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COMBINED LOADING CRITERIA FOR TITANIUM RISERS

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ABSTRACT

DNV has, in cooperation with partners from the industry, carried out a joint industry project with the aim to develop recommended practice with respect to the design of titanium risers. As a part of this work, calibrated design formulae for combined loading have been established. The considered load situation is a combination of internal overpressure, bending moment and axial force.

The data basis for the calibration study encompasses results from 12 finite element simulations with varying diameter to thickness ratio and internal pressure exposed to bending moment and axial force.

With the design equation for steel risers, taken from the DNV Offshore Standard (OS-F201) *Dynamic Risers*, as a basis, the titanium data basis was investigated using state-of-the art methodology with an uncertainty modeling for load effects in compliance with recent research and development projects for risers and pipeline design.

The outcome of this work is a design equation with reliability based calibration of safety factors that comply with the overall safety objective in the above offshore standard.

INTRODUCTION

Successful design of titanium risers has been carried out for a limited number of cases over the last decade. In lack of relevant codes, the principles of standards such as DNV-OS-F101 *Submarine Pipelines* and the later DNV-OS-F201 *Dynamic Risers* have been used. Significant modifications are, however, necessary in order to use these standards for titanium risers. The fact that no established design codes for titanium risers are available, introduces extensive demands to the design process. The general impression is that the lack of a good and reliable titanium design code is one of the obstacles to utilizing titanium risers in a reliable and economical way.

DATA BASIS

Based on previous work, see e.g. Refs. Vitali (1999) and Bjørset (1999), an FE model was established in order to investigate the bending moment capacity of titanium risers with internal overpressure.

A total of 24 FE models were analyzed, 12 of these simulating a titanium material and 12 simulating a steel material. The latter were run for reference and comparison with previous work, e.g. Ref. Vitali (1999). Within each of the two series, the effects of varying diameter to thickness ratio and internal overpressure were investigated according to the following analysis matrix:

Table 1 Test matrix.

$q_h \rightarrow$ / $D/t \downarrow$	0.0	0.4	0.8
15	t-15-0 s-15-0	t-15-4 s-15-4	t-15-8 s-15-8
20	t-20-0 s-20-0	t-20-4 s-20-4	t-20-8 s-20-8
25	t-25-0 s-25-0	t-25-4 s-25-4	t-25-8 s-25-8
30	t-30-0 s-30-0	t-30-4 s-30-4	t-30-8 s-30-8

The letter "t" refers to titanium and the letter "s" stands for steel. D is the outer diameter of the pipe, t is the wall thickness whereas q_h is the hoop to yield stress ratio σ_h/f_y. E.g.: the notation t-25-4 refers to a titanium material model with D/t = 25 and q_h = 0.4.

The two material types, both assumed to be isotropic, were modeled according to the Ramberg-Osgood relation:

$$\varepsilon = \frac{\sigma}{E} \cdot \left[1 + \alpha \cdot \left(\frac{\sigma}{\sigma_R} \right)^{n-1} \right]$$

with input parameters as given in Table 2 below.

Table 2 Material input parameters.

Parameter & Unit		*Titanium*	*Steel*
E	[MPa]	$1.05 \cdot 10^5$	$2.07 \cdot 10^5$
ν	[-]	0.3	0.3
σ_R	[MPa]	766.8	429.1
α	[-]	3/7	3/7
n	[-]	44.016	27.48

These values yield the following reference values:

Titanium: f_y = 759.0MPa @ ε = 0.923% ,
f_u = 828.0MPa @ ε = 10.0%,
Steel: f_y = 448.0MPa @ ε = 0.5%, and
f_u = 530.2MPa @ ε = 20.0%.

Compared to steel, titanium is characterized by 70% higher yield stress but half the modulus of elasticity. The yield stress for titanium is taken at 0.2% plastic strain.

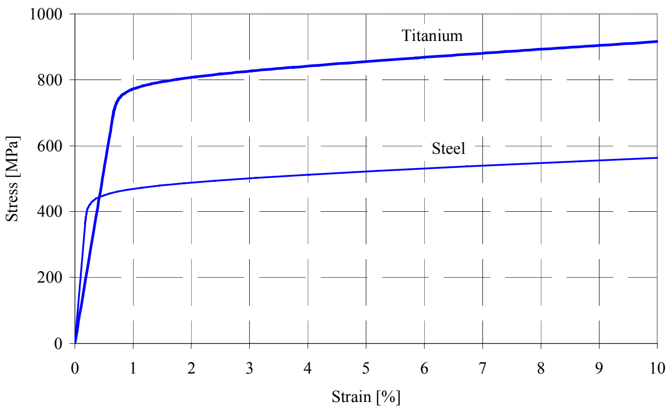

Figure 1 Material curves for titanium and steel, true stress and logarithmic strain.

