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CONSIDERING THE INFO-GAP APPROACH TO ROBUST DECISIONS UNDER SEVERE UNCERTAINTY IN THE CONTEXT OF ENVIRONMENTALLY BENIGN DESIGN

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ABSTRACT

Information-Gap Decision Theory (IGDT), an approach to robust decision making under severe uncertainty, is considered in the context of a simple life cycle engineering example. IGDT offers a path to a decision in the class of problems where only a nominal estimate is available for some uncertain life cycle variable that affects performance, and where there is some unknown amount of discrepancy between that estimate and the variable's actual value. Instead of seeking maximized performance, the decision rule inherent to IGDT prefers designs with maximum immunity (info-gap robustness) to the size that the unknown discrepancy could take. This robustness aspiration is subject to a constraint of achieving better than some minimal requirement for performance. In this paper, an automotive oil filter selection design example, which involves several types of severe uncertainty, is formulated and solved using an IDGT approach. Particular attention is paid to the complexities of assessing preference for robustness to multiple severe uncertainties simultaneously. The strengths and limitations of the approach are discussed mainly in the context of environmentally benign design and manufacture.

1. INTRODUCTION

Consumers and legislators are becoming increasingly aware of the costs to society that result from the environmental impact of products. In response, designers must be increasingly concerned with reducing the environmental impact of their products. The goal of reducing environmental impact fits under the broad category of environmentally benign design.

In order to design environmentally benign products, designers need a means for assessing the impact of their products before they are deployed. All products and processes in some way affect the environment during their entire, and often long, life spans. Consequently, evaluating environmental impacts has traditionally been addressed with life cycle assessment (LCA) methods [1]. A key characteristic of LCA is that often only very limited information and knowledge is available, resulting in large uncertainty, as summarized by Ross [2] and Björklund [3].

When information is very sparse, a decision maker may want to make a robust decision—that is, a decision that will yield a reasonably satisfactory result over a large range of realizations of the uncertain parameters. One means of identifying robust designs is information-gap decision theory (IGDT), developed by Ben-Haim [4]. A detailed introduction to IGDT is presented in Section 3, but a brief overview is presented next.

In IGDT, it is assumed that a decision maker has available a nominal, but suspect, estimate of an uncertain quantity. IGDT presents an approach to making design decisions when there is a gap of unknown size between the uncertain quantity's true value (which could be known but is not) and the available nominal estimate. IGDT models the size of this gap as an uncertainty parameter, α. In IGDT, the design decision maker confronts this gap by employing a performance-satisficing (rather than the traditional performance-maximizing) decision policy that seeks to maximize robustness to uncertainty. The decision maker must specify a satisficing, critical performance level—a "good enough", minimally acceptable level of performance in a worst case scenario—and accordingly choose the design that, subject to this minimum requirement, allows for the largest information gap, i.e., the largest α.

IGDT emerged from work on convex set-based models of uncertainty [5-7] and has been applied to flood management [8], water resources management [9], correlation studies between experimental tests and simulations [10], structural design [11, 12], and biological conservation management [13]. Additionally, IGDT is often mentioned in passing in papers on uncertainty in engineering design, but the authors are not aware of any previous detailed discussions of IGDT in environmentally benign design. The goal of this paper is to examine the applicability of IGDT to environmental benign design and LCA when there is severe uncertainty in assessing the loads and impacts that a design has on the environment.

In Section 2, the context of the study is set by describing a life-cycle analysis and discussing uncertainty representations. In Section 3, a detailed introduction to IGDT is presented. In Section 4, the specific oil filter selection example of an environmentally benign design problem is introduced. In Section 5, the design problem is solved under various assumptions about the numbers of types of uncertain parameters. Finally, Section 6 contains a discussion of results.

2. PERSPECTIVE AND CONTEXT

LCA is a broad topic, and there are many methods for representing uncertainty in design decisions. Section 2.1 establishes the perspective taken in this paper towards LCA, including a brief discussion of sources of uncertainty in the analysis. In Section 2.2, attention is turned towards the relationship between IGDT and familiar methods for representing uncertainty.

2.1. Life-cycle analysis perspective

In general, multi-criteria LCA evaluations that include environmental performance can be decomposed as depicted in Figure 1. Similar decompositions have been proposed (for example [14, 15]), though none are identical in form or scope to the structure presented here. Components are grouped, as indicated by dashed-lines in the figure, using Hofstetter's concept of "spheres" of knowledge and reasoning about environmental evaluation [14]:

- Technosphere: description of the product and its life cycle and an inventory of loads (e.g. emissions)
- Ecosphere: modeling of changes to the environment
- Valuesphere: modeling of the perceived seriousness or importance of changes to the environment

Any of the components in Figure 1 can be a source of uncertainty. In the technosphere, it will be assumed that negligible uncertainty exists about the *form* (e.g., volume, mass, and material content) for any given concept and design variables, though in reality, information about what materials are used can be limited, especially when suppliers provide subcomponents. There can be considerable uncertainty about the *life cycle events* (e.g., frequency of service, properties of the material-cycling, energy-supply infrastructures, customer usage behavior, and actual disposal paths). Uncertainty in the *inventory of loads*, in turn, is dependent on the uncertainty in form and life cycle events.

Ecosphere components typically involve considerably more uncertainty. Environmental *effects* (e.g., ozone layer depletion, carcinogenesis, and toxic stress) are related to inventory first through fate analyses, and then exposure and effect analyses [16]. Fate analyses are simpler for point-source loads, but become complex for products that are sold, used, and disposed of over a wide spatial and temporal range. Exposure and effect analyses are data intensive, involve simplified models, and may have limited applicability depending on how actual conditions deviate or fluctuate. Similar forms of uncertainty affect analysis of *damage*, e.g., ecosystem or human health impairment, and resource depletion with respect to available reserves.

Uncertainty also arises in the valuesphere. The valuesphere attempts to model the decision maker's preferences. This involves capturing tradeoffs between different environmental impacts (e.g. what amount of non-renewable resource depletion is equivalent to species loss)

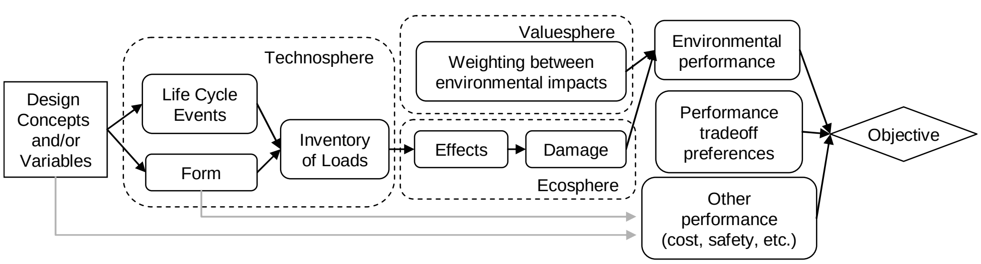

Figure 1: The components of an environmental analysis

and between environmental impacts and other design goals (e.g. cost and reliability). Many factors add to this uncertainty, including lack of information about values, failure to reach consensus, and the potential for values to shift in the future.

