# Evolution of Alfvén Eigenmodes during the Sawtooth Cycle at ASDEX Upgrade

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# Introduction

In recent experiments conducted at ASDEX Upgrade, modes with frequencies between those of toroidicity-induced Alfvén eigenmodes (TAEs) and the kink instability have been observed using the soft X-ray diagnostic [1]. These modes seem to exist close to the q = 1 surface and have been observed during ICRH shots. It is proposed that these modes are beta-induced Alfvén eigenmodes (BAEs). BAEs are electromagnetic,  $n \neq 0$  perturbations, driven by energetic particles, usually located in the plasma core [2]. BAEs occur in the low frequency gap caused by coupling between Alfvén waves and sound waves, at frequencies below the Alfvén continuum BAE minimum. Geodesic curvature and compressibility cause the change from cylindrical symmetry that creates a low-frequency gap which increases with beta.

### **Experimental Observations**

What appear to be BAEs have been observed in certain AUG discharges during sawtooth activity. Shot 25549 is a medium/high density shot with a plasma current of 800kA, that exhibits L-mode behaviour. For the time-slice in question only ICRH is present and there is continuous sawtooth activity throughout the



Figure 1: Mode activity during sawtooth cycle.

period of 4.38MW flat-top ICRH power. Figure 1 shows mode activity, measured using the SXR diagnostic, taking place during a sawtooth cycle between t = 2.125s and t = 2.165s. The above mode frequency measurements are taken using a SXR channel directed towards the plasma core, with tangency radius approximately between  $\rho_{pol} = 0.25$  and  $\rho_{pol} = 0.35$ . The modes that are visible approximately between 60kHz and 80kHz are seen to dip signifigantly in frequency during the first half of the sawtooth cycle before chirping upwards. Definite splittings in the mode frequency are observed and from SXR measurements these are found to correspond

to toroidal mode numbers n = 3, 4, 5 and 6, in order of decreasing frequency.

It is proposed that there is a strong relationship between the evolution of the core plasma temperature during the sawtooth cycle and this mode behaviour. In this case, the q = 1 surface is inferred from ECE measurements to lie at approximately  $\rho_{pol} = 0.35$ . This also gives an estimate for the sawtooth inversion radius, which is the radial location where the temperature is ex-



**Figure 2:** Evolution of electron temperature gradient near q = 1 surface.

pected to remain constant throughout the sawtooth cycle, and which is usually located slightly outside the q = 1 surface. Figure 2 shows the difference in temperature between ECE channel 55 at the q = 1 surface and ECE channels immediately surrounding it over time.

As the locations of the ECE measurements are essentially fixed over the duration of the sawtooth cycle, the changes in temperature differences appear as an initial steepening of the local temperature gradient followed by a flattening as the cycle reaches it's end. This local flattening is due to sawtooth precursor oscillations directly preceding the crash which serve to expel particles and decrease the gradient.



**Figure 3:** Fitted  $T_e$  profiles near q = 1 surface.

#### **Numerical Analysis**

In order to investigate this behaviour, a kinetic balooning mode dispersion relation was rederived for the gyrokinetic model underlying the eigenvalue code LIGKA [3] [2]. This derivation is also based on a Fourier expansion in the poloidal angle but keeps the full resonances, i.e. is valid for low q. Keeping the  $m \pm 1$ -sidebands, retaining the geodesic curvature and the sound wave coupling by an appropriate approximation of the propagator integrals, leads to:

$$\frac{\omega^2}{\omega_A^2} (1 - \frac{\omega_{*p}}{\omega}) - \frac{(k_{\parallel m}^2 + k_{\parallel m+1}^2) \pm \sqrt{(k_{\parallel m}^2 - k_{\parallel m+1}^2)^2 + 4\varepsilon^2 \rho_{pol}^2 k_{\parallel m}^2 k_{\parallel m+1}^2}}{2(1 - \varepsilon^2 \rho_{pol}^2)} = 2(\sqrt{\frac{2}{1 + \kappa^2}}) \frac{v_{thi}^2}{R_0^2 \omega_A^2} (-[H(x_{m-1}) + H(x_{m+1})] + \frac{\tau[\frac{N^m(x_{m-1})N^{m-1}(x_{m-1})}{D(x_{m-1})} + \frac{N^m(x_{m+1})N^{m+1}(x_{m+1})}{D(x_{m+1})}])$$
  