Note that the intention is not to compare the actual strength for the two materials, but rather reveal possible differences in the general behavior when the material strength is almost doubled whereas the elasticity is halved.

The general-purpose structural computer code ABAQUS, Ref. Hibbit (1998) has been used to perform the simulations. A typical model is shown in Figure 2. The mesh consists of 24 shell elements (S4R) around half the circumference and 46 elements in the longitudinal direction extending a length equal to 3 riser diameters = $3 \cdot D$. A uniform mesh is used in the circumferential direction. Longitudinally a finer mesh is used near the mid cross section (where the buckle is expected to occur and the moment is recorded) and a coarser mesh closer to the other end. Totally the mesh contains 1584 elements and 1729 nodes. Seven integration points through the thickness are used.

One master node is defined at either end, i.e. node no. 10001 at $x_1 = x_2 = x_3 = 0.0$ and node no. 10002 at $x_1 = x_2 = 0.0$, $x_3 = 3 \cdot D$. When u_i refers to translations and θ_i to rotations (i = 1,2,3), the boundary conditions for node no. 10001 is $u_1 = u_3 = \theta_1 = \theta_2 = \theta_3 = 0.0$. Similarly for node no. 10002, the boundary conditions are $u_1 = u_2 = \theta_2 = \theta_3 = 0.0$ whereas the bending moment in the riser is introduced with an imposed rotation in degree of freedom θ_1 in node 10002. The shell element nodes at either end ($x_1 = 0.0$ and $x_1 = 3 \cdot D$) are kinematically coupled to their respective master node through the degrees of freedoms u_3 and θ_2. Finally, a longitudinal symmetry plane is introduced to further reduce the model size by introducing the boundary conditions $u_1 = \theta_2 = \theta_3 = 0.0$ along the two edges $x_2 = \pm 0.5 \cdot D$. (Reference is made to Figure 2.)

As mentioned above, the curvature is introduced by rotating the appropriate rotational degree of freedom in master node no. 10002. The internal pressure is introduced in a load step prior to this rotation which means that the riser is fully pressurized during the rotation.

The objective of the present analyses is to estimate the bending moment capacity for the various conditions, i.e. variations in slenderness and internal pressure. This moment has been retrieved directly from the master node no. 10001. The average strain over $0.0 \leq x_3 \leq 1 \cdot D$ has been estimated by the following procedure:

- First the rotation θ_1^{1D} of the cross section at $x_3 = 1 \cdot D$ is determined by $(u_3^{top} - u_3^{bottom})/D$ where u_3^{top} and u_3^{bottom} refers to the longitudinal displacement at the top and bottom (symmetry) edges one diameter from node no. 10001.
- Then the curvature is estimated by dividing this rotation over the true distance that it accumulates: $\kappa^{1D} = \theta^{1D}/(D + u_3^{mid})$ where u_3^{mid} is the mean longitudinal displacement of the nodes at $x_3 = D$.
- Finally the average strain is found by multiplying this curvature by the radius of the riser such that $\varepsilon^{1D} = \kappa^{1D} \cdot D/2$.

