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ON THE INVESTIGATION OF MACHINE TOOL CHATTER IN THE MILLING PROCESS

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ABSTRACT

In this paper, the chatter phenomenon is investigated through a single degree of freedom model of the milling process. In this regard, the non-linear equation of motion obtained from modeling of the milling process, which is a time-periodic delay differential equation, is simulated, and by changing the parameters: spindle speed and depth of cut, and assuming constant quantities for other parameters of the system the stable and instable points for the system are gained according to these two parameters by numerical method. In the end, the stability chart for this system is plotted and the approximate boundaries between the stability and instability regions are obtained numerically.

Key words: Modelling, Simulation, machine tool chatter, regenerative effect

1 INTRODUCTION

Industrial Competition augmentation in today's developed technology has driven the manufacturers' attention to increase speed and accuracy in manufacturing more than the past. Machining Operations are one of the most widely used manufacturing processes [1, 2]. One of the most important problems in this field is the vibration occurrence and controlling it in machining. In every machine tool design, it is attempted to reduce its undesirable effects of vibration including noise, poor surface finish, reduced dimensional accuracy, and shortened machine tool life as much as possible[2,3]. One of the most prevalent vibrations in machining is a phenomenon called chatter. Chatter can be simply described as the self-excited, large amplitude periodic relative vibration between the tool and the work-piece.[3,10]

The history of machine tool chatter goes back almost 100 years, when Taylor described machine tool chatter as "the most obscure and delicate of all problems facing the machinist" in ASME conference in 1907 [4]. After the extensive work of Tlusty et al [5], Tobias [3] and Merrit [6] the so-called regenerative effect has become the most commonly accepted explanation for machine tool chatter [7]. In fact, the phase difference between the waves produced by the vibration between the tool and the work-piece, caused by any external or internal perturbation, and the waves on the work-piece surface cut during the previous revolution, results in the chip thickness variation at the tool's edge. This leads to the cutting force variation that excites the structure and the regenerative effect occurs [10]. Additionally, two other mechanisms for machine tool chatter has been recognized: mode coupling and velocity dependent effect.[10] However, between these three mechanisms, the regenerative effect is the most important and influential one and is mostly considered. [10]

The necessity for preventing chatter and the control of it justifies the significance of its dynamic investigation and analysis. According to analysis of dynamic process of chatter and predicting the conditions of chatter occurrence, several analytical and numerical methods have been presented [9,10]. Predictive models for machining operations can provide favorable circumstances to ameliorate the process efficiencies and dimensional precision. Dynamic machining models give the manufacturers the opportunity to predict regions of stable and unstable cutting for a large combination of process parameters. This permits these models to be used in place of costly trial and error for process optimization [11].

For mathematical description of a system that regenerative effect plays roll in it, delay differential equations (DDEs) are used. The DDE describes a system where the present rate of

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change of state depends on a past value of the state. This effect is exerted by the term having the delay. [10, 12]
In the case of milling, the direction of the cutting force is changing with the tool rotation, and as each tooth enters and leaves the work-piece, the cutting process confronts with interruption. As a result, the equation of motion should be a DDE with a time periodic coefficient [11]. The amount of the time delay should be equal to the activity time of one tooth.
Beside this, numerous types of nonlinearities can affect the dynamic behavior of milling processes. These nonlinearities can be regenerative or arisen from the intermittent nature of the cut itself. They evidence that many milling vibrations may actually be chaotic and new schemes controlling chaotic vibrations might be applicable. These kinds of vibrations can arise simultaneously with more familiar dynamic phenomena such as chatter. [10]
In this paper the dynamic instability and chatter phenomenon have been studied in a single degree of freedom mechanical model of the milling process. For this purpose, the time-periodic delay differential equation of motion obtained from modeling of the milling process, which is non-linear, is simulated.
As the objective of this paper is simple qualitative investigation of chatter and regenerative effect, in the next chapters, the equation of motion extracted from the mechanical model is numerically simulated and with utilization of the results, a picture of the system behavior in stability and instability regions including chatter arising areas has been obtained.

