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AXISYMMETRIC STAGNATION POINT MHD FLOW OVER A POROUS PLATE WITH HEAT TRANSFER

Hazem Ali Attia
Department of Mathematics, College of Science,
King Saud University (Al-Qasseem Branch),
P.O. Box 237, Buraidah 81999,
Kingdom of Saudi Arabia

ABSTRACT

The steady axisymmetric hydromagnetic flow of an incompressible viscous electrically conducting fluid impinging on a porous flat plate with heat transfer are investigated. An external uniform magnetic field and a uniform suction or injection are applied normal to the plate which is maintained at a constant temperature. Numerical solution for the governing nonlinear equations is obtained.

KEYWORDS

Stagnation point flow, hydromagnetic flow, heat transfer, numerical solution

INTRODUCTION

The viscous fluid motion generated by a stagnation point flow impinging on a flat plate is a classical problem in fluid mechanics. The stagnation point flow, first examined by Hiemenz [1], is of great technical importance, for example in the prediction of skin-friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. The effect of uniform suction on the stagnation point flow is considered by Preston [2]. In hydromagnetics, the problem of Hiemenz flow was solved using approximate methods in [3,4].

The study of heat transfer in boundary layer flows is of importance in many engineering applications such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, thermal recovery of oil, etc. [5]. Massoudi and Ramezan [5] used a perturbation technique to solve for the stagnation point flow and heat transfer of a non-Newtonian fluid of second grade. Their analysis is valid only for small values of the parameter that determines the behavior of the non-Newtonian fluid. Later Massoudi and Ramezan [6] extended the problem to nonisothermal surface. Garg [8] improved the solution obtained by Massoudi [6] by computing numerically the flow characteristics for any value of the non-Newtonian parameter using a pseudo-similarity solution. In references [5-7], the authors considered the case of two-dimensional stagnation point flow of a non-conducting fluid.

The purpose of the present paper is to study the steady laminar flow of an incompressible viscous electrically conducting fluid at an axisymmetric three-dimensional stagnation point impinging on a porous flat plate with heat transfer. The fluid is acted upon by an external uniform magnetic field and a uniform suction or injection directed normal to the plane of the wall. The magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected [8]. The wall and stream

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temperatures are assumed to be constants. A numerical solution is obtained for the governing momentum and energy equations using finite difference approximations which takes into account the asymptotic boundary conditions. The numerical solution computes the flow and heat characteristics for any value of the Hartmann number, the suction or injection parameter and Prandtl number.

FORMULATION OF THE PROBLEM

Consider the axisymmetric three-dimensional stagnation point flow of a viscous incompressible electrically conducting fluid impinging perpendicular to a porous plate. This is an example of a potential flow which arrives from the entire space above the plate and impinges on a flat wall placed at z=0, flows away radially in all directions. (u,w) are the components of velocity at any point (r,φ,z) for the viscous flow and the velocity component in the φ-direction vanishes whereas (U,W) are the velocity components for the potential flow. A uniform magnetic field B_o and a uniform suction or injection with a transpiration velocity at the boundary of the plate given by $-w_o$ for suction and w_o for injection are applied normal to the plane.

Then, for the three-dimensional steady state flow, the continuity and momentum equations, using the usual boundary layer approximations [9] and by introducing Lorentz force, reduce to

$$\partial u/\partial r + u/r + \partial w/\partial z = 0, \quad (1)$$

$$u\partial u/\partial r + w\partial u/\partial z = -1/\rho\ \partial p/\partial r + \upsilon\ (\partial^2 u/\partial r^2 + 1/r\partial u/\partial r - u/r^2 + \partial^2 u/\partial z^2) + \sigma B_o^2/\rho\ (U(r)-u), \quad (2)$$
(2)

$$u\partial w/\partial r + w\partial w/\partial z = -1/\rho\ \partial p/\partial z + \upsilon\ (\partial^2 w/\partial r^2 + 1/r\partial w/\partial r + \partial^2 w/\partial z^2), \quad (3)$$

where $\partial/\partial\varphi=0$, ρ, υ and σ are, respectively, the density, the kinematic viscosity, and the electric conductivity of the fluid and U(r) is the radial component of the inviscid potential flow velocity above the boundary layer formed over the plate surface. The boundary conditions for the velocity problem, assuming the absence of magnetic field in the potential flow region, are given by,