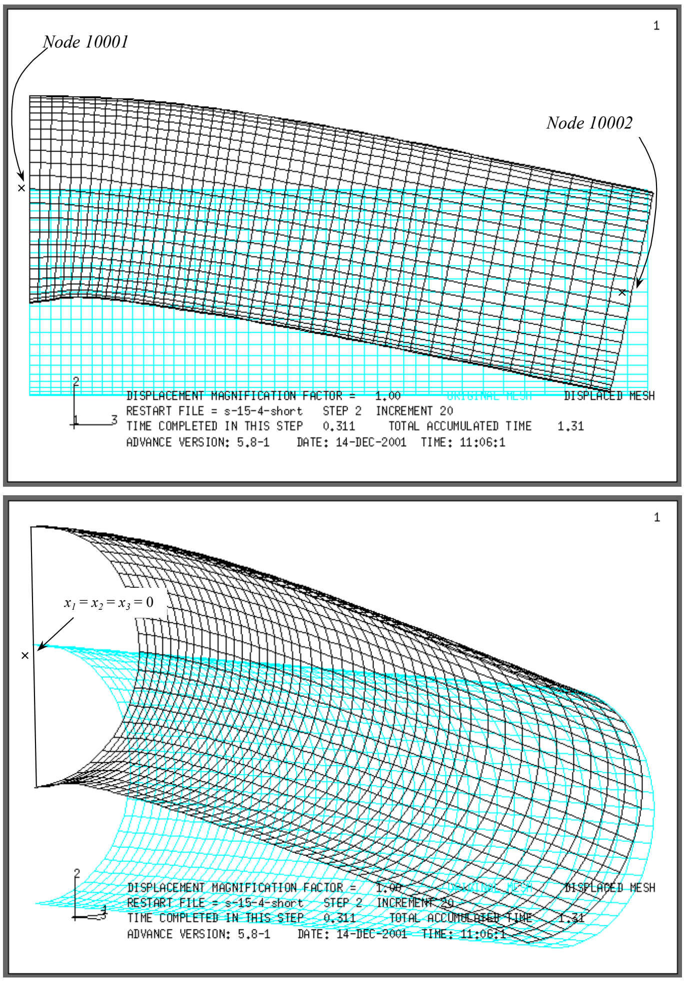

Figure 2 Finite Element model, initial and deformed configuration.

The results are presented as strain-moment relations in Figures 3 to 6. The lower elastic stiffness of the titanium models are easily recognized, whereas their bending moment capacity is significantly higher due to the higher yield stress of titanium.

It is also worthwhile to note that strain at which maximum bending moment occur increases with increased internal pressure and wall thickness.

The analyses do not take account for material strain limits and these are assumed to be lower for titanium than for steel. Work is ongoing in this area, but in practical design it will most likely not be a problem because equipment is not designed to operate, even in survival conditions, at such high strains.

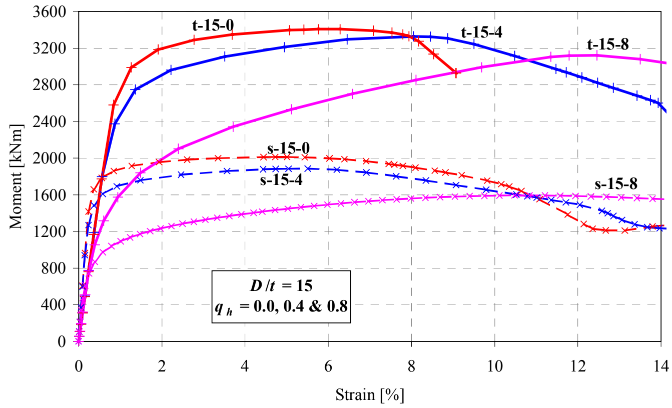

Figure 3 Moment versus strain for models with $D/t = 15$.

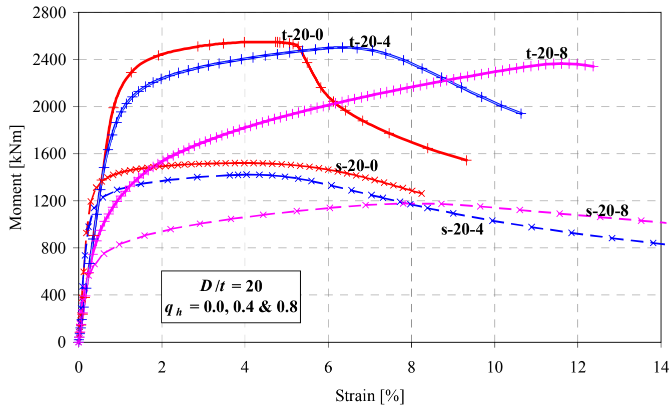

Figure 4 Moment versus strain for models with $D/t = 20$.

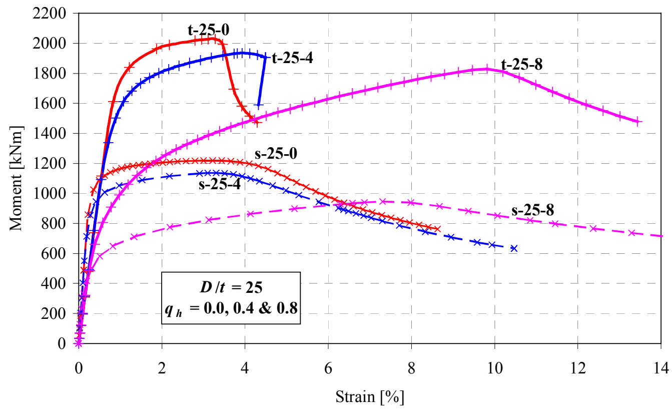

Figure 5 Moment versus strain for models with $D/t = 25$.