2. MODELLING

The detailed mechanical model of the milling process is shown in Fig.1. [7, 9] The mass of the tool *(m)*, the damping coefficient *c* and the spring stiffness *k* can be determined by the modal analysis of the machine tool. *x* is the displacement of the centre of the tool relative to the work-piece. As the structure is supposed to be flexible only in the *x* direction, the model is single degree of freedom. When the structure is flexible mostly in one direction while it is likely to be rigid in the orthogonal direction, this kind of model is appropriate. [7]

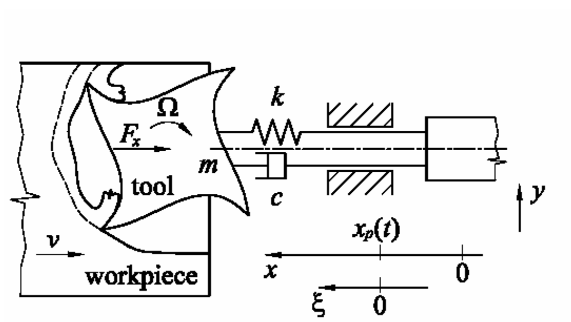


Fig.1. Regenerative mechanical model for milling [7]

By assuming the prescribed feed motion uniform with a constant speed *v* of the work-piece, according to Newton's law, the equation of motion is:

$$m\ddot{x}(t) = -F_x - kx(t) - c\dot{x}(t) \qquad (2.1)$$

The cutting force F_x can be calculated by determination of its components acting on an active tooth (number *j*):
The tangential component of the cutting force can be approximated by:

$$F_{jt} = Kw(f\sin\varphi_j)^{x_F} \qquad (2.2)$$

Where *K* is the cutting coefficient, *w* is the depth of cut; *f* is the feed per tooth and φ_j refers to the angular position of the tool. x_F is a small constant. $x_F = 0.8$ is a typical value.
The normal component of the cutting force acting on the *j*th tooth is usually estimated as:

$$F_{jn} = 0.3F_{jt} \qquad (2.3)$$

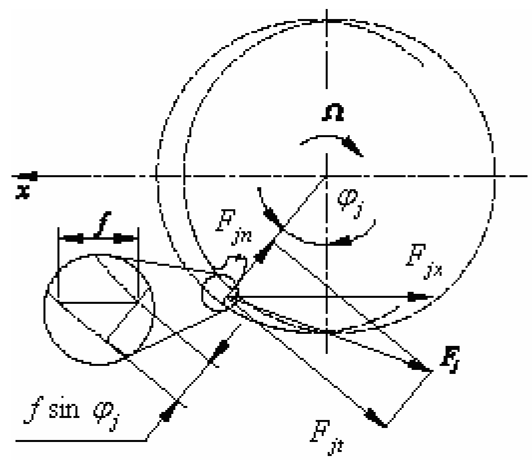


Fig.2. Cutting force components [7]

The *x* component of the cutting force is shown in Fig.2 is:

$$F_{jx} = g_j(t)(F_{jt}\cos\varphi_j + F_{jn}\sin\varphi_j) \qquad (2.4)$$

$g_j(t)$ acts as a switching function. It is equal to one of the *j*th tooth is active, and is zero if it is not.

If the spindle speed (Ω) is given in *r.p.m*, the tooth path period will be $\tau = 60/z\Omega$ (s) where *z* is the number of the teeth [7]. The feed is equal to the difference of the present and the delayed position of the tool, plus the distance covered by the work piece relative to the tool in the time of each activity:

$$f = x(t) - x(t-\tau) + v\tau \qquad (2.5)$$

The angular position of each tool is $\varphi_j = \Omega t + j\theta$ where $\theta = 2\pi/z$. [7, 9]

As a result, the *x* component of the cutting force acting on the tool is given by the sum of F_{jx} for all *j*:

$$F_x = wq(t)(x(t) - x(t-\tau) + v\tau)^{x_F} \qquad (2.6)$$

Where

$$q(t) = K[\sum_{j=1}^{z} g_j(t)\sin^{x_F}(\Omega t + j\theta) \qquad (2.7)$$

$$\left(\cos(\Omega t + j\theta) + 0.3\sin(\Omega t + j\theta)\right)]$$ [7, 9]