A complete and high quality LCA must appropriately account for the various uncertainties identified in this section. The following section examines the information requirements of different uncertainty formalisms.

2.2. Uncertainty modeling approaches

Different formalisms for modeling uncertainty require different assumptions on available information. In this section, we specifically discuss IGDT, interval methods [17, 18], decision analysis with basic sensitivity analysis [19, 20], traditional statistical decision theory [21], and probability bounds analysis (PBA) [22, 23]. The purpose of this discussion is to highlight these assumptions and to place IGDT with respect to the others. There is no intent, in this paper, of comparing the validity of these assumptions.

A decision maker's goal should be to choose the representation that is most appropriate for the available information and the assumptions that he or she is willing to make. On one hand, a good decision requires using all of the information that is available and not using or assuming information that is not present [22, 24]. On the other hand, including more information requires the use of more complex uncertainty models and formalisms, thus incurring additional costs. The principles of information economics [25] indicate that if these costs outweigh the benefits of the improved decision, one would be better off with a simpler representation.

In their simplest "uniform bounded" form, uncertainty models in information gap decision theory require only that a nominal value be identified. Given that, the models can represent extreme uncertainty about the location of the true value relative to this nominal. In a sense, this is the least possible information requirement—a single point with unknown uncertainty on either side.

In interval methods, specific bounds on the uncertainties are required. In some ways, this requires more information than an IGDT approach, which leaves uncertainty bounds unspecified. However, with interval methods there is no need to identify a nominal center point, so the requirements are inherently different, and could also be considered slightly reduced.

In basic decision analysis, nominal values are assumed to be known, and the problem is solved using these values. Then a standard sensitivity analysis (such as with a tornado diagram [19]) is performed to explore the effects of uncertainty (in the form of bounded intervals around the nominal values). Generally this method assumes independence between uncertain quantities and irrelevance of higher-order interactions.

In traditional statistical decision theory, it is assumed that all uncertainties can be characterized using precise probability distributions. Various assumptions or scenarios can lead to these probabilities, such as large historical databases, well-elicited beliefs, or well-founded prior distributions. We believe that these requirements (more information or assumptions) are stronger than for the other methods.

In PBA, uncertainty is represented using a structure called a probability-box, or p-box. A p-box generalizes both probability theory and interval methods, and it provides the flexibility of combining the two, meaning that certain imprecise probabilities [26] can be represented. Because a p-box is more general than both interval methods and probability, its information requirements are as low as the lowest of the two, but its applicability (without throwing information away) is greater than either. (A comparison of PBA and basic decision analysis, using a similar EBDM case study, is provided in another paper at this conference [27].)

The preceding analysis has placed IGDT as the least demanding approach to modeling uncertainty considered. However, as will be shown, a different decision approach is needed to utilize this deficient information. In the remainder of this paper, the applicability of IGDT to a particular example of environmentally benign design is explored. The emphasis is not on the applicability of the assumption of severe uncertainty, but rather on the generality of IGDT to handle various scenarios of problems that involve severe uncertainties.

3. IGDT CONCEPTS AND COMPONENTS

The goal of many decision approaches is to optimize some measure of system performance. In contrast, the goal in IGDT is to optimize a *robustness function* subject to a satisficing critical constraint on performance. Satisficing means accepting "good enough" performance in exchange for the potential to attain other objectives, especially when only idealized models or limited information is available [28]. Using IGDT, one satisfices performance to increase immunity to errors due to unavailable information (either due to ignorance or concern about extremely rare events) about an uncertain variable. The foundations of IGDT are explained in detail in [4]. A summary follows.

The main components needed for an info-gap analysis are:

- u, the uncertain variable for which some nominal $\tilde{u}$ is available
- q, some design variable(s)
- A performance (or "reward") model, $R(q,u)$, of system response whose output is a performance attribute of interest.

- r_c, a critical satisficing value of performance that must be guaranteed (met or exceeded); alternatively: a failure criterion.

When a larger (rather than smaller) reward $R(q,u)$ is desirable, the critical value r_c is defined such that for all q and u, the critical satisficing constraint requires:

$$R(q,u) \geq r_c \qquad (1)$$

In IGDT, it is assumed that u is an uncertain variable for which the decision maker can estimate a nominal value $\tilde{u}$ but cannot quantify the discrepancy (approximation error) between u and $\tilde{u}$. Thus, as shown in Figure 2, u is represented as family of nested, convex sets centered[1] around the nominal value, $\tilde{u}$. The size of each set is characterized by the uncertainty parameter, α. The parameter α differs from parameters in other uncertainty formalisms because, due to lack of information, α is not specified; instead, robustness to that parameter's unknown size is calculated as a function of r_c and chosen by the decision maker, as explained shortly.

Mathematically, a simple uniformly bounded info-gap can be defined as:

$$u = \mathcal{U}(\alpha,\tilde{u}) = \left\{ u : \left| u - \tilde{u} \right| \leq \alpha \right\}, \alpha \geq 0 \qquad (2)$$

or, fractionally, normalized with respect to the nominal, as:

$$u = \mathcal{U}(\alpha,\tilde{u}) = \left\{ u : \frac{\left| u - \tilde{u} \right|}{\tilde{u}} \leq \alpha \right\}, \alpha \geq 0 \qquad (3)$$

Info-gap models are defined based on information about how the bounds on the uncertain variable grow. Besides the uniform bound model of Eq. (2) and Figure 2, info-gaps can be bounded using various types of envelopes as discussed in [4]. If u is itself a function or model, then integral, Fourier, or other types of bounds can be defined.

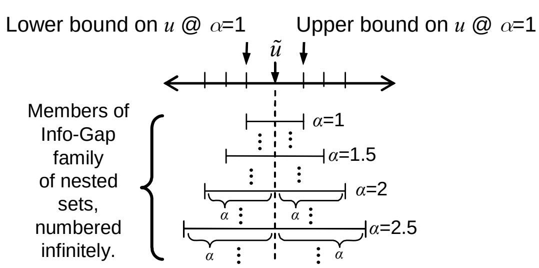

Figure 2: Representing unbounded uncertainty as an α–parameterized family of nested sets

[1] The info-gap model, parameterized from its center, has two ends of interest for each set in the family, as seen in Figure 2. The focus of this paper will only be on the bound that creates the worst consequence to performance. However, IGDT can consider the "favorable" end of the interval when using an *opportuneness function* [4], not discussed herein.