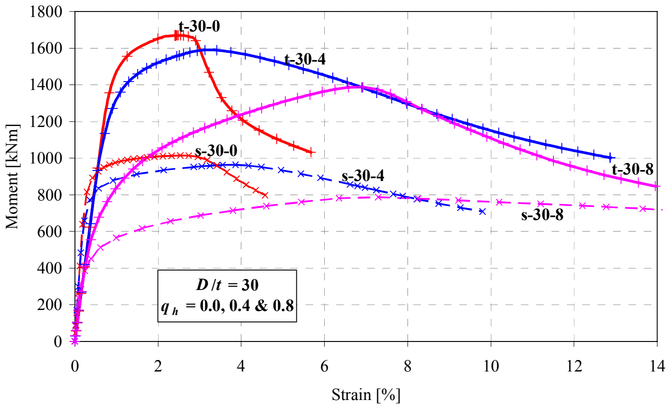

Figure 6 Moment versus strain for models with $D/t = 30$.

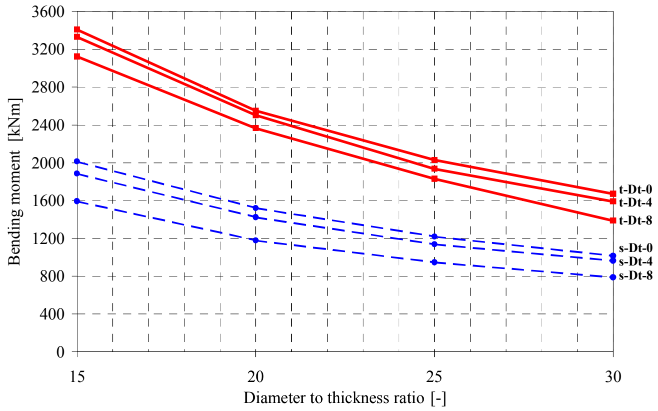

Figure 8 Maximum bending moment from FE analyses versus diameter to thickness ratio.

DISCUSSION OF FEA RESULTS

Figures 7 and 8 show the bending moment capacities as functions of internal pressure q_h (= $\sigma_h/SMYS$) and slenderness D/t, respectively.

As expected, the bending moment capacity for the titanium models is larger than that for steel due to the higher yield stress of titanium. Furthermore, it is seen that for both materials the capacity decreases slightly with increasing internal pressure, Ref. Figure 7, and decreases significantly with decreasing wall thickness, Ref. Figure 8. These relations are as expected.

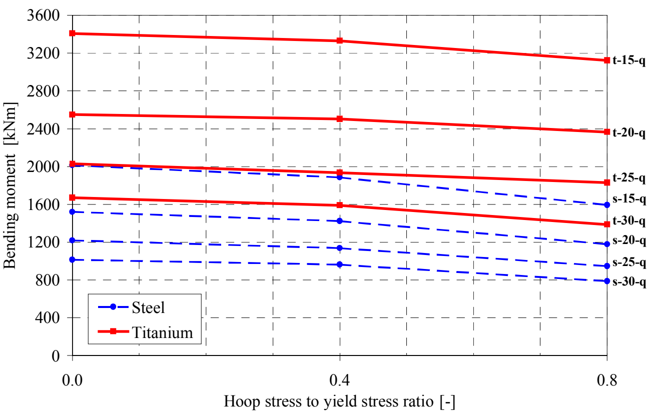

Figure 7 Maximum bending moment from FE analyses versus internal pressure.

EXISTING DESIGN CRITERION FOR STEEL

The design criterion in DNV-OS-F201 for load interaction with internal overpressure reads:

$$\{\gamma_{SC}\cdot\gamma_m\}\left\{\left(\frac{|M_d|}{M_k}\cdot\sqrt{1-\left(\frac{p_{ld}-p_e}{p_b}\right)^2}\right)+\left(\frac{T_{ed}}{T_k}\right)^2\right\}+\left(\frac{p_{ld}-p_e}{p_b}\right)^2\leq 1$$

where M_d, T_{ed} and (p_{ld} - p_e) are the applied bending moment, the effective axial force and the internal overpressure, respectively. M_k, T_k and p_b represent respective capacities whereas γ_{SC} and γ_m are safety factors. Reference is made to DNV-OS-F201 for further details.

In the following, the safety factors are ignored and the internal over pressure (p_{ld} - p_e) is denoted p_i. This leaves an expression for the capacity CAP_k equal to

$$CAP_k=\frac{1}{\frac{|M_d|}{M_k}\cdot\sqrt{1-\left(\frac{p_i}{p_b}\right)^2}+\left(\frac{T_{ed}}{T_k}\right)^2+\left(\frac{p_i}{p_b}\right)^2}$$

The survival criterion is then that $CAP_k \geq 1$. However, this expression appears to under-predict the true capacity when it is based on the bending moment capacity defined by $M_k = f_y\cdot\alpha_c\cdot(D-t)^2 t$. The true capacity can be expressed by

$$CAP_t = CAP_k\cdot X_{\lim}$$

where $X_{\lim}$ is a bias correction factor, which is sometimes referred to as a model uncertainty factor. The survival criterion then needs to be redefined to $CAP_t \geq 1$.