where  $x_m = \frac{\omega}{k_{\parallel m} v_{th}}$ ,  $v_{thi}^2 = \frac{2T_i}{m_i}$ ,  $\omega_{*p} = \omega_{*n} + \omega_{*T} = \frac{T_i}{eB} k_\theta (\frac{\nabla n}{n})(1 + \eta)$  with  $\eta = \frac{\nabla T}{T} / \frac{\nabla n}{n}$  and  $D, H$   
and  $N$  representing the sound wave, geodesic curvature and coupling terms respectively. The  
equation has been modified in order to take account of plasma elongation effects [4] and the in-

fluence of  $Z_{eff}$ . TAE coupling is not important here but has been kept for completeness.  $\kappa$  was taken to be 1.67 and a  $Z_{eff}$  of 1.6 was chosen. In order to obtain reliable input profiles for this analysis, equilibrium reconstructions were carried out for a number of timepoints to determine the electron temperature and density as well as realistic q-profiles that evolve with the sawtooth cycle. This was done using the CLISTE code [5].  $q_0$  was assumed to decrease from  $q_0 = 1.00$  at the beginning of the cycle to  $q_0 \approx 0.95$  at the end of the cycle, just before the next sawtooth crash. This q-profile evolution is shown in figure 4. As the modes are observed at the same location after the crash, the q = 1 surface was assumed fixed at approximately  $\rho_{pol} = 0.35$  [6].

In order to model the local temperature gradient changes near the q = 1 surface, measurements taken from individual ECE channels at given timepoints were splined into the time averaged temperature profiles. These are shown in figure 3. The effects of density variations during the sawtooth cycle were not included in the present analysis and the density profile was assumed to remain constant throughout.

#### Results

Figure 5 shows the BAE accumulation points for various toroidal mode numbers at the maximum and minimum temperature gradients. It is evident how the BAE frequency minima chirp upwards in time at the end of the sawtooth cycle and how modes with higher toroidal mode numbers experience a greater downward shift. Figure 6 compares results obtained numerically to 0.170.160.150.140.120.20.230.34



**Figure 4:** Evolution of *q*-profile during sawtooth cycle.





experimental observations for mode numbers n = 3 - 6 and t = 2.125s-2.160s. It can be seen that the numerical behaviour agrees well qualitatively with the experimental, both exhibiting a dip in frequency followed by an unchirp and higher mode number modes experiencing a more pronounced frequency down-shift. A Doppler shift of  $nf_{rot}$  was added to the numerical results, where *n* is the toroidal mode number and  $f_{rot} = 3$ kHz is the plasma rotation frequency, inferred from the sawtooth precursor frequency. The BAE frequency was also subject to a downshift of approximately 20% as a result of trapped particle effects [7] [2]. It should also be noted that the numerical results correspond to the frequency of the Alfvén continuum minimum. The BAEs themselves will exist at a frequency below the minimum and so, the numerical results should be subject to a downward frequency shift. Quantitatively, the numerical results are found to be several kHz above the experimental ones. This could be a result of the omission of density gradient shifts, an over-estimation of  $Z_{eff}$  effects or an under-estimation of trapped particle effects.

Due to the significant temperature gradient variation near the q = 1 surface, diamagnetic effects become very important close to the BAE minimum as  $\omega_{*p} \propto \nabla T$ .  $\omega_{*p}$ varies approximately between one and two thirds of the BAE frequency, depending on the mode number. Thus, it strongly affects the dispersion relation. When the the mode and diamagnetic frequency become comparable, for higher toroidal mode numbers, the BAE



**Figure 6:** Comparison of experimental and numerical results for BAE frequencies.

branch appears to couple to the ion diamagnetic branch, as seen in figure 6. A further correlation between electron temperature and BAE frequency is seen at approximately t = 2.138s, where a partial sawtooth crash seems to occur. This results in a small jump in the temperature of some of the ECE channels, accompanied by a sudden up-chirp in the observed mode frequency.

# Conclusion

A strong correlation has been observed experimentally between changing temperature gradients and mode frequencies during the sawtooth cycle and this has been confirmed numerically. It has been suggested that this is related to the diamagnetic frequency which becomes an appreciable fraction of the mode frequency in this situation. It is hoped to extend this analysis in the future by including variations in the density profile during the sawtooth cycle and to apply it to the investigation of beta-induced Alfvén-acoustic eigenmodes (BAAEs).

# References

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