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An Experimental Verification of Laser-Velocimeter Sampling Bias and Its Correction

The existence of "sampling bias" in individual-realization laser velocimeter measurements is experimentally verified and shown to be independent of sample rate. The experiments were performed in a simple two-stream mixing shear flow with the standard for comparison being laser-velocimeter results obtained under continuous-wave conditions. It is also demonstrated that the errors resulting from sampling bias can be removed by a proper interpretation of the sampling statistics. In addition, data obtained in a shock-induced separated flow and in the near-wake of airfoils are presented, both bias-corrected and uncorrected, to illustrate the effects of sampling bias in the extreme.

Introduction

The laser velocimeter makes it possible to measure the flow fields of extremely complex flows – measurements that would be impossible to make with any other technique. Of particular importance has been the application of the laser velocimeter to flows in which the turbulence exceeds that for which hot-wire anemometry can be expected to provide accurate measurements. Such flows are common in almost every engineering discipline concerned with the flow of fluids (e.g., aerodynamics, propulsion, and combustion).

As the ability to predict such complicated flow fields improves, it will become increasingly important that the experimental data used to assess and extend predictive methods be of the highest accuracy possible. Properly applied, the laser velocimeter technique can provide, in principle, accurate measurements of mean velocities and higher-order turbulence quantities even when the local turbulence level is infinite – a situation common to turbulent separated flows. There exists, however, considerable controversy whether corrections to the measurements must be made for a bias toward higher velocities when the measurements are obtained under conditions of relatively low particle concentration (i.e., conditions for which only one particle is present in the laser velocimeter sensing volume at any instant of time); these conditions usually prevail in airflow applications.

This bias toward higher velocities is commonly referred to as "sampling bias," and it is argued that it occurs because the particle arrival rate (hence, sampling rate) is dependent on the local instantaneous speed of the fluid at the sensing volume. The errors attributable to this form of biasing are only significant when the turbulence level becomes extremely high; however, it is precisely for these conditions that the potential utility of the laser velocimeter technique is the greatest.

The controversy is not so much about how to correct for sampling bias but whether a correction is warranted or not. This uncertainty has remained because of lack of conclusive experimental evidence to support or disprove the existence of this bias. As a consequence, some laser velocimetry data that are reported have been corrected for sampling bias and some have not; the latter is the most prevalent, possibly because correcting for sampling bias requires more effort in the experiment.

In this paper, experimental evidence is presented for a simple free-shear layer flow which clearly demonstrates the existence of sampling bias and how the data can be accurately corrected for this effect by a proper weighting of the velocity samples. The results are pertinent to the case in which signal processing is accomplished with a burst-period counter that uses a fixed number of fringe crossings to effect a measurement. The standard for comparison in assessing the presence of sampling bias was laser velocimeter measurements obtained at high seeding concentration levels, for which several particles were always present in the laser-velocimeter sensing volume. In addition, data obtained in a shock-induced separated flow and in the near-wake of a conventional and supercritical airfoil are presented bias-corrected and uncorrected to illustrate the effects of sampling bias in the extreme.

Background

In many practical applications of laser velocimetry in which air is the fluid media, the concentration of light-scattering particles is such that no more than one particle is present in the sensing volume at any instant. Under these conditions, the mode of operation is often referred to as individual-realization (IR) laser velocimetry to distinguish it from the continuous-wave (CW) mode, in which at least several particles are always present in the sensing volume. This latter mode of operation is prevalent in water-flow applications for which high seeding concentrations can easily be generated. At the other end of the spectrum are the applications in unseeded

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wind-tunnel environments in which the particle concentration is so sparse that the percentage of time that a particle exists in the sensing volume is extremely small relative to the percentage of time that the sensing volume is void of particles.

In the individual-realization mode, the collection of velocity information is statistical in nature, with the sampling being dependent on the occurrences of particles crossing the sensing volume. In early applications of individual-realization laser velocimetry, it was thought that the sampling of the local velocity field was random and unbiased. Consequently, statistical estimators for the mean and variances were used which assumed that the sampling was unbiased.

In 1973, well after the inception of individual-realization laser velocimetry, McLaughlin and Teiderman [1] postulated that the sampling was not totally random but biased in turbulent flows, with the probability of sampling high-velocity particles being greater than that of sampling low-velocity particles. Their arguments were based on the assumptions that the particles were homogeneously distributed in the fluid, that all particles crossing the sensing volume had an equal probability of producing a validated signal independent of speed or trajectory through the sensing volume, and that only one velocity sample was obtained for each particle transit. To show theoretically the dependence of particle arrival rate on the instantaneous speed of the flow at the sensing volume, and hence sampling bias, they also assumed that the concentration of particles was sufficiently high that the average time between particle occurrences was small compared to the time scale of the turbulence.