It is assumed that the true capacity is achieved from the FE analyses. This implies that the maximum bending moment M_{FE}

estimated from the FE models can be taken as a measure of the true bending moment capacity. Hence, M_{FE} can be substituted for M_d in the expression for CAP_k, and further substitution into the redefined survival criterion, which is turned into an equality by

$$CAP_k \cdot X_{\lim} = 1.0$$

at failure, leads to the following expression for $X_{\lim}$:

$$X_{\lim} = \frac{|M_{FE}|}{M_k} \cdot \sqrt{1 - \frac{3}{4} q_h^2} + \frac{3}{4} q_h^2,$$

$$q_h = \frac{\sigma_h}{f_y} = \frac{p_i}{f_y} \cdot \frac{D-t}{2 \cdot t}$$

A perfect match between the estimated bending moment and the design equation would imply that X_{lim} equals unity. (Note that the applied effective axial tension is set to zero. A small tension is introduced in the FE models due to the rotation of the riser, however, this force is small enough to be ignored.)

The maximum bending moments estimated by the FE models were substituted in the expression for X_{lim} above together with the applied internal pressure. The obtained values are presented numerically in Table 3 (together with mean values and spreading).

From this it appears that the FE results confirms that the design equation works in excellent manner for steel, giving a minimum value of 0.984 which corresponds to an underestimation of the capacity equal to 1.6% and a maximum value of 1.074 which corresponds to an overestimation of 7.4%. The mean value for the 12 steel models yields a mean value only 1.8% above unity with a coefficient of variation (CoV) equal to 2.65%. No strong dependency with internal pressure or with slenderness is apparent.

For titanium the picture is different. The mean value of X_{lim} for the 12 titanium models is 1.085 which means that the capacity is overestimated by 8.5%, (compared to 1.8% for steel). Furthermore, the results for titanium reveals that the overestimation increases for decreasing D/t ratios. This results in a higher spreading for the titanium models with CoV = 4.54% (versus 2.65% for the steel models).

Table 3 Data basis from FE analyses.

Steel					
	q_h =0.0	q_h=0.4	q_h=0.8	mean	CoV
D/t=15	1.074	1.003	1.019	1.032	3.60 %
D/t=20	1.052	0.990	1.004	1.016	3.21 %
D/t=25	1.040	0.984	1.006	1.010	2.83 %
D/t=30	1.032	0.999	1.009	1.013	1.70 %
mean	1.050	0.994	1.009	1.018	
CoV	1.73 %	0.87 %	0.69 %		2.65 %

Titanium, based on design criterion for steel					
	q_h =0.0	q_h=0.4	q_h=0.8	mean	CoV
D/t=15	1.111	1.104	1.166	1.127	3.01 %
D/t=20	1.074	1.079	1.157	1.103	4.18 %
D/t=25	1.051	1.035	1.128	1.071	4.62 %
D/t=30	1.028	1.016	1.068	1.038	2.60 %
mean	1.066	1.059	1.130	1.085	
CoV	3.34 %	3.79 %	3.92 %		4.54 %

PROPOSED DESIGN CRITERION FOR TITANIUM

Mørk et al. (2001) dealt with titanium risers with *external* overpressure with a data base including results from both physical tests and numerical simulations. The recommended design equation from that study reads:

$$\{\gamma_{SC} \cdot \gamma_m\}^2 \left\{ \left(\frac{|M_d|}{\alpha_t \cdot M_k} \right) + \left(\frac{T_{ed}}{\alpha_t \cdot T_k} \right)^2 \right\}^2 + \{\gamma_{SC} \cdot \gamma_m\}^2 \left(\frac{p_e - p_{min}}{\alpha_p \cdot p_c} \right)^2 \leq 1$$

with a factor α_t applied to the bending moment and axial force capacities given by

$$\alpha_t = 1.1 - \frac{D}{150 \cdot t}$$

and a factor α_p applied to the collapse capacity given by:

$$\alpha_p = 1.15$$

The factor α_t equals unity for D/t equal to 15 and decreases for thinner walls. This implies that the expressions for the bending moment capacity for titanium and steel are identical for D/t = 15. For thinner risers, a slightly lower relative bending moment capacity will be predicted for titanium than for steel. The factor α_p greater than unity implies that the equation allows a larger capacity for titanium for pure collapse. The bending moment – external pressure interaction for steel and titanium are compared graphically in Figure 9.

