{m} w_{ij}x_j + I_i = -\partial J/\partial x_i\,,\quad (i=1,2,\Lambda\;,N) \tag{4}$$

where J is the cost function of the problem and N is the number of free variables. Note that the symbol 'Λ' in this paper is completely equivalent to symbol '$\cdots$'.

III. NNTCTG-BASED SIGNAL DETECTOR

Here we consider a digital communication system over the intersymbol interference (ISI) and additive Gaussian noise channel and make the assumption of the time-invariant channel during at least *n* symbol intervals. The actual ISI channel together with baseband Nyquist filters in the transmitter and receiver can be modeled as a finite impulse response filter of length $L+1$ whose impulse response is given by $h(k)=h_k$. Here L is the number of symbol intervals over which the ISI spans and hence $h(k)=0$ for $k\notin[0,L]$. Hence, the received signal is produced by

$$r(k)=\sum_i r_k\delta(k-i)=\sum_{i=0}^{L}u_{k-i}h_i+n(k) \tag{5}$$

where $u(k)=\sum_i u_i\delta(k-i)$, $\delta(k)$ is the Kronecker delta function and $n(k)$ is a white Gaussian noise of zero-mean and finite variance.

If the signaling alphabet $\alpha=\{\alpha_k\}$, $k=1,2,\Lambda\;,M$, and sequence $\mathbf{u}_n=\{u_i\}, i=0,1,2,\Lambda\;,n-1$, correspond to a finite set of numbers and the degree of freedom, respectively, then there are M^n possible combinations over which the maximum-likelihood sequence estimator (MLSE) computes the cost function. It selects a sequence as a best estimate of the transmitted sequence, that maximizes the conditional *a posterior* probabilities $p(\mathbf{r}\,|\mathbf{u}\,)$, where $\mathbf{r}=[r_1,\ r_2,\ \Lambda\ ,\ r_{n-1}]^T$ is the received signal vector, $\mathbf{u}=[u_1,\ u_2,\ \Lambda,\ u_{n-1}]^T$ is the transmitted sequence vector and superscript T indicates transpose operation. For a sufficiently large n, the MLSE algorithm is to choose a sequence from M^n possible sequences, that maximizes a scalar cost function

$$J = -\sum_{k=0}^{n-1}\left| r_k - \sum_{i=0}^{L} h_{k-i}u_i \right|^2 \tag{6}$$

After proper simplification, the minimization of (6) is equivalent to minimize

$$J = \tfrac{1}{2}\mathbf{u}^H\mathbf{M}\mathbf{u} - \Re e\left\{\mathbf{u}^H\mathbf{z}\right\} \tag{7}$$

where $\mathbf{u}=[u_1,\ u_2,\ \Lambda,\ u_{n-1}]^T=\mathbf{u}_I+j\mathbf{u}_Q$, $\mathbf{u}\in\{\alpha_1,\ \alpha_2,\ \Lambda\ ,\ \alpha_M\}^n$, $\alpha_i\in C$, $\mathbf{z}=[z_1,\ z_2,\ \Lambda,\ z_{n-1}]^T=\mathbf{z}_I+j\mathbf{z}_Q,\ z_i\in C^n\ \mathbf{M}\in C^{n\times n}$.

here subscript I and Q of above variables respectively denote the real part and imaginary part of the corresponding variables. $\mathbf{z}=\mathbf{r}*\mathbf{h}$ is cross-correlation between the received signal and channel impulse response, $\mathbf{M}$ is the correlation matrix of transmission channel.

By cumbersome derivatives, we can have the following objective function

$$J = \tfrac{1}{2}\bar{\mathbf{u}}^T\bar{\mathbf{M}}\bar{\mathbf{u}} - \bar{\mathbf{u}}^T\bar{\mathbf{z}} \tag{8}$$

where $\bar{\mathbf{u}}=\begin{bmatrix}\mathbf{u}_I\\ \mathbf{u}_Q\end{bmatrix}$, $\bar{\mathbf{M}}=\begin{bmatrix}\mathbf{M}_I & \mathbf{M}_Q^T\\ \mathbf{M}_Q & \mathbf{M}_I\end{bmatrix}$, $\bar{\mathbf{z}}=\begin{bmatrix}\mathbf{z}_I\\ \mathbf{z}_Q\end{bmatrix}$.

It was shown from (8) that the cost function to be minimized in the MLSE has the same quadratic form as the Lyapunov function associated with the network. If the cost function is mapped onto the network then the desired estimate is obtained at the output. However, for a combinatorial optimization it suffers from the local minimum problem as in Hopfield neural networks[2]. Optimum or near-optimum solutions can be obtained by applying various annealing approaches[5-8]. As an efficient method for MLSE of digital signal, a chaotic-annealed network is proposed.

When the MLSE cost function of (8) is directly mapped onto a neural network, one difficulty may arise. For $m_0\geq 0$, the matrix $\mathbf{M}$ is positive semidefinite and $\boldsymbol{J}$ is a convex function of output. Correspondingly, the continuous-valued steady-state output $\mathbf{x}\in D^{2n}$ may occur although the desired binary output can also be obtained by using additional limiting devices at the output. However, in order to reduce the circuit complexity and the effect of noise, a network with combinatorial solutions is highly desirable. So, we need an additional constraint energy

$$E_c = \mu(\mathbf{x}+\mathbf{1})^T(\mathbf{x}-\mathbf{1}) \tag{9}$$

where $\mathbf{1}$ is $2n$-dimension constant vector with element of one and μ is a constant. The constraint energy satisfies $E_c\geq 0, \mathbf{x}\in D^{2n}$, where the equality holds only if

$\mathbf{x} \in \{-1,\ +1\}^{2n}$. If we neglect the constant term , the final modified objective function for MLSE problem is given by

$$\bar{J} = \tfrac{1}{2}\mathbf{x}^T(\mathbf{M} + \mu\mathbf{E})\mathbf{x} \ \ - \mathbf{x}^T\mathbf{z} \qquad (10)$$

where **E** is a unity matrix. The parameter μ controls the shape of energy landscape and is chosen according to the maximum eigenvalue of **M** so that $\bar{J}$ is a concave function. In many actual applications, μ can be coarsely selected as $\mu < -m_0$.