From the main IGDT components, u, $R(q,u)$, and r_c, a robustness function $\hat{\alpha}(q,r_c)$ can be defined that maximizes the size that the uncertainty parameter α can take and still satisfy the critical constraint of Eq. (1). This constraint is embedded into the *robustness function*, defined mathematically as an optimization problem:

$$\hat{\alpha}(q,r_c) = \max \left\{ \alpha : \min_{u \in U(\alpha,\tilde{u})} R(q,u) \geq r_c \right\} \qquad (4)$$

The info-gap robustness $\hat{\alpha}(q,r_c)$ is "the maximum tolerable α so that all u [in the info-gap model's family of sets] up to uncertainty size α satisfy the minimum requirement for survival" [4]. Eq. (4) is formulated for cases where larger values of performance are better. If smaller performance is desired, as when the objective involves stress, cost, or environmental impact, a maximization should replace the inner minimization:

$$\hat{\alpha}(q,r_c) = \max \left\{ \alpha : \max_{u \in U(\alpha,\tilde{u})} R(q,u) \leq r_c \right\} \qquad (5)$$

In either case, the "hat" on the symbol for robustness, $\hat{\alpha}$, distinguishes it from the uncertainty size α. The actual value of α is unknown, but one can still determine how much robustness, $\hat{\alpha}$, to unknown uncertainty bounds can be achieved by choosing a satisficing design rather than a risky, pure reward optimizing design. The design q that yields the largest robustness $\hat{\alpha}(q,r_c)$ for a given r_c is the "robustness-optimal" design, denoted $\hat{q}(r_c)$.

If the satisficing critical constraint, r_c, is flexible, the decision maker can examine the effect that relaxing the constraint has on opportunity for info-gap robustness. By graphically plotting robustness to uncertainty versus the level of critical performance, one can view a tradeoff. Relaxing one's requirement for minimum satisficing performance *sometimes* takes advantage of an accelerating payoff in robustness to info-gap uncertainty. Graphical tradeoff analyses can be used to elicit tradeoff preferences and make decisions.

To review, the typical steps to finding a satisficing, robust-optimal design using IGDT include translating the severely uncertain information into an info-gap model, defining the reward function, *R(q,u)*, choosing a critical level of guaranteed performance, r_c, and finding the robust-optimal design, $\hat{q}(r_c)$. If the requirement for critical performance is flexible, one can take the additional step of plotting the relationship between r_c and $\hat{\alpha}(r_c)$. Additional explanation of IGDT is done via example in Section 5.

4. OIL FILTER DESIGN PROBLEM INTRODUCTION

About 250 million light duty oil filters are discarded (and not recycled) in the United States each year, while about 250 million more are recycled [29]. The environmental impact of these filters can be substantial, as disposable filters contain large amounts of steel, aluminum, or plastic, depending on the style of filter.

In this example, it is assumed that an automobile manufacturer wants to reduce the environmental impact of oil filters from its cars by designing a more environmentally benign filter. Naturally, some simplifications and assumptions are introduced in the problem. For example, the exact parameters for the problem are chosen to be realistic, but they do not represent hard, real-world data. Consequently, the emphasis is not on the actual decision outcome (i.e. the chosen filter), but rather on the decision and analysis *process*.

4.1. Types of oil filters

In this simplified model, shown in Figure 3, an oil filter is comprised of five main components: housing, top cap, filter, inner support, and bottom cap. support, and bottom cap. The housing, top cap, and bottom cap make up the *casing*, and the inner support and filter make up the *cartridge*. Two different types of oil filters are considered, as summarized in Table 1. The dimensions of all components have been specified for the appropriate balance of strength and weight and are therefore fixed.

Filter	Material	Discarded parts
SEC	Steel	All (casing and cartridge)
TASO	Aluminum	Cartridge only

Table 1. Types of filters

Engineers have developed two competing concepts for the new design. The first filter considered is the steel easy change (SEC) filter. For an SEC filter, the structural components are made of steel and the filter of cellulose. The entire filter is designed to be replaced at once; it is simply unscrewed from the engine and the discarded or recycled. The second type of filter is the take-apart spin-on (TASO) filter. A TASO filter's structural elements are made out of aluminum and when the filter is replaced, only the cartridge is discarded; the casing is designed to last for the lifetime of the engine and is reused when the filter is changed. The environmental performance of both alternatives is considered over a vehicle's entire lifetime, which relates to F, the total number of filters used over the lifetime.

4.2. Environmental impact calculation

It is assumed that the primary environmental impact of an oil filter arises due to the construction, transportation, and disposal of the casing and cartridge components shown in Figure 3. Other substances, such as oil residue and rubber seals are generally equivalent in both filter types, and therefore do not contribute to the selection decision.

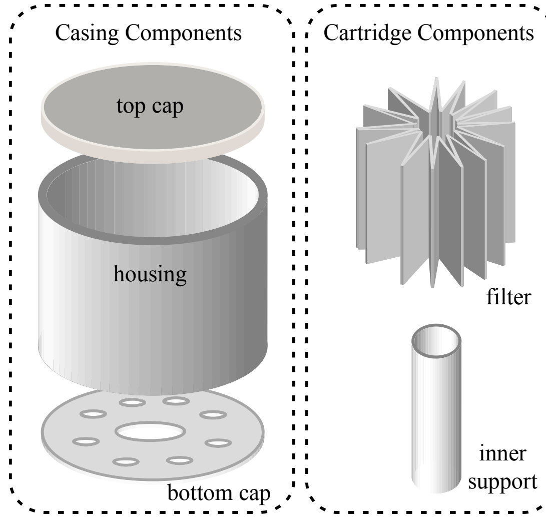

Figure 3. Oil filter schematic diagram

Eco-indicator 99 is an impact assessment method for life-cycle analysis in which particular scores, measured in millipoints (mPt), are assigned to specific materials and processes [16]. In this paper, only those environmental impacts that increase in direct proportion to mass are considered. For simplicity, these impacts-per-mass for different stages of the life cycle (mining, processing, disposal, recycling, etc.) will be summed for each component and referred to, for simplicity, as Eco-indicator *rates*, or simply *ecorates*, subsequently.

In the Eco-indicator methodology, the ecorates are tabulated as shown in Figure 4 by considering the three spheres of knowledge and reasoning noted earlier. The rates are tabulated for various products or by-products of manufacturing processes and product life-cycles. For each of the items in a potential load inventory (part of the technosphere), there are associated environmental effects in the ecosphere. For example, the release of CFCs into the environment depletes the ozone layer. In some cases these effects are clearly understood, and in other cases there is uncertainty as to how strong the effects are.

Each effect, in turn, has particular *damages* associated with it. These damage estimates are often more uncertain than the effects. For example, consider the current debate surrounding the damages that result from increased greenhouse gas emissions—how much are they damaging the ecosystem? There is not universal agreement, and hence significant uncertainty, as to the true damages.

Once the ecosphere aspects are modeled, one evaluates how much he or she actually cares about these damages relative to other damages. This is a valuesphere question. The value that someone or some society places on a particular damage can vary with factors such as culture, religion, and geographic location. For example, assuming greenhouse gas emissions are causing global warming and raising sea levels, how much does one care about these damages compared to damages caused by acid rain?

The Eco-indicator construct provides a baseline for comparing the environmental impact of different materials across all of the spheres. However, the uncertainties in the ecosphere (effects and damages) and valuesphere combine to yield a very large uncertainty in the ecorates. It thus initially appears that IGDT is an appropriate match for ecorates and offers the promise of robust decision making in the presence of these large uncertainties. The following examples explore this promise, but first more details about the example problem are presented.