$u(r,0)=0,\ w(r,0)=-w_o$, for suction or $w(r,0)=w_o$ for injection, (4a)

$$u(r,\infty)=U(r)=ar,\ \ w(r,\infty)=W(z)=-2az,\ \ p(r,\infty)=P_o - \rho a^2/2\ (r^2 + z^2) \quad (4b)$$

where 'a' is a constant. The temperature distribution can be found from the energy equation which may be written as (neglecting the dissipation) [10],

$$\rho\ c_p\ (u\partial T/\partial r + w\partial T/\partial z)= k\ \partial^2 T/\partial z^2 \quad (5)$$

where T is the temperature of the fluid, c_p is the specific heat capacity at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the temperature problem are given by

$$T(r,0)=T_w,\ \ T(r,\infty)=T_\infty \quad (6)$$

By introducing the following dimensionless variables and parameters

$\zeta=\sqrt{a/\upsilon}\ z,\ \ u(r,z)=ar\ \varphi'(\zeta),\ w(r,z)=-\ \sqrt{a\upsilon}\ \varphi(\zeta),$
$\theta(\zeta)=(T-T_\infty)/(T_w-T_\infty)$

$H_a^2= \sigma B_o^2/\rho a$, H_a is the modified Hartmann number,

$Pr=\mu c_p/k$, Pr is the Prandtl number,

$A= \varphi(0)=\pm w_o/\sqrt{a\upsilon}$, A is the suction parameter; A>0 for suction and A<0 for injection,
the governing Eqs. (1) to (5), respectively, reduce to

$$\varphi''' + 2\ \varphi\varphi'' - \varphi'^2 + Ha^2\ (1- \varphi') +1 = 0, \quad (7)$$

$$\varphi(0)=A,\ \ \varphi'(0)=0,\ \ \varphi'(\infty)=1, \quad (8)$$

$$\theta'' + Pr\ \varphi\ \theta' =0, \quad (9)$$

$$\theta(0)=1,\ \ \theta(\infty)=0, \quad (10)$$

where prime denotes differentiation with respect to ζ.

The flow Eqs. (7) and (8) are solved numerically using finite differences. A quasi-linearization technique [3] is first applied to

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replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence. The quasi-linearized form of Eq. (7) is,

$$\varphi'''_{n+1} + 2\,\varphi_n\,\varphi''_{n+1} + 2\,\varphi_{n+1}\,\varphi''_n - 2\,\varphi_n\,\varphi''_n - 2\,\varphi'_n\,\varphi'_{n+1} + \varphi'^2_n + Ha^2\,(1-\varphi'_{n+1}) + 1 = 0$$

where the subscript n or n+1 represents the n^{th} or $(n+1)^{th}$ approximation to the solution. Then, the Crank-Nicolson implicit method is used to replace the different terms by their second order central difference approximations [3]. An iterative scheme is used to solve the quasi-linearized system of difference equations. The solution for the non-magnetic and Newtonian case is chosen as an initial guess and the iterations are continued till convergence within prescribed accuracy. Finally, the resulting block tridiagonal system was solved using generalized Thomas' algorithm.

The energy Eq. (9) is a linear second order ordinary differential equation with variable coefficient, $\varphi(\zeta)$, which is known from the solution of the flow Eqs. (7) and (8) and the Prandtl number Pr is assumed constant. Having determined the function $\varphi(\zeta)$, Eqs. (9) and (10) are solved numerically using central differences for the derivatives and Thomas algorithm for the solution of the set of discritized equations. The resulting system of equations has to be solved in the infinite domain $0<\zeta<\infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid-independence studies show that the computational domain $0<\zeta<\zeta_\infty$ can be divided into intervals each of uniform step size which equals 0.02. This reduces the number of points between $0<\zeta<\zeta_\infty$ without sacrificing accuracy. The value $\zeta_\infty=10$ was found to be adequate for all the ranges of parameters studied here.

RESULTS AND DISCUSSION

Figures 1 and 2 present the velocity profiles $\varphi(\zeta)$ and $\varphi'(\zeta)$ respectively for various values of Ha and A. The figures show that increasing the parameter Ha or A increases both $\varphi(\zeta)$ and $\varphi'(\zeta)$. The figures indicate also that the effect of Ha on $\varphi(\zeta)$ and $\varphi'(\zeta)$ is more pronounced for smaller values of A (case of injection). Figure 1 shows that increasing the magnetic field or the injection velocity (decreasing A) decreases the velocity boundary layer thickness.

Figure 3 presents the temperature profile $\theta(\zeta)$ for various values of A and Ha and Pr=0.5. The figure indicates that the thermal boundary layer thickness decreases as A or Ha increases. This emphasizes the influence of the injected flow in the cooling process and the role of the magnetic field in controlling the rate of this cooling. The action of fluid injection (A<0) is to fill the space immediately adjacent to the disk with fluid having nearly the same temperature as that of the disk. As the injection becomes stronger, so that does the blanket extend to greater distances from the surface. As shown in Fig. 3, these effects are manifested by the progressive flattening of the temperature profile adjacent to the disk. Thus, the injected flow forms an effective insulating layer, decreasing the heat transfer from the disk. Suction, on the other hand, serves the function of bringing large quantities of ambient fluid into the immediate neighborhood of the disk surface. As a consequence of the increased heat-consuming ability of this augment flow, the temperature drops quickly as we proceed away from the disk. The presence of fluid at near-ambient temperature close to the surface increases the heat transfer. Also, it is clear from the figure that the magnetic field has an apparent effect on $\theta(\zeta)$ for small values of A while its influence can be neglected in the case of large A. Figure 4 presents the temperature profiles for various values of Pr and A and for Ha=1. The figure brings out clearly the effect of the Prandtl number on the thermal boundary layer thickness. Increasing Pr or A decreases the thermal boundary layer thickness. The effect of A on $\theta(\zeta)$ is more pronounced for larger values of Pr. In general, the velocity boundary layer is thicker than the thermal boundary layer.