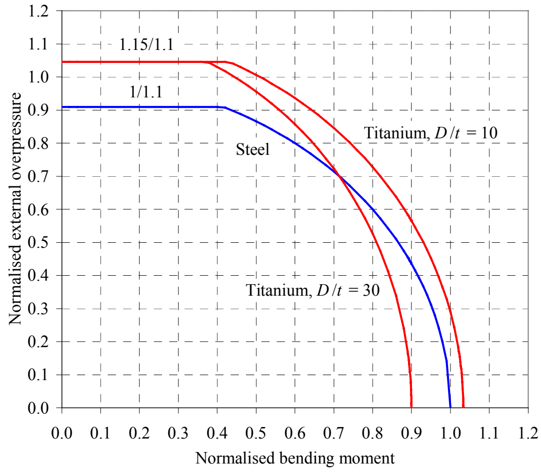

Figure 9 Pressure – moment interaction.

The factor α_t proposed in Mørk et. al. (2001) for titanium risers with external overpressure accounts for the observation made that the capacity for titanium (when using the design equation for steel) is decreasing with decreasing wall thickness. A similar observation has been made in the present study for internal overpressure.

In order to maintain compatibility with the previously proposed design criterion for risers with external overpressure, it was decided in the present study that the α_t factor should also be applied to the bending moment and axial force capacities for risers with internal overpressure.

Table 3 presents the model uncertainty factor, predicted by the FE models, for titanium using the existing design criterion for steel, i.e. without the α_t factor. Table 4 below presents the same load capacity predictions for titanium including this factor. By comparison it is seen that the α_t factor does reduce the spreading in the results and increases the mean value to between 1.12 and 1.18 depending on the internal pressure.

Table 4 Data basis from FE analyses, titanium including the α_t factor.

Titanium including α_t					
	q_h =0.0	q_h=0.4	q_h=0.8	mean	CoV
D/t=15	1.111	1.104	1.166	1.127	3.01 %
D/t=20	1.111	1.112	1.180	1.134	3.47 %
D/t=25	1.126	1.101	1.174	1.133	3.29 %
D/t=30	1.142	1.116	1.133	1.131	1.18 %
mean	1.123	1.108	1.163	1.131	
CoV	1.32 %	0.64 %	1.79 %		2.47 %

The FE results produced in the present work, with internal overpressure, indicate that the capacity of risers would be underestimated even when including the α_t factor. Based on this observation, the condition factor γ_c is introduced. The FE results indicate that the condition factor should be less than or equal to unity. Hence, the following design criterion for titanium risers with internal overpressure is proposed:

$$\gamma_c \cdot \{\gamma_{SC} \cdot \gamma_m\} \left\{ \left(\frac{|M_d|}{\alpha_t \cdot M_k} \cdot \sqrt{1 - \left(\frac{p_{ld} - p_e}{p_b} \right)^2} \right) + \left(\frac{T_{ed}}{\alpha_t \cdot T_k} \right)^2 \right\} + \left(\frac{p_{ld} - p_e}{p_b} \right)^2 \leq 1$$

Hence, rather than modifying α_t to gain an unbiased capacity prediction α_t is maintained in order to achieve compatibility between design criteria for external and internal overpressure. The positive bias in the capacity is accounted for in the subsequent calibration study and the condition factor γ_c is calibrated to comply with the safety objective in DNV-OS-F201.

CALIBRATION OF DESIGN CRITERION

The limit state function can be defined as

$$g(\mathbf{X}) = X_{\text{lim}} - \frac{M_{applied,S}}{\alpha_t M_{K,S}} \sqrt{1 - \left(\frac{p_{i,S}}{\alpha_{c,S} p_{b,S}} \right)^2} - \left(\frac{p_{i,S}}{\alpha_{c,S} p_{b,S}} \right)^2$$

in which $M_{applied}$ denotes the applied bending moment, M_K denotes the moment capacity, and subscript S signifies stochastic variables. α_t is, in principle, a stochastic variable, but is treated as deterministic here. **X** denotes the vector of stochastic variables. X_{lim} is a model uncertainty factor, which would have been exactly equal to unity if the model had been ideal. As the model is not ideal, but encumbered with uncertainties owing to simplifications and idealisations made, X_{lim} becomes a stochastic variable rather than a fixed constant.