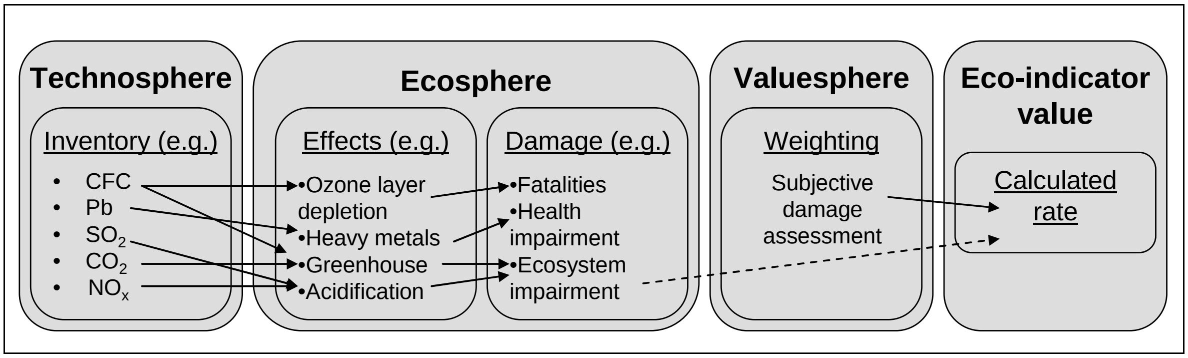

Figure 4. Eco-indicator calculation. Adapted from [16]

The Eco-indicator methods condense ecosphere and valuesphere information for individual materials on a per-mass basis. In order to calculate the actual impact of a process or product, the total mass of materials present—the inventory or technosphere information—must be determined. In this example, we assume that the filter casings and cartridges can each be parameterized per filter, and thus the impact of each can be summarized with one mass and ecorate. The specific assumed data are shown in Table 2. The impact I_c of a given component c can be calculated as:

$$I_c = mass_c \cdot ecorate_c \tag{6}$$

The total environmental impact over a vehicle's lifetime depends on the number of filters F used, which is categorized as a life cycle event in the terminology of Figure 1. The quantity F is uncertain because not every vehicle is in service for the same number of miles, and car owners change the filters with difference frequencies. When using LCA in practice it is important to communicate fully to decision makers what masses and remaining assumptions were used in determining the Eco-indicator scores. However, because this paper is not intended to be an actual recommendation of a filter, the rest of this step will be skipped for brevity.

		mass, kg	ecorate, millipoints/kg
TASO	Cartridge	$m_{cr,T}$=0.071	$e_{cr,T}$=5.07
	Casing	$m_{cs,T}$=1.841	$e_{cs,T}$=17.10
SEC	Cartridge	$m_{cr,S}$=0.075	$e_{cr,S}$=5.50
	Casing	$m_{cs,S}$=0.817	$e_{cs,S}$=1.78

Table 2. Mass and impact-per-mass for all components

Assembling these components into equations for environmental performance, the total impacts of the filters are:

$$\begin{aligned} I_{TASO} &= I_{casing} + \left(I_{cartridge} \cdot F\right) \\ &= (m_{cs,T} \cdot e_{cs,T}) + (m_{cr,T} \cdot e_{cr,T} \cdot F) \end{aligned} \tag{7}$$

$$\begin{aligned} I_{SEC} &= \left(I_{casing} + I_{cartridge}\right) \cdot F \\ &= (m_{cs,S} \cdot e_{cs,S} + m_{cr,S} \cdot e_{cr,S}) \cdot F \end{aligned} \tag{8}$$

An essential difference between the environmental impact of the designs is their casings: the TASO incurs a high one-time load whereas the SEC incurs a smaller load every time the filter is changed. For small F, the SEC filter has a smaller impact, but as F increases, the impact of replacing the casing with the SEC filter will exceed the one-time impact of the TASO's casing. The TASO casing has a higher impact because it is contains more material—it is built to last as long as the car's engine—than the SEC filter and because the material is aluminum, which is more resource intensive per unit weight than steel. In contrast, the SEC filter is made of steel (with a lower impact per mass) and contains less material since its lifetime is shorter.

The oil filter selection problem introduced in this section will now be used to explore the application of IGDT to various scenarios regarding the uncertainty in the ecorates and the number of filters used over the vehicle's lifetime.

5. OIL FILTER DESIGN DECISIONS WITH INFO-GAP THEORY

In the following series of examples, info-gap models and analyses of uncertainty will be explained for increasingly complicated situations. It is shown that different analysis approaches require different considerations and demands on the decision maker to form preferences for tradeoffs in critical performance versus robustness to uncertainty. The examples include, progressively:

- Section 5.1: One uncertainty that affects both design alternatives. Specifically, the number of filters F used over the vehicle's lifetime.
- Section 5.2: One uncertainty that has the same units and type but a different nominal for each alternative. Specifically, the *ecorate* of the casing material for each alternative is considered uncertain.
- Section 5.3: Two unrelated uncertainties evaluated first using a combined uncertainty parameter, α, and second using separate uncertainty parameters, as will be explained. Specifically, both the ecorates of the casings and the number of filters F are assumed to be uncertain.

5.1. Uncertain Number of Filters Used in Lifetime

The first example assumes severely deficient information regarding the average number of filters used over an engine's life. Despite recommendation by the manufacturer of a particular mileage period between filter changes, uncertainty about the frequency with which the customers will actually change their filters, coupled with uncertainty about the average lifetime of their cars, makes the actual average number of lifetime filter changes severely uncertain. The decision maker wishes to evaluate the selection decision without collecting further information, and decides to use the IGDT approach to do so.

The decision maker takes the attitude that settling for some guaranteed lower-bound on performance is acceptable and preferable to risky, (but higher) optimized performance that relies on the veracity of unfounded assumptions about the uncertainty. Accordingly, the decision maker seeks the design alternative with maximum robustness to the unknown gap between the unknown *actual* number of filters and a nominal *estimate*. The desire to maximize the size to which the discrepancy can grow is subject to a satisficing critical constraint that defines a largest environmental load that can be accepted, one that is sub-optimal with respect to the solution with no uncertainty, yet "good enough" given its robustness to uncertainty.

For this first example problem, an info-gap analysis will be explained in detail. Subsequent examples are variants of this problem, so only their formulation differences and results will be presented.

5.1.1. Info-gap model

It is assumed that the design firm has experience making filters for vehicles owned by customers in the industrial sector who schedule regular maintenance and change filters with predictable frequency. On average, those customers use 17 filters over the life of an engine. However, the design company wishes to expand its business with a new filter design targeting the public sector. Customers in that sector are expected to have less predictable maintenance behavior, and the degree to which their change frequency will deviate from that of industrial customers is unknown.