This assumption left open to question the sampling statistics for many applications in which the average time between particle realizations is large compared to the time scale of the turbulence. In reference [2], for example, it was suggested that the biased sampling situation discussed in reference [1] was not present at very sparse seeding conditions. It has also been proposed [3] that the sample biasing toward higher velocity particles proposed in reference [1] is either totally or partially eliminated by a compensating effect. This compensating effect is based on the argument that slower moving particles produce signals of higher amplitude than faster moving particles, as a result of the response characteristics of the detector, and hence a higher probability of producing a validated output. It has also been argued that the presence or absence of sampling bias is dependent on the type of signal processor used to extract the velocity information.

The number of papers cited in reference [4] that deal with laser velocimeter sampling bias indicates the degree of controversy that still remains on this subject.

Unfortunately, at the turbulence levels at which sampling bias becomes significant, the appropriateness of using either pitot pressure probe or hot-wire anemometer measurements

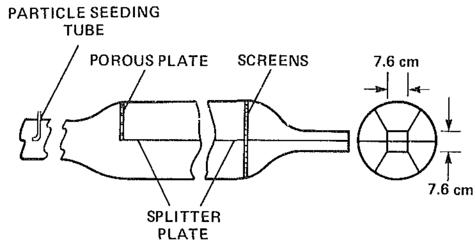

Fig. 1 Schematic of two-stream mixing flow model

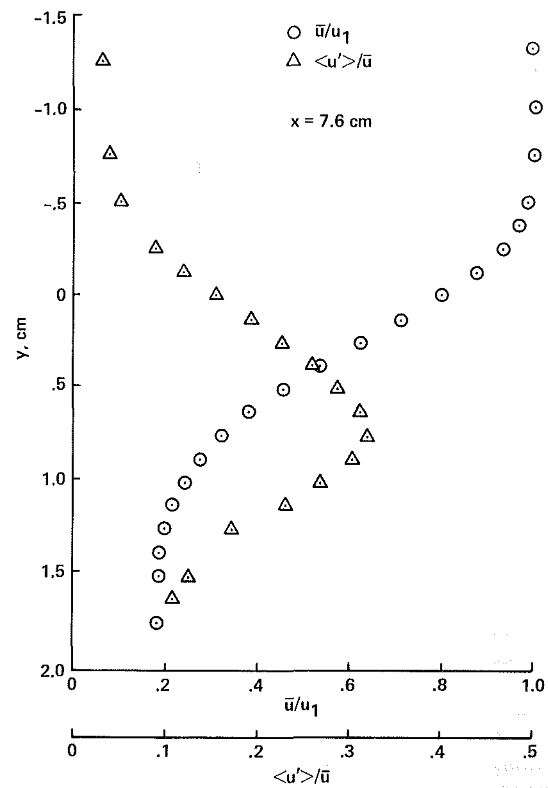

Fig. 2 Shear-layer mean velocity and turbulence intensity distributions

as a standard for comparison must be questioned. This lack of another measurement technique that could provide a standard of comparison has been the primary reason sampling bias has not been experimentally confirmed. Moreover, there are many factors besides the sampling bias proposed in reference [1] that can affect the measurement accuracy of a laser velocimeter system. If extreme care is not taken, conclusions

Nomenclature

- c = chord of model
- d = diameter of sensing volume
- l = length of sensing volume
- N = total number of velocity realizations
- p = probability density function
- P = probability distribution function
- u = velocity component in streamwise direction
- v = velocity component in normal direction
- V = velocity vector, $u\hat{i} + v\hat{j} + w\hat{k}$
- w = velocity component in cross-stream direction
- x = streamwise distance from splitter plate
- y = vertical distance from splitter plate
- λ = mean particle occurrence rate
- τ = interarrival time between velocity samples
- τ_t = integral time scale of turbulence
- ω = weighting factor for sampling bias

Subscripts

- CW = continuous-wave quantity
- e = boundary-layer edge conditions
- i = ith velocity realization
- UNW = unweighted quantity
- 1 = conditions at faster stream
- 1D = one-dimensional weighted quantity
- 2D = two-dimensional weighted quantity

Superscripts

- $(\ )'$ = fluctuating quantity
- $(\bar{\ })$ = averaged quantity
- $\langle ' \rangle$ = rms value of quantity

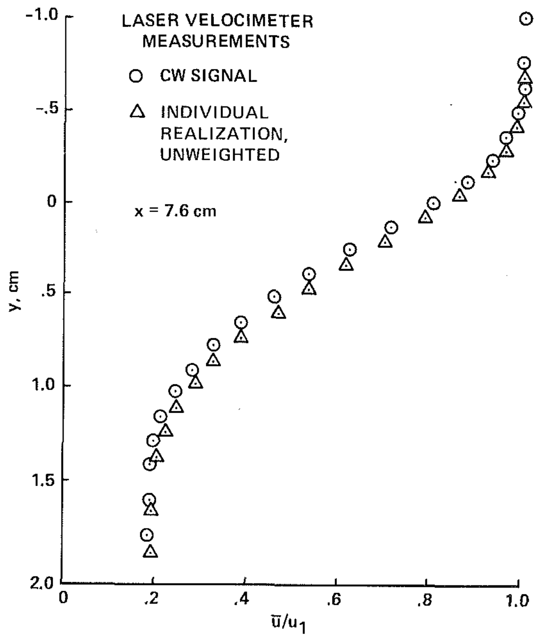

Fig. 3 Comparison of continuous-wave and individual-realization (unweighted) mean-velocity measurements