The limit state function may, after some algebra, be recast into the following non-dimensional form:

$$g(\mathbf{X}) = X_{\text{lim}} - \frac{1}{\gamma_c \gamma_{SC} \gamma_m} \cdot \frac{X_F + X_E q_M}{\gamma_F + \gamma_E q_M} \cdot \frac{1}{X_y X_t X_D{}^2} \times$$

$$\sqrt{\left(1 - \frac{3}{4}\left(\frac{X_q q_h}{\alpha_c{}^2 X_\alpha}\right)^2\right) \cdot \left(1 - \frac{3}{4}\left(\frac{q_h}{\alpha_c{}^2}\right)^2\right)} - \frac{3}{4}\left(\frac{X_q q_h}{\alpha_c{}^2 X_\alpha}\right)^2$$

in which $q_M = M_E/M_F$ is the load ratio, where M_E denotes the bending moment due to environmental loads and M_F denotes the bending moment due to functional loads. There are two associated partial safety factors on load, γ_E and γ_F, respectively.

The following nine normalised stochastic variables have been introduced herein

$$X_F = \frac{M_{F,S}}{M_F} \qquad X_E = \frac{M_{E,S}}{M_E} \qquad X_y = \frac{\sigma_y}{f_y}$$

$$X_U = \frac{\sigma_U}{f_u} \qquad X_t = \frac{t_S}{t} \qquad X_D = \frac{D_S}{D}$$

$$X_P = \frac{p_{i,S}}{p_i} \qquad X_q = \frac{X_P}{X_y X_t} \qquad X_\alpha = \frac{\alpha_{c,S}}{\alpha_c}$$

Note that when the ratio f_y/f_U is set equal to 1.18, which is assumed here, then X_α can be expressed as

$$X_\alpha = \frac{X_y(1-\beta) + X_U \beta \cdot 1.18}{X_y(1-\beta) + X_y \beta \cdot 1.18}$$

Other symbols used are

$$q_h = \frac{p_i}{p_b} \cdot \frac{2}{\sqrt{3}}$$

$$\beta = (0.4 + q_h)(60 - \frac{D}{t})/45 \quad \text{for } 15 < D/t < 60$$

$$\alpha_c = (1-\beta) + \beta \frac{f_U}{f_y}$$

$f_U = SMTS$ with σ_U as its stochastic counterpart

$f_y = SMYS$ with σ_y as its stochastic counterpart

The probability distributions assigned to the stochastic variables are summarized in Table 5.

Table 5: Distribution models for normalized variables

Vari-able	*Purpose*	*Distribution(μ;CoV)*
X_F	Functional bending moment	N(1.0;0.07)
X_E	Environmental bending moment	G(0.75;0.25)
X_y	Yield stress, σ_y	N(1.06;0.03)
X_U	Tensile strength, σ_u	N(1.08;0.025)
X_t	Thickness	N(1.0;0.03)
X_D	Diameter (fixed)	1.0
X_P	Pressure	G(1.02;0.02)
X_α	Strain hardening	Function of X_y and X_u
X_{lim}	Model uncertainty	N(1.1;0.0545)

The distribution models for functional and environmental bending moments are extracted from Mørk et al. (2001). The distribution models for the yield stress and tensile strength variables are based on DNV in-house test data for seamless titanium pipes. The distribution models for the wall thickness and for the pressure variable are based on a SUPERB recommendation.

The distribution parameters for the model uncertainty factor X_{lim} have been estimated by calibrating results by the capacity model with results obtained for the "true" capacity as assessed by FEM calculations. Based on this, the mean value of X_{lim} has been conservatively set equal to 1.1. Owing to uncertainties known to be present in the FEM model, the standard deviation of X_{lim} has been set to 0.06, which is a little higher than the value indicated by the results of the calibration of X_{lim}. The standard deviation of 0.06 corresponds to the CoV of 0.0545 quoted in the table.

A first-order reliability method (FORM) as described in Madsen et al. (1986), is used to calculate the probability of failure

$$P_F = P[g(\mathbf{X}) < 0]$$

based on the stochastic models assumed for the set of governing stochastic variables **X**. With the limit state function $g(\mathbf{X})$ formulated as quoted above, expressed in terms of the safety factors γ_F, γ_E, γ_c, γ_{SC} and γ_m, the calibration exercise is reduced to adjust the values of γ_F, γ_E, γ_c, γ_{SC} and γ_m in such a manner that the probability of failure P_F that results from the FORM analysis equals the requirement to P_F specified for the safety class in question. An infinite number of combinations of γ_F, γ_E, γ_c, γ_{SC} and γ_m exist that will meet the requirement to P_F. It is common to prescribe the values of some of the safety factors and thereby reduce the calibration exercise further to determine the remaining factors. Here, the load factors γ_F and γ_E are prescribed together with the material factors γ_{SC} and γ_m, and the calibration is hence reduced to determine the value of one partial safety only, viz. the condition factor γ_c.