However, there are short moments that the tool leaves the work piece. When it happens, the chip thickness that is here equal to feed becomes negative, which is something that physically does not exist. At this point the cutting force becomes zero and the regenerative effect is 'switched off'. The free tool outside the work-piece begins a simple oscillating motion that is damped, and the tool will soon return to the work-piece. This is an important nonlinear part of the cutting force variation. [10]

To consider this effect here, the nonlinear equation of motion has been presented in two conditions as follows:

If $x(t) - x(t-\tau) + v\tau > 0$,

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -wq(t)(x(t) - x(t-\tau) + v\tau)^{x_F} \quad (2.8)$$

If $x(t) - x(t-\tau) + v\tau \le 0$

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0$$

As it can be seen, this equation is not continuous. In the next chapter the stability of the equation (2.8) is investigated by numerical simulation through different points.

The nonlinear equation (2.6) represents the motion of the machine tool relative to workspace. For theoretical stability analysis of the system the equation is linearized about the unperturbed motion and after obtaining a linear time periodic equation for perturbation, the stability of this equation has been investigated through different frequencies by Insperger, Stépán, Mann and Bayly [7].The theoretical approach is not the objective of this paper, however the linearization proposed in reference [7] is briefly outlined below:

We can consider the tool motion in the form of:

$$x(t) = x_p(t) + \xi(t) \quad (2.9)$$

Where $x_p(t) = x_p(t+\tau)$ is a τ periodic motion which is the ideal motion with no self-excited motion arising. $\xi(t)$ is the perturbation. Substituting equation (2.9) in (2.6):

$$m\ddot{x}_p(t) + c\dot{x}_p(t) + kx_p(t) + m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) =$$

$$-wq(t)(v\tau + \xi(t) - \xi(t-\tau))^{x_F} \quad (2.10)$$

In the ideal case the perturbation doesn't exist ($\xi(t) \equiv 0$) and the tool moves according to $x(t) = x_p(t)$ .For this case one can write this ordinary differential equation:

$$m\ddot{x}_p(t) + c\dot{x}_p(t) + kx_p(t) = -w(v\tau)^{x_F} q(t) \quad (2.11)$$

For linear stability analysis, the variational system of equation (2.6) is determined about the combined linear periodical motion $x_p(t)$ .Expanding the nonlinear term in equation (2.10) into Taylor series with respect to $\xi(t)$ and neglecting the higher order terms, gives the equation:

$$m\ddot{x}_p(t) + c\dot{x}_p(t) + kx_p(t) + m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) =$$
$$-w(v\tau)^{x_F} q(t) - wx_F(v\tau)^{x_F-1} q(t)(v\tau + \xi(t) - \xi(t-\tau))^{x_F} \quad (2.12)$$

Using equation (2.11) and (2.12) a linear periodic equation is obtained for perturbation:

$$m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) = -w(v\tau)^{x_F-1} q(t)(\xi(t) - \xi(t-\tau)) \quad (2.13)$$

When this equation loses stability or resonance occurs in equation (2.11) Chatter arises.

3 SIMULATIONS AND INSPECTION OF THE RESULTS

As mentioned in the previous chapter, in order to inquire the stability of them system the equation (2.8) has been numerically simulated. For this numerical simulation, the following amounts have been used for the system parameters:

$m = 2.586kg, k = 2.2 \times 10^6 N/m, c = 18.13 Ns/m,$

$K = 1.9 \times 10^9 N/m^{1+x^F}, x_F = 0.8$ [7]

In the calculations, the tool is assumed to have four teeth and the work-piece width, as shown in Fig.3, is exactly chosen in a way that every time two teeth will be active. In the computer program it has been supposed that the origin of the time corresponds to the start of the second tooth activity; i.e. when $0 \le t < \tau$, $g_1(t)$ and $g_2(t)$ are equal to one, though $g_3(t)$ and $g_4(t)$ are zero. At the time τ, the first tooth leaves the work-piece, while the third tooth enters it. Consequently, in this situation $g_1(t)$ changes to zero but $g_3(t)$ become one. Thus, this procedure continues.