Thus, the info-gap model for this example can be specified with the knowledge that:

- The nominal value of oil filters used over an engine's lifetime is $\tilde{F}=17$, taken from information on maintenance rates in the industrial sector.
- The growth of deviation around nominal can be expressed mathematically as a simple, uniformly-bounded interval.

Combining the uncertainty parameter, α, with this sparse information, the info-gap model for lifetime filter usage is:

$$F(\alpha,\tilde{F})=\left\{F:\left|F-\tilde{F}\right|\le\alpha\right\},\ \alpha\ge 0 \tag{9}$$

The form of this particular info-gap model can also be expressed more simply:

$$\tilde{F}-\alpha\le F\le\tilde{F}+\alpha \tag{10}$$

5.1.2. Reward function and satisficing critical value

The other two components needed for an info-gap decision analysis are the reward function and satisficing critical value for performance. The reward functions for environmental impact, based on Table 2 and Eqs. (7) and (8), of the TASO and SEC designs are respectively:

$$R(q_1,u)=I(TASO,F)=31.48+(0.36\cdot F),\ [\text{mPt}] \tag{11}$$

$$R(q_2,u)=I(SEC,F)=(1.46+0.41)\cdot F,\ [\text{mPt}] \tag{12}$$

The designer chooses a critical value of $I_{critical}$=40mPt, which is the highest level of environmental impact deemed tolerable. In this problem, the decision maker seeks to minimize impact, so the inequality in Eq. (1) is reversed and the critical constraint is given as:

$$I(alt,F)\le I_{critical} \tag{13}$$

For convenience, the variable *alt* is used to represent the discrete design alternatives, TASO and SEC.

5.1.3. Info-gap robustness function

Of main interest in an info-gap analysis is what largest amount of robustness to uncertainty, $\hat{\alpha}(q,r_c)$, is achievable. This robustness is the largest amount of uncertainty α that can be sustained by a design alternative q while still guaranteeing, at worst, achievement of the chosen critical performance level r_c. Expressed in the form of Eq. (5), the info-gap robustness for this example is:

$$\hat{\alpha}(alt, I_{critical}) = \max\left\{\alpha : \max_{F \in U(\alpha, \tilde{F})} I(alt, F) \le I_{critical}\right\} \quad (14)$$

For this particular problem, finding the expression for $\hat{\alpha}$ for either design alternative is relatively simple. First, the uncertain variable F in Eqs. (11) and (12) is replaced with $\tilde{F} + \alpha$, the side of the parameterized info-gap model associated with worse performance, e.g.:

$$I(TASO, F) = 31.48 + (0.36 \cdot (\tilde{F} + \alpha)) \quad (15)$$

With this equation form, one can solve for α and calculate $\alpha(alt, I_{critical})$, equivalent in this case to info-gap robustness, $\hat{\alpha}(alt, I_{critical})$. When the reward function, info-gap model, and/or design space q assume more complicated forms, the optimization problem embedded in Eq. (14) can be more difficult to solve.

For the critical level $I_{critical}$=40mPt, $\hat{\alpha}(TASO, 40mPt)$ =6.7 filters and $\hat{\alpha}(SEC, 40mPt)$ =4.5 filters. One design is preferable to another when it can assure performance at or better than the critical requirement *amidst a greater amount of deviation α*. In this case, the TASO is "robust-optimal" and preferred to the SEC because the TASO filter can meet the critical impact constraint for a larger amount of uncertainty than the SEC filter can.

5.1.4. Analysis of Robustness-Performance Tradeoff

Analysis of preference for the trade between robustness and critical (acceptable) performance is facilitated by examining a tradeoff plot. This plot is shown in Figure 5 and discussed next. Rather than solving for and plotting robustness, $\hat{\alpha}(q, r_c)$, the performance Eqs. (11) and (12) are plotted. Critical levels of performance can be chosen along the vertical axis, with the corresponding robustness found as the horizontal distance from the vertical axis to the performance line for each design alternative.

The designer, not knowing the estimation error α, is tasked with choosing a point on the vertical axis corresponding to his or her demanded level of satisficing performance. In some applications, the r_c value may be strongly dictated by external factors. In other applications, the decision maker has the flexibility to relax their choice of critical performance level in order to gain more robustness. The decision maker can explore this tradeoff graphically in Figure 5 by examining the maximum robustness achievable for different values $I_{critical}$. In this example, the design having *maximum* robustness is the one whose performance function plot is the farthest to the right at a given critical performance level.

The plot in Figure 5 is instrumental in understanding how design preference changes as the demand for minimally acceptable performance is relaxed further away from the performance-optimal level. For example, at critical satisficing level discussed earlier, $I_{critical}$=40mPt, it can be seen that $\hat{\alpha}(TASO)$=6.7 filters and $\hat{\alpha}(SEC)$=4.5 filters. If an impact as aggressively low as 31.7mPt were demanded, only the SEC would satisfy the constraint, and even then, there would be no tolerance for error, α, in estimating the number of filter changes. Thus, $\hat{\alpha}(SEC, 31.7mPt)$=0. Until the critical requirement is relaxed (i.e., moved up the axis) as far as 37.6mPt, SEC is still the only viable option, with its tolerance for error growing linearly. At $I_{critical} = 37.6mPt$, TASO is now a viable design, but offers no info-gap robustness. TASO's robustness eventually overtakes SEC at $I_{critical}$=39mPt, where the performance lines cross in Figure 5 and the preferred design changes. The decision maker must explore these tradeoffs and determine what feasible combinations of robustness and critical performance are preferable.

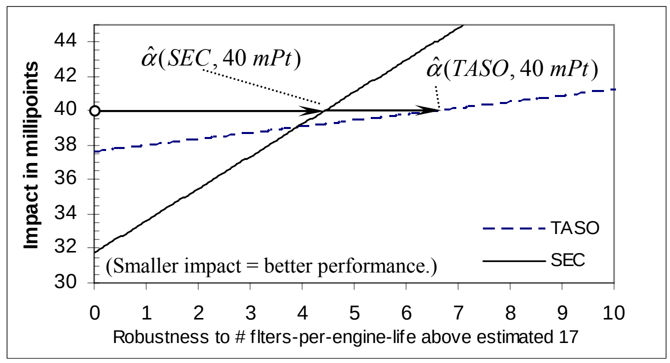

Figure 5. Environmental impact versus robustness

The following knowledge is gained in this simple example:

- If the decision maker can accept a worst-case environmental impact as high as or higher (which indicates worse performance in this example) than 39mPt, then the TASO design is preferable because it can endure the highest amount of error above the nominal guess and still satisfy the performance constraint. Moreover, the rate at which info-gap robustness is gained with incremental relaxation of the $I_{critical}$ demand (i.e., the line slope) is faster for TASO than SEC, making TASO even more attractive past 39mPt of demand.
- If there were no uncertainty, SEC would outperform TASO by a difference of 5.9 mPt; however, if *in reality* the deviation above the nominal estimate of 17 filter changes grew as high as 6 changes, for a total of 23 changes, TASO would then instead outperform SEC by 3.2 mPt.
- The designer, not knowing what the uncertain variable actually will be, can use the info-gap analysis and plot in Figure 5 to get a handle on what a decision change entails under the satisficing decision rule. If it seems reasonable that the average number of filter changes could deviate more than 4 above the estimate of 17, and that a relaxed demand of 7.3 mPt is reasonable, then the designer should choose the more robust TASO. Past that decision-switch point, the TASO option takes

advantage of greater robustness-per-incremental-relax-in-demand, as indicated by TASO line's flatter slope. It is up to the decision maker to sort out his or her preference for robustness versus guaranteed achievement of, at worst, some critical level of performance.