The calibration scope covers expected relevant combinations of

1. hoop stress utilization q_h (a range 0.5-0.8 is assumed)
2. D/t ratio (a range 15-30 is assumed)
3. load ratio q_M (a range 0-5 is assumed)

The calibration has been carried out for the low, normal and high safety classes, which for the ULS correspond to target failure probabilities of 10^{-3}, 10^{-4} and 10^{-5}, respectively. The two partial safety factors for load have been assigned values according to OS-F201, viz. γ_F = 1.1 and γ_E = 1.3. Also the values of the two partial safety factors for resistance have been assigned values according to OS-F201. These values are reproduced in Table 5. The third partial safety factor for resistance is the condition factor γ_c, which results from the calibration. The calibration is carried out by means of a first-order reliability analysis with the limit state function and probability distributions quoted above. The value of γ_c as input to this analysis is then adjusted until the failure probability resulting from the analysis meets the specified requirement to the target failure probability for the safety class in question. Over the range of combinations (q_h, D/t, q_M), some variation in the resulting necessary γ_c value is produced. A synthesis of the obtained results for γ_c by this calibration suggests requirements to γ_c as quoted in Table 6.

Table 6 Results of safety factor calibration.

Safety factor	*Safety class*		
	Low	*Normal*	*High*
γ_m	1.15	1.15	1.15
γ_{SC}	1.04	1.14	1.26
γ_c	0.90	0.95	1.00

Figures 10 and 11 shows graphically the model uncertainty factor predicted by the FE models for both titanium and steel, using γ_c = 0.95 for safety class normal.

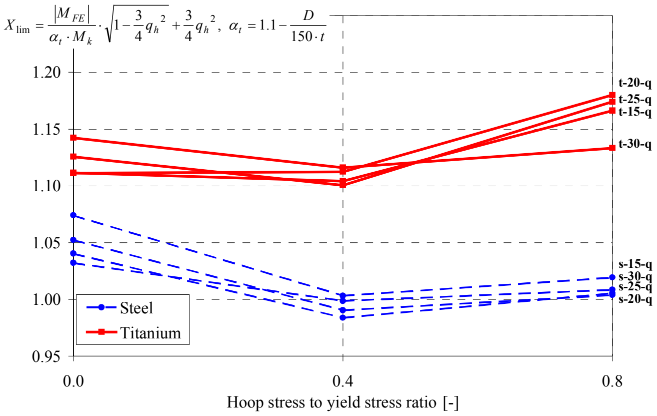

Figure 10 Model uncertainty versus internal pressure, including the factors α_t and γ_C for titanium.

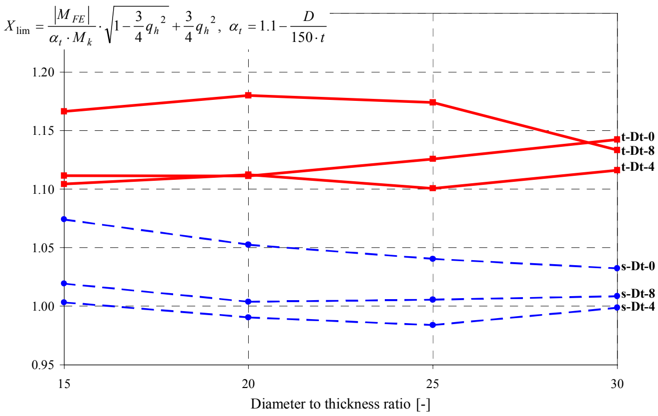

Figure 11 Model uncertainty versus diameter to thickness ratio, including the factors α_t and γ_C for titanium.

RECOMMENDATION

The following design criterion is recommended for titanium risers with $D/t \le 30$:

$$\gamma_c \cdot \{\gamma_{SC} \cdot \gamma_m\} \left\{ \left(\frac{|M_d|}{\alpha_t \cdot M_k} \cdot \sqrt{1 - \left(\frac{p_{ld} - p_e}{p_b} \right)^2} \right) + \left(\frac{T_{ed}}{\alpha_t \cdot T_k} \right)^2 \right\} + \left(\frac{p_{ld} - p_e}{p_b} \right)^2 \le 1$$

The modifications from the corresponding steel equation, as given in DNV-OS-F201, is represented by the titanium correction factor α_t and the condition factor γ_c:

$$\alpha_t = 1.1 - \frac{D}{150 \cdot t}$$

γ_c	*Safety Class*
0.90	Low
0.95	Normal
1.00	High

The α_t factor will ensure that the capacity is consistent with varying geometry, i.e. D/t, whereas the condition factor γ_c is recommended to account for the fact that the capacity for titanium is underpredicted, but the variability of the results are higher compared to steel. Hence, it is a function of the safety class. (Note that the γ_c factor applies to titanium risers only, not to steel risers.)

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