According to the amounts mentioned for the parameters *m, c, k, K* and *z* and application of the initial conditions $x(0) = 0, v(0) = 0.2m/s$ the calculations have been performed for different amounts of spindle speed (Ω) and the depth of cut (w), and for each case the diagram of total displacement $(x(t))$ with respect to time, and the relevant phase diagram have been plotted.

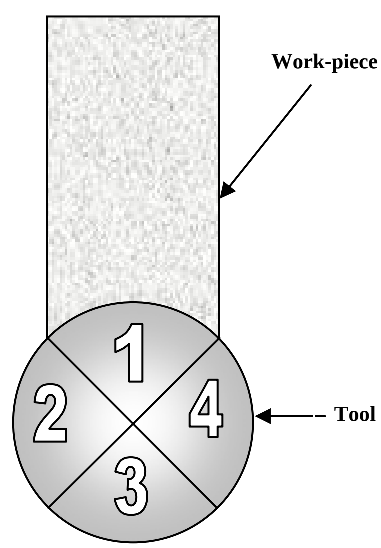
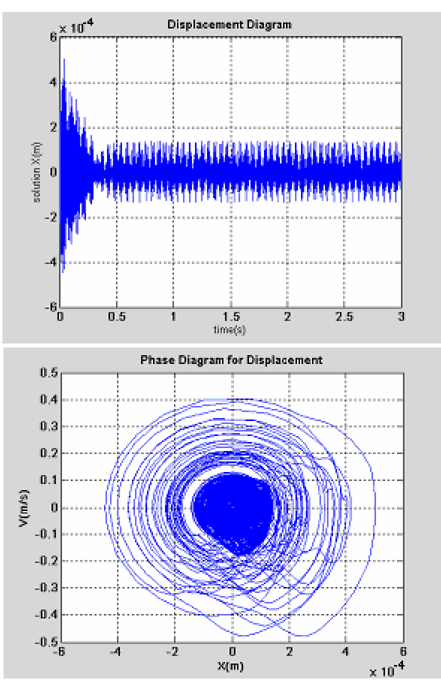


Fig.3. The situation of the teeth and the work-piece

In every plotted diagram, there is a transient state which is affected by the initial conditions and lasts only for a short time. Since the rate of the system behavior changes in this state is great, the pertinent section in the phase diagram appears as thinly scattered trajectories.
After termination of this state, the rate of the variations in the system function becomes small and the system behavior tends to an attractor specified in the phase diagram by very close trajectories forming a dense region.
Now, regarding to the initial conditions applied for solving the equation, the obtained diagrams are investigated:
The diagrams can be generally divided into two groups. It is observed that the vibration amplitude in some of the diagrams (except in the transient state) is low, while in other diagrams the amplitude has a clear difference with the first group, sometimes with a divergence, that it is much higher and even sometimes it reaches to millimeters. So, it can be concluded that in the latter group chatter has occurred.
Here, the border of *0.5 mm* is determined between the stability and instability, which is practically reasonable. In the cases that the vibration amplitude is approximately in this extent, it can be said that the point is located in the environs of the stability and instability borders.
As an example, in the case of $w = 2mm, \Omega = 3100r.p.m$ that refers to a stable point, after finish of the transient state which is affected by the initial conditions, the vibration amplitude decreases and a kind of convergence can be observed in the displacement diagram. Relevantly, in the phase diagram the outsider trajectories of the diagram that belong to the higher amplitude vibration, as they correspond to the transient state, appear sparse.



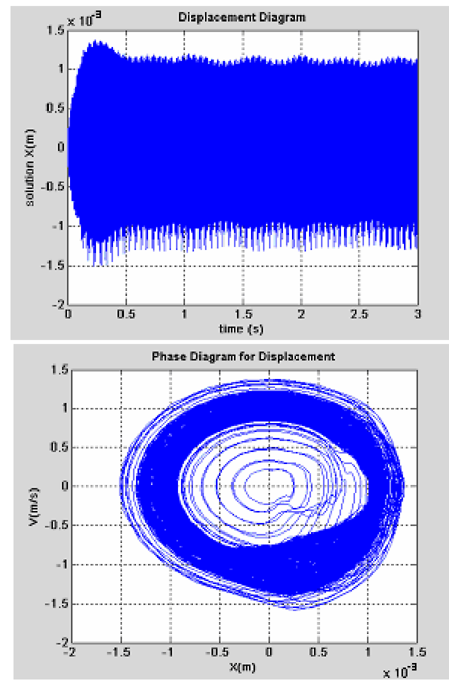
(a)