5.2. Example 2: Uncertain environmental impact

In this section, an info-gap analysis is performed assuming extreme uncertainty in the ecorates for the filter casings. The rationale for considering ecorates as extremely uncertain was discussed in Section 4.2. In this section, the previously unknown number of filters will be considered known in order to isolate the effects of uncertainty in the estimates of the casing ecorates. Similarly, the ecorates for the cartridges will be assumed known in order to facilitate illustration of the main ideas. An illustration of the more complicated case of multiple uncertainties is postponed until Section 5.3.

Whereas the lifetime number of filters affected both TASO and SEC alternatives, the ecorates, while having similar properties, have different nominal values for each design alternative because each is made from a different material. The values for ecorates, which were previously exact, are now used as the nominal values, i.e., $\tilde{e}_{case,TASO}$ =17.1 mPt/kg and $\tilde{e}_{case,SEC}$ =1.78 mPt/kg. We note that the units of the two are the same, and thus they can be expressed using a common α. Using the uniform bounded form as before:

$$e_{case}(\alpha,\tilde{e}_{material}) = \left\{ e_{material} : \left| e_{material} - \tilde{e}_{material} \right| \le \alpha \right\},\ \alpha \ge 0 \quad (16)$$

Or, more simply stated, for the side of the info-gap model that creates worse performance:

$$e_{material} \le \tilde{e}_{material} + \alpha \quad (17)$$

Substituting $e_{material}$ with $\tilde{e}_{material} + \alpha$ in the original performance function of Eqs. (7) and (8), a new plot of performance versus maximum error, α, in estimating $e_{material}$ is presented in Figure 6.

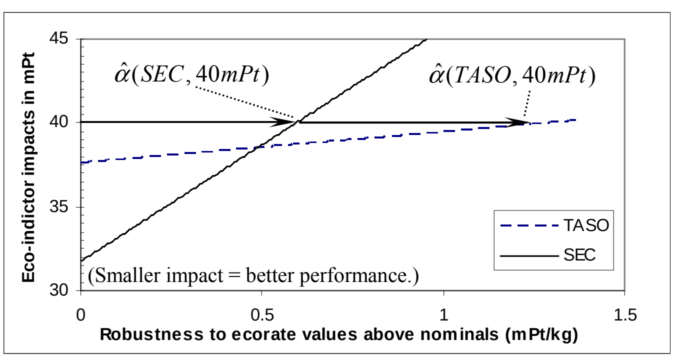

Figure 6. Tradeoff between impact and robustness to error in estimating the ecorate of the casing materials.

In the analysis of Section 5.1, each design alternative endured the same uncertain quantity, F, so α was clearly the same for each alternative. This made comparisons between the robustness of the SEC filter and TASO filter straightforward.

In Figure 6, consider the comparison of the robustness of the TASO and SEC filters for a critical impact of 40mPt. At this critical impact, the TASO filter allows for a larger α than SEC. However, a particular α for SEC is not necessarily equivalent to the same α for TASO. To clarify, as defined in Figure 6, the units of the two α's are the same, but the meaning is not necessary equivalent. For example, is a deviation of 1mPt/kg from the nominal value of 17.1 mPt/kg for the TASO casing the same as a 1mPt/kg deviation from the nominal value of 1.78mPt/kg for the SEC casing? We believe this to be a highly restrictive assumption because the underlying causes of the uncertainty could be different.

An alternative way to compare two uncertainties with different nominal values would be to use the percent deviations from the nominal. However, this still assumes that the uncertainties tend to deviate in the same percentages in reality, and that the decision maker cares equally about equal percentage deviations. These restrictive demands on assumptions are examined in more detail in the following section, in which both the ecorates for the casings and the number of filters F are assumed to be uncertain.

5.3. Example 3: Multiple severe uncertainties

Two approaches for evaluating multiple, different uncertainties are explained in this section. In each case, both the number of filters, F, and ecorates of the casings, e_{cs}, will be considered uncertain and represented with info-gap models. The first approach uses an existing method that parameterizes all uncertain quantities in terms of a single uncertainty parameter, α, expressed as percent error above nominal estimates. The second approach will use a separate α for each different uncertainty, facilitating an evaluation of tradeoffs between the robustnesses and critical performance, as well as between the robustnesses themselves, as will be explained graphically.

5.3.1. Using a single uncertainty-parameter, α

The standard info-gap approach to evaluating multiple severely uncertain nominal estimates involves parameterizing each uncertain quantity's info-gap model in terms of the same uncertainty parameter [13]. To make the units of each α the same, the info-gap model for each uncertain quantity is typically formulated in terms of *fractional* deviation from their respective nominal values, as was introduced in Eq. (3).

To evaluate the potential for robustness to both F and e_{cs}, the following info-gap models need to be defined in terms of one α:

$$F(\alpha,\tilde{F}) = \left\{ F : \left| \frac{F-\tilde{F}}{\tilde{F}} \right| \le \alpha \right\},\ \alpha \ge 0 \qquad (18)$$

$$e_{case}(\alpha,\tilde{e}_{material}) = \left\{ e_{material} : \left| \frac{e_{material}-\tilde{e}_{material}}{\tilde{e}_{material}} \right| \le \alpha \right\},\ \alpha \ge 0 \qquad (19)$$

Normalizing deviation with respect to the nominal gives the uncertainty parameter, α, units of percent error (%). To determine the effects of growth in α on environmental impact, the uncertain F and e_{cs} can be substituted in Eqs. (7) and (8) with $\tilde{F}(1+\alpha)$ and $\tilde{e}_{material}(1+\alpha)$, respectively. Over a range of α values, the plot of performance versus error is shown in Figure 7:

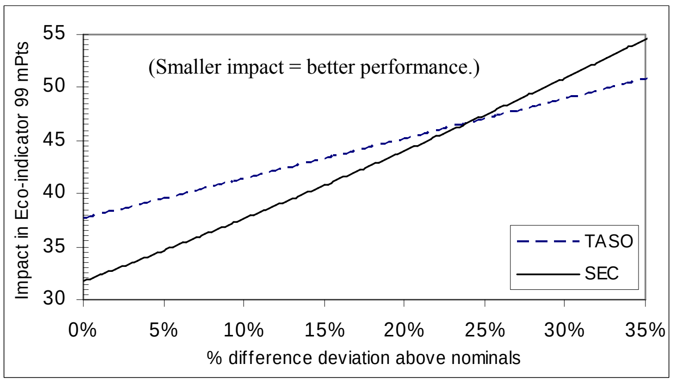

Figure 7: Info-gap growth, for all uncertainties simultaneously, versus environmental impact.