When the objective function of (10) mapped onto the network by (4), the synaptic weights and neuron bias of the proposed neural network detector can be determined by

$$W = -\mathbf{M} - \mu\mathbf{E}, \qquad I = \mathbf{z} \qquad (11)$$

where W is the synaptic connection weights matrix and I is the neuron bias vector of the neural network.

Hence, the MLSE problem can be easily solved by the NNTCTG based receiver.

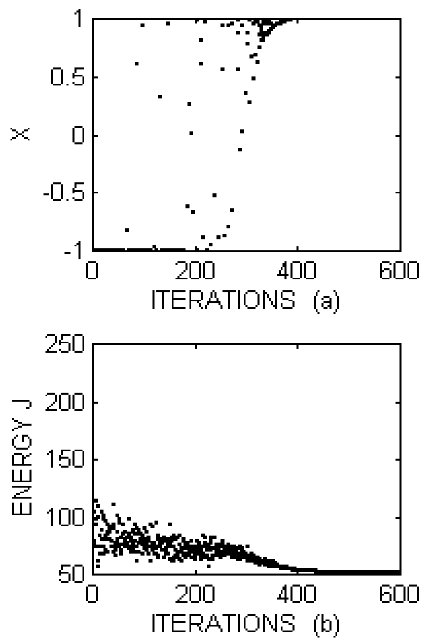

Fig.1 One of the output trajectories and energy evolution plot of the network at SNR=10 dB

IV. SIMULATION RESULTS

As was known that the signaling alphabet $\alpha = \{\alpha_k\},\ k = 1,2,\Lambda\ , M$ depends on the modulation techniques employed. For the sake of simplicity, we perform the simulation experiments of a simple binary communication system with two ISI channels by solving the dynamic equations in (1)-(3). In this case, the transmitted signals are binary vectors. Randomly generated data sequence $U_n = \{u_0,\ u_1,\ \Lambda\ ,\ u_{n-1}\}$ is convolved with a channel response *h(k)* which is assumed to be known exactly in our simulations.

Experiment 1: The simulation is conducted on a binary communication system with the ISI channel given by

$$H_m(z) = \frac{1}{\sqrt{1.25}}\left(1 + 0.5z^{-1}\right). \quad \text{therefore, } h(0)=0.8944,\ h(1)=0.4472;\ h(k)=0, \text{ if } k \neq 0,1 \tag{12}$$

So the autocorrelation elements of above channel can be given by $m_0 = 1.0$, $m_1 = m_{-1} = 0.4$, $m_k = 0$ for $|k| \geq 2$.

Fig.1 shows one of the output trajectories and energy decrease plot of the network at SNR=10 dB. For each signal-to-noise ratio (SNR) value 100 simulation runs are performed independently with the sequence length of 1000. Fig.2 shows the probability of error of detection of the NNTCTG-based signal detector. For comparison with the existing detection methods, we also plot the probability of the error of HNN detector and Viterbi algorithm in Fig.2. The results shown in Fig.2 demonstrate that our approach is better in the probability of error than HNN detector (by dash-dotted line) and is less efficient than the Viterbi algorithm(VA) (by dashed-line) at the moderate values of SNR.

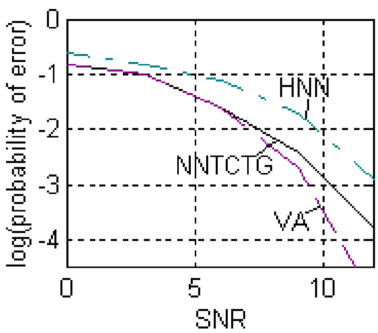

Fig.2 Curves of the probability of error for several signal detection methods in digital communications

Experiment 2: Consider communication system with the ISI channel as

$$H_m(z) = \frac{1}{\sqrt{1.25}}\left(0.5 + z^{-1}\right). \quad \text{i.e., } h(0)=0.4472,\ h(1)=0.8944;\ h(k)=0, \text{ if } k \neq 0,1 \tag{13}$$

Its autocorrelation matrix is same as one in experiment 1. In Fig.3 we plot the curves of the average probability of the error of several signal detection methods. Fig.3 shows that our approach is best one for channel in (13) among the three methods.

In summary, the simulation results shown in Fig.1 and Fig.3 demonstrate that our proposed approach is very efficient and has good robust.

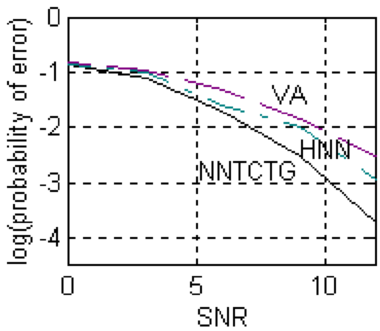

Fig.3 Curves of the probability of error for several signal detection methods over channel of (13)

V. CONCLUSION

A neural network detection model for signals in digital communications is proposed in this paper. Although there are so many local optima in this problem, the proposed NNTCTG-based signal detector not only can easily reach the global optimum or its neighborhood after a transiently chaotic process but also has low computation complexity. Numerical simulation results show that our method is an more efficient and robust technique for implementing MLSE receiver for digital signals in communications than existing methods.

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Biography

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