(b)

Fig.4. (a) The displacement diagram and (b) the corresponding phase diagram of the case: $w = 2mm, \Omega = 3100r.p.m$

But after the expiration of the transient state, as the system behavior variations become small, the trajectories get very close to each other. Consequently, the central part of the diagram that refers to lower amplitude vibration is shown as dense trajectories that form an attractor.

When the amplitude of vibration is great and the system faces instability, the situation is converse. The case of $w = 2mm, \Omega = 3400r.p.m$ is an example.



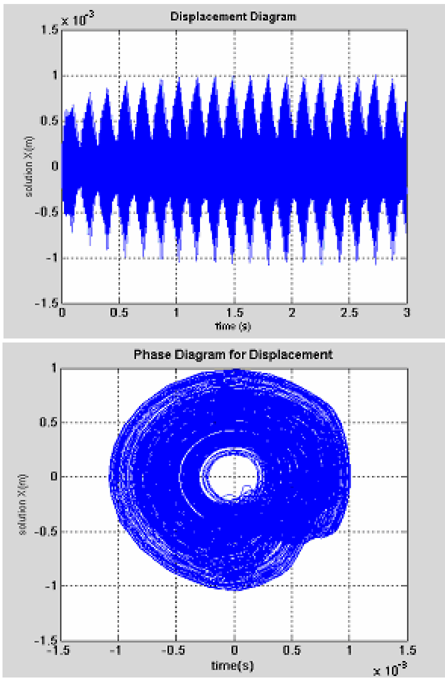
(a)

(b)

Fig.5. (a) The displacement diagram and (b) the corresponding phase diagram of the case: $w = 2mm, \Omega = 3400r.p.m$

In this group of diagrams, in the transient state the vibration amplitude is comparatively small. But it rapidly increases until it gets to a state that the changes of the system behavior become small. As a result, in the displacement diagram a kind of divergence appears. Pertinently, in the phase diagram the central region appears as sparse trajectories, but the dense appearing attractors locates in the outsider region.
Evidently, these characteristics are qualitative. As an example, the length of the transient state is not the same in all the diagrams.
As another example, in some of the diagrams, such as those corresponding to the case $w = 2mm, \Omega = 3300r.p.m$ it can be seen that in the displacement and the phase diagram, there is no obvious distinction between the transient state and the attractor. Only in the transient state the maximum amplitude increases gradually.
Also, as it appears in the displacement diagram, the rate of the amplitude variation neither in the transient state nor after that is small. However, the system follows a common pattern periodically.

It is evident that the result is a large area of the attractor in the phase diagram, because it covers a wide range of amplitudes.



(a)

(b)

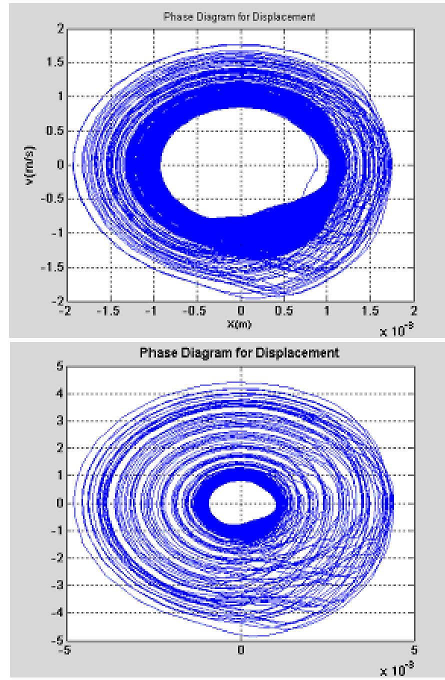
Fig.6. (a) The displacement diagram and (b) the corresponding phase diagram of the case: $w = 2mm, \Omega = 3300 r.p.m$

More over, separate from general behavior of the system, there are some insignificant convergences or divergences in some of the displacement diagrams, which cause small changes in the corresponding phase diagrams.