As indicated in the figure above, the performance optimal reward (which assumes the condition of no uncertainty, where $\tilde{F} = 17$ is correct), is 31.7mPt with the SEC filter. Up to a sacrifice of 14.7mPt above this optimal, the decision will not change—the SEC filter remains the better choice. However, at 46.4mPt, the two curves cross. The decision maker would need to contemplate the worth of that relaxed demand on performance in light of the amount of robustness (enduring a maximum of 23.7% relative error in *all* of the uncertainties simultaneously) attainable at the decision switch point. Additionally, the difference in the slopes of the lines corresponds to how much faster robustness would be gained past the decision-switch point.

While this approach provides some decision-making power, it requires the strong assumption that all uncertain variables will experience the same extreme degree of estimation error at the same time. We believe that such knowledge would not be available for this EBDM scenario. Moreover, any knowledge the decision maker might have about the relative growth of different estimation errors separately could not be utilized.

5.3.2. Using two uncertainty parameters, α_i

An alternative technique, which to the authors' knowledge is novel, can be used to evaluate the preceding scenario using two separate uncertainty parameters instead of a single one. This technique enables a decision maker to identify, for a given level of satisficing critical performance, any dominance of one design alternative over another in terms of their capacities to afford robustness to each uncertain quantity *separately*. When robustnesses to two severely uncertain variables are sought, they must trade off (like a Pareto curve) if their respective deviations from nominal each separately worsen performance.

This approach separates uncertainty into two parameters: in this example, α_F and α_e, for the uncertain F and e_{cs}, respectively. Because these parameters are separate, the previous need to use a fractional info-gap with consistent units of percent error is now optional. Instead, simple uniform info gaps will be defined for F and e_{cs}:

$$F(\alpha_F,\tilde{F}) = \left\{ F : \left| F-\tilde{F} \right| \le \alpha_F \right\},\ \alpha_F \ge 0 \qquad (20)$$

$$e_{case}(\alpha_e,\tilde{e}_{material}) = \left\{ e_{material} : \left| e_{material}-\tilde{e}_{material} \right| \le \alpha_e \right\},\ \alpha_e \ge 0 \qquad (21)$$

As before, the functions for environmental performance of each design alternative are calculated using Eqs. (7) and (8). But whereas preceding evaluations compared one performance function, *Impact*, to one uncertainty parameter, α, each on their own axes, an additional axis is needed for the additional uncertainty parameter. Because of the difficulty of assessing relationships between three dimensions in a single plot, the next set of figures shows the tradeoffs in attainable levels of robustness to error for progressively satisficing (i.e., higher allowable) critical performance levels of $I_{critical}$ of 38.16 mPt, 38.48 mPt, 38.72 mPt, and 39.0 mPt in Figure 8 through Figure 11, respectively.

As before, lines in these graphs correspond to the TASO and SEC design alternatives, but now they represent the tradeoff between what levels of robustness to each uncertain quantity can maximally be achieved. This is similar in concept to a Pareto tradeoff curve, where improvement in one goal is only attainable through sacrifice of another. For instance, when the minimally acceptable performance cutoff is set at 38.16 mPt, as in Figure 8, the SEC option can endure either: 0.46 mPt/kg of error in the estimate of its ecorate and no error in the estimate of the lifetime number of filters used; or can endure 3.4 filters worth of error above the lifetime estimate and no error in the estimate ecorate; or any combination in between along the solid line in Figure 8.

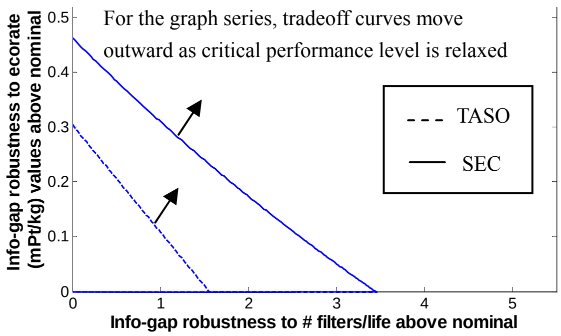

Figure 8. Tradeoff between robustnesses, $I_{critical} = 38.16$ mPt

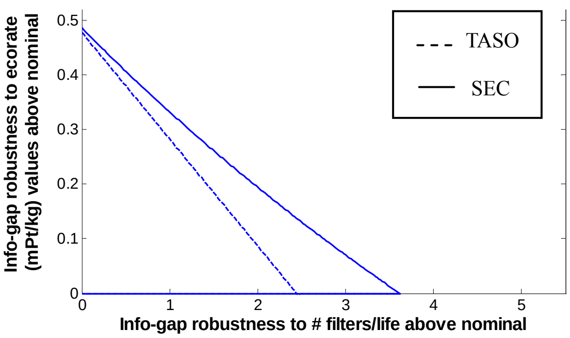

Figure 9. Tradeoff between robustnesses, $I_{critical} = 38.48$ mPt

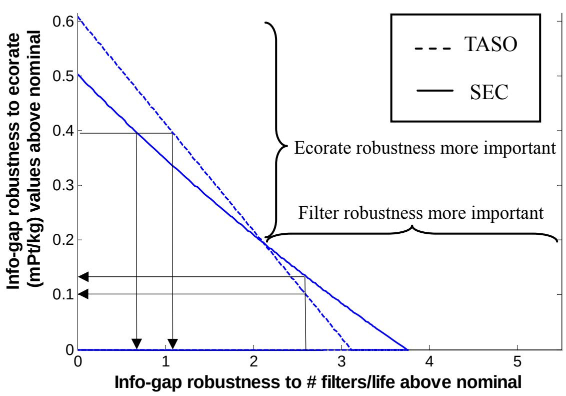

Figure 10. Tradeoff between robustnesses, $I_{critical} = 38.72$ mPt

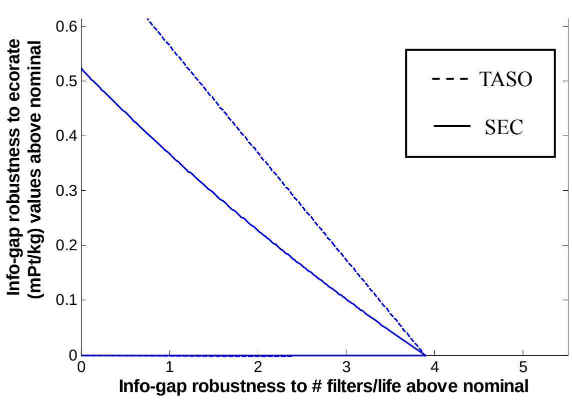

Figure 11. Tradeoff between robustnesses, $I_{critical} = 39.00$ mPt

The following is a narrative of the implication of changes seen in the plots as demand for performance is relaxed. As shown in Figure 8, the SEC option clearly always provides more robustness to the both uncertainties than the TASO does. In Figure 9, at a satisficing critical performance of 38.48 mPt (0.32 mPt more impact allowed than the previous figure), the TASO option has almost caught up in its ability to provide robustness to error in the *ecorate* estimate, but SEC is still preferable for any tradeoff between robustnesses.