CONCLUSION

The axisymmetric three-dimensional stagnation point flow of a viscous incompressible electrically conducting fluid with heat transfer is studied. A numerical solution for the governing equations is obtained which allows the computation of the flow and heat transfer characteristics for various values of the Hartmann

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number Ha, the suction parameter A, and Prandtl number Pr. The results indicate that increasing the parameter Ha or A decreases both the velocity and thermal boundary layer thickness.

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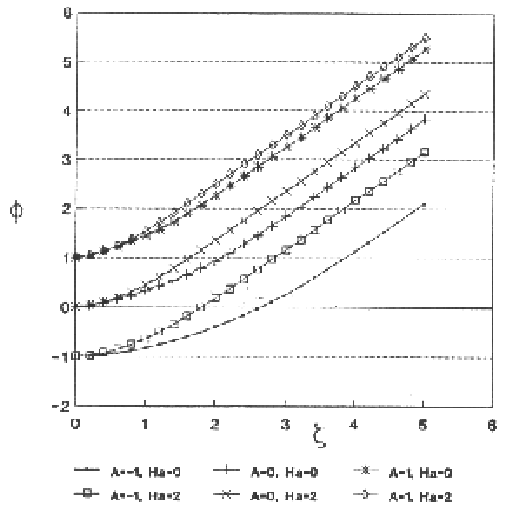

Fig. 1 Variation of ϕ for various values of A and Ha

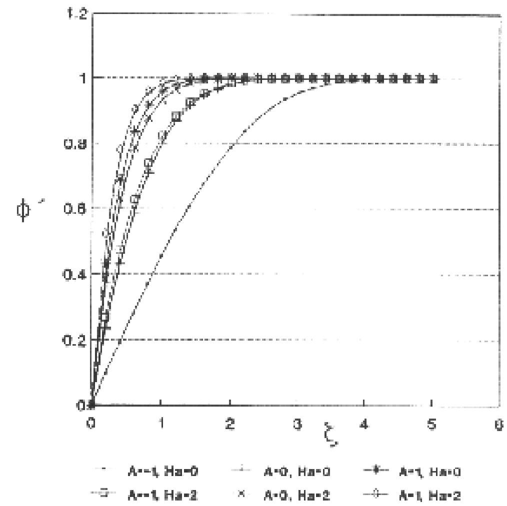

Fig. 2 Variation of ϕ' for various values of A and Ha

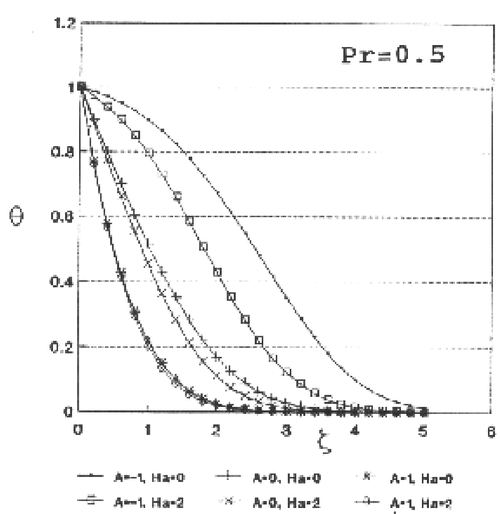

Fig. 3 Variation of θ for various values of A and Ha

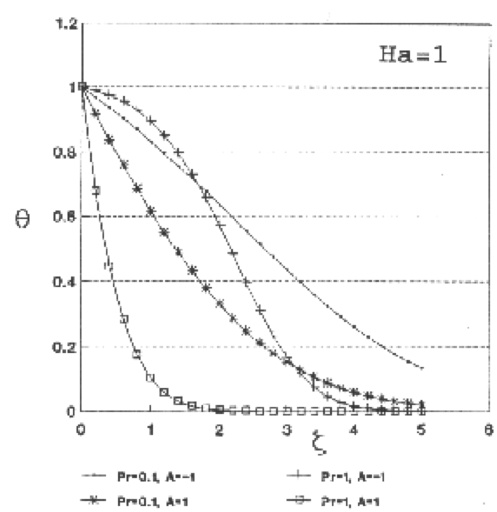

Fig. 4 Variation of θ for various values of A and Pr

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