It can be shown that the system behavior after the transient state is dependent from the initial conditions applied to the system. In other words, by applying any initial conditions, the same attractor will be reached.

Here, initial conditions for different cases have been changed, and expectedly the attractor in the new results coincides to the previous ones. It is evident that there is a unique attractor that is independent from the initial conditions. As an example, for the case $w = 2mm, \Omega = 3400 r.p.m$, the Initial conditions once have been changed from $x(0) = 0, v(0) = 0.2 m/s$ to $x(0) = 0.001m$, and $v(0) = 0.25 m/s$ and once to $x(0) = 0.001m$, $v(0) = 1.5 m/s$ which the phase diagrams are shown in Fig.7 (a) and (b) respectively. As it can be seen the attractors are the same, but behavior of the system in the transient state changes.

Beside this, by plotting the power spectrum for the diagrams the existence of one dominant frequency for the system vibration can be shown, which is higher than other frequencies. This concludes that the pattern of the system behavior is quasi-periodic.



(a)

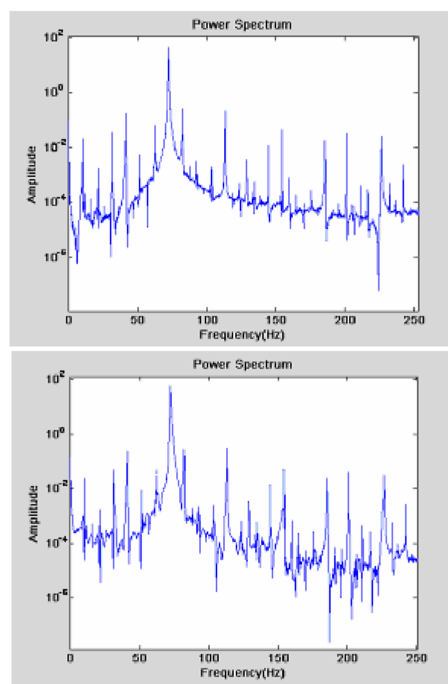
(b)

Fig.7.Phase diagram for the case $w = 2mm, \Omega = 3400 r.p.m$,with the initial conditions of

(a): $x(0) = 0.001m, v(0) = .25 m/s$ and

(b): $x(0) = 0.001m, v(0) = 1.5 m/s$

Here, the power spectrum has been plotted for two previous diagrams which are shown in Fig.8 (a) and (b).



(a)

(b)

Fig 8.(a). Power spectrum related to Fig 7.(a)
Fig 8.(b). Power spectrum related to Fig 7.(b)

As discussed in section 2, the nonlinear equation of motion is presented as a conditional equation (2.8). Otherwise, there will be large amplitude vibrations in the results that in fact never happen in reality. The condition stated in equation (2.8) omits these unreal situations from the results, and make them more realistic. As an example, the displacement diagram of the case $w = 2mm, \Omega = 3400 r.p.m$ without considering the conditions has been plotted, in Fig.9. As it can be seen, the vibration amplitude is much higher than the previously investigated diagrams, which is not practical. Since much earlier than the vibration amplitude raises this high, the tool will loose its contact with the work piece.

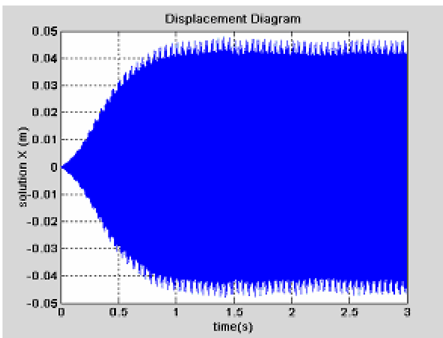


Fig.9. The displacement diagram for the case $w = 2mm, \Omega = 3400 r.p.m$ without the conditional consideration

Based on the obtained results, the stability chart of this mechanical system with the specified parameters can be approximately plotted numerically. This chart is in the form of a lobe diagram, in which depth of cut has been drawn with respect to spindle speed. By finding the stability and instability borders, the whole area of the diagram can be divided into two stable and unstable sections. The stability chart plotted for this system is shown in Fig.10.