In Figure 10, determining a preference for a design alternative now requires an understanding of how to trade between achieving robustness to one uncertainty or the other, which, in the case of severe uncertainties, may not be possible. As indicated by the two horizontal arrows in Figure 10, when a need for robustness to error greater than 2.15 filter changes (above nominal) takes priority over the need for robustness to ecorate error, the SEC filter can still afford more robustness to uncertainty in filter changes *and* the ecorate than TASO can. On the other hand, as indicated by the vertical arrows, if a need for robustness to error greater than 0.2 mPt/kg in the ecorate takes priority, the TASO filter can afford more robustness to uncertainty in the ecorate *and* number of filter changes than SEC can. In Figure 11, where the demand for small impact is relaxed further to 39.0 mPt, TASO now becomes the clear choice for any tradeoff between robustnesses.

Before, when evaluating the tradeoff between robustness and performance in two-dimensions, there was a distinct point, at some level of critical performance, where the robust-optimal design switched. However, with two uncertainties considered simultaneously, as explored in this section, there is now a range of satisficing critical performance levels, e.g., $I_{critical}$=[38.5, 39.0] mPt, for which the choice is indeterminate unless a tradeoff between the multiple α values can be expressed. Though small in this example, if the range were large, the tradeoffs between robustnesses would become important. To our knowledge, this idea has not been considered in previous IGDT examples in the literature and represents an area for future investigation.

6. DISCUSSION

Observations from the preceding examples are now generalized into a discussion about possible applications of information gap decision theory in the context of more general environmentally benign design applications. The prospects and need for future work is mentioned throughout.

6.1. When IGDT is warranted

In certain situations, the info-gap design analysis approach can eliminate the need for further data collection by facilitating decision making under extreme uncertainty (i.e., when estimation error cannot be quantified). For instance, if a switch in design choice (e.g., from SEC to TASO) requires a small sacrifice in guaranteed performance yet affords a reasonably large amount of extra robustness to error in a nominal estimate, one could decide to switch their choice without collecting more information. A item for future work is to quantify the information cost savings that IGDT analyses generate.

Although info-gap models are meant for use when less information is available than is required by the other uncertainty representations discussed in Section 2.2, it seems possible that there are still "gray areas" where, given the available information, it is difficult to know which approach will produce the best results. For example, in Section 5.1 it was assumed that the uncertainty in the number of filters was extreme. A strong argument could be made that this uncertainty could be bound with an interval, such as $F = [5, 40]$. Which is a better approach, IGDT or interval analysis? Future work will include experiments comparing IGDT results to those of other approaches with different information, assumptions, and values, with an aim towards eventually developing guidelines for when IGDT would be more appropriate or less expensive to apply.

6.2. Intuitiveness of evaluating severe uncertainty and satisficing critical performance

The IGDT approach requires that the decision maker be able to evaluate the acceptability of some satisficing level of critical performance in light of the corresponding gain in robustness to an info-gap of unknown size. In the examples in this paper, we assumed that the decision maker could state a preference for some acceptable size in the Eco-indicator 99 measure of environmental impact, specifically $I_{critical}$=40mPt. Although Eco-indicator scoring is grounded in reality, with one millipoint corresponding to 1/1000000 of the environmental load of a European citizen over 1 year, the Eco-indicator 99 construct was primarily developed to compare options relatively, not absolutely [16]. Whether or not a decision maker would find it reasonable to state one's preference for an absolute millipoint score with that reference point in mind is left to future study. It is noted that a similar challenge exists when performance is expressed in terms of utility.

Similarly, IGDT requires a decision maker to have a *relative* understanding of the magnitude of deviation around an uncertain quantity's nominal estimate, but not all uncertainty severities are equally easy to assess. In this example, it is probably easier to understand the severity of error in number of lifetime filter changes in Section 5.1 than to understand the severity of particular errors in an ecorate in the analysis of Section 5.2. This problem was compounded in Section 5.2 because there were uncertain ecorates whose actual values differ for different materials. Difficulty assessing the severity of an uncertainty makes trading off critical performance to gain robustness difficult, perhaps prohibitively so. A discussion of calibration and judgment of tradeoffs is considered in an entire chapter by Ben-Haim [4], but more experimentation is needed to determine the efficacy of such techniques in environmentally benign design problems.

6.3. Considerations when using IGDT for multiple uncertainties

In general, analyzing the relationships between critical performance, info-gap robustness, and the robust-optimal design increases in difficulty whenever any of them have multiple dimensions. In Section 5.3, it was shown that having multiple uncertainties made visualizing and understanding tradeoffs more involved. The established technique of parameterizing all uncertainties with a single α was shown in Section 5.3.1 to be feasible but restrictive, as all errors had to be defined as normalized by their nominals as well as all growing at the same rates.

The novel technique presented in Section 5.3.2 of assigning a separate uncertainty parameter α_i to each uncertain quantity revealed that there are possible ranges of indeterminacy that are not identified when uncertainties are lumped into a single α. However, a three-dimensional viewing method was needed to facilitate an understanding of relationships. For this multi-α method to be usable for examples with more uncertainties or more than two design alternatives, mathematically generalized expressions for indeterminacy ranges, or perhaps better visualization techniques, are necessary. In parallel, future work is needed to determine how large the ranges of indeterminacy are, as well as whether or not a designer could successfully tradeoff robustnesses between different uncertainties, making decisions possible in the ranges of indeterminacy.

6.4. Other future areas to investigate

There are other opportunities for future work besides those mentioned in previous sections. With the goal in mind to integrate economic assessments into environmentally benign design, support for multi-objective problems is necessary. The existing multi-criteria techniques used in info-gap decision theory [4], which involve defining goals preemptively, may have practical limitations similar in nature to those found when designing for multiple uncertainties. Also, the implications of IGDT need to be considered for a wider variety of the uncertainties across the components originally laid out in Figure 1; in this paper, only life cycle events and proxies for real environmental impact were explored. In addition, only uncertain variables have been examined, not uncertain models, for which IGDT may be more useful, given that model error may be more difficult to quantify. Finally, it is the expectation of the authors that more careful and structured experiments comparing uncertainty formalisms can be used as the foundation for a framework for systematic treatment of the typical uncertainties encountered in environmentally benign design.

7. SUMMARY

Information-gap decision theory (IGDT), developed by Ben Haim [4], seeks to assist a decision maker in making decisions that yield satisfactory performance in the presence of severe uncertainty. The examples examined in the paper have shown that IGDT has promise for expanding decision making capabilities under severe uncertainty in environmentally benign design problems. However, assessing one's preference for robustness versus critical reward becomes more complex as the nature and number of uncertainties increase. A clearer demarcation of the effectiveness of info-gap in practical situations, as well as closer examination of the method with respect to other robustness approaches, is left to future work.

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