4 CONCLUDING REMARKS

In this paper, in order to study the chatter phenomenon, according to single degree of freedom model of milling process, the nonlinear equation of motion of the machine tool relative to the work piece has been simulated numerically, and the calculations have been performed for different amounts of spindle speed and the depth of cut.

Each situation can present a point in the *spindle speed-depth of cut* diagram. (The stability chart) By determining the stable and unstable points and passing a curve through the border points, the approximate stability chart for the system can be obtained. In Figure 10 the stability chart of the system with the parameters mentioned in part 3 has been plotted between *2000* and *8000 r.p.m.*

The unstable points have been concentrated in finite areas which form an unstable region.

It is apparent that in every spindle speed, increasing the depth of cut increases the vibration amplitude, While the increase of spindle speed in a constant depth of cut, does not always leads to vibration amplitude increase.

As this method can show the system behavior in each point, it can be used for the approximate function prediction of a system with specific parameters.

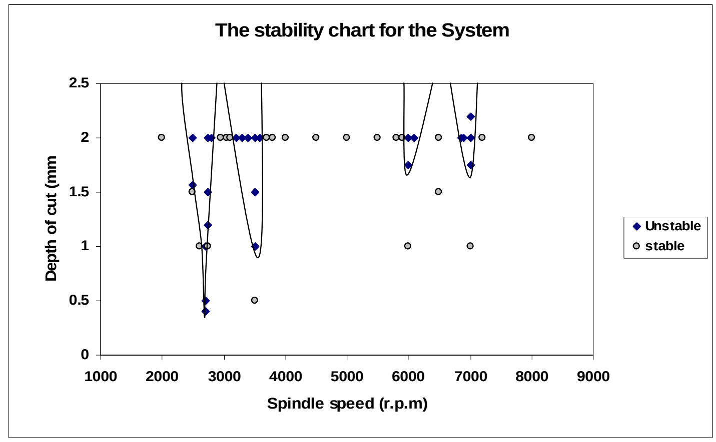


Fig.10. The stability chart for the system

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5 REFERENCES

[1] J. Tlusty, Manufacturing Processes and Equipment, Prentice Hall, New Jersey ,2000

[2] Robert C. Creese, Introduction to Manufacturing Processes and Materials, CRC Press,1999

[3] S.A. Tobias, *Machine Tool Vibration*, Blackie, London, 1965.

[4] F.W. Taylor, “On the art of cutting metals”, *Transactions of ASME* 28 (1907) pp. 31-350

[5] F. Koenigsberger, J. Tlusty, Structure of Machine Tools, Pergamon Press, Oxford, 1971

[6] H.E. Merrit,” Theory of Self-Excited Machine Tool Chatter”, *ASME Journal of Engineering for Industry*,87/4 (1965)pp. 447-454

[7] T. Insperger, G. Stépán, P. V. Bayly, B. P. Mann, “Multiple chatter frequencies in milling processes”, *Journal of Sound and Vibration* 262 (2003)pp. 333-345

[8] N. Sri Namachchivaya, R. Beddini, “Spindle Speed Variation for the Suppression of Regenerative Chatter”, *Journal of Nonlinear Science* 13 (2003) pp. 265-288.

[9] T. Insperger, G. Stépán, P. V. Bayly, "Stability of up-milling and down-milling, part 1: Alternative analytical methods", *International Journal of Machine Tools and Manufacture* 43 (2003) pp. 25-34.

[10] F.C. Moon, *Dynamics and chaos in Manufacturing Processes*, Wiley, New York, 1998

[11] T. Insperger, G. Stépán, P. V. Bayly, "Stability of up-milling and down-milling, part 2: experimental verification", *International Journal of Machine Tools and Manufacture* 43 (2003) pp. 35-40.

[12] F. Maghami Asl, A. Galip Ulsoy, "Analysis of a system of Linear Delay Differential Equation", *Journal of Dynamic Systems, Measurements and controls* 125 (2003)pp. 215-315