can be drawn regarding sampling bias that are really the results of some other elements of the experiment. For example, particle-concentration gradients, such as that present for a seeded air jet issuing into an environment of different particle concentration, can produce errors as large as that expected from sampling bias. The use of stationary fringes in a highly turbulent flow region also can result in measurement errors if the signal processor requires some minimum number of fringe crossings to produce a validated output. Velocity-gradient effects owing to insufficient spatial resolution, particle-lag effects, inadequate signal-to-noise ratio for accurate signal processing, and poor detector response are other sources of measurement error which can result in improper conclusions.

An apparent confirmation of sampling bias was reported by Quigley and Teidermann [5] for a two-dimensional (2D) channel flow in which the standard for comparison was the near-wall velocity gradient as inferred from the streamwise pressure gradient. However, in a later study of sampling-rate effects on sampling bias in the same facility, Bogard and Teidermann in reference [4] were unable to reproduce the earlier results of reference [5]. It would appear that the aspect ratio of the channel was insufficient for the assumption of a spanwise constant wall shear.

The approach in the present study was to use the laser velocimeter technique as its own standard for comparison. Theoretically, if particles are always present in the sensing volume, the sampling statistics are no longer dependent on particle arrival rate and sampling bias is absent. This concept was applied to obtain data free of sample bias for comparison with individual-realization results.

Experimental Approach

To experimentally assess the sampling-bias effect, a flow field was desired that had regions of high turbulence but for which a uniform concentration of seeding particles could be assured. Based on these requirements, a free-shear layer developed by two streams of unequal velocity was selected. By imposing a large difference in the velocities of the two streams, significant turbulence intensities could be generated. Contrary to a single-jet experiment, the realization of uniform seeding concentration across the shear layer was straightforward with this two-stream mixing model. Shown in Fig. 1 is a sketch of the flow model. A velocity ratio of 5:1 was generated by using a porous plate to impede the air flow in the upper stream. The mean velocity and turbulence intensity distributions for a measurement station 7.6 cm downstream of the splitter plate is shown in Fig. 2. As is conventionally done, the faster stream is shown as the upper stream in Fig. 2. As seen from Fig. 2, local turbulence levels approaching 35% are generated in this shear flow. The experiments were conducted with the faster stream traveling at 30 m/sec.

A two-color laser velocimeter system [6] designed for transonic boundary-layer studies in the Ames 2- by 2-ft Transonic Wind Tunnel was used in the study. This system uses a 40-MHz Bragg-cell shift in both colors to assure the minimum number of fringe crossings required for burst counters regardless of particle trajectory. To obtain a frequency offset more appropriate for the low speeds of the present experiment, the signals were mixed electronically. The same fringe spacing as that used in transonic testing – about 18 μm was used. Thus, the maximum Doppler frequency was only 1.7 MHz, whereas frequency offsets ranging between 2 and 4 MHz were used. With the 4-W argon-ion laser and off-axis forward light-scatter collection, the system can detect particles in the 1-μm range, even at transonic conditions. The present experiment with its low speeds (narrower bandwidth) and absence of flare sources, such as windows or solid surfaces, resulted in very high signal-to-noise ratios and high data rates when sufficient seed material was injected into the plenum.

The sensing volume diameter for this laser system is about 200 μm, well within the resolution requirements of the shear layer, which was nominally 1.25 cm thick. Because of the small angle (≈ 2 deg) between incident beams used to establish a large fringe spacing for transonic testing, the sensing volume length was determined by the collection optics rather than by the transmitting optics. For the off-axis collection angle of 10 deg and the nominally large spatial filter of 1 mm at the detectors, the sensing volume was about 6 mm in length. Although this may appear excessive, the laser velocimeter's instantaneous spatial resolution in the individual-realization mode is determined by the particle's trajectory through the sensing volume, not by the total length of the sensing volume. Thus, fluctuations from turbulence scales much smaller than the sensing-volume length can be resolved. What must be insured is that the time-averaged quantities to be measured (e.g., $\bar{u}$, $\langle u' \rangle$, and $\overline{u'v'}$) not vary along the length of the sensing volume. For the two-dimensional shear layer of this experiment, the sensing-volume dimensions were more than adequate to meet this requirement.

The photodetector outputs were processed with burst-period counters that use eight fringe crossings to determine the period of the signal. The counters employ both a 5/8 comparison and a three-level validation circuit to minimize erroneous period readings. This latter circuit permits a validated output, only if for all eight fringe crossings the signal passes through a positive threshold, a zero level, and a negative threshold in the proper sequence.

The output from the two burst counters was recorded in two different ways. In one mode of operation, the digital output of the counters was fed directly to a desk-top computer via a multiplexer. The multiplexer could be operated in such a manner to insure that the validated data from the two velocity channels were from the same particle; it also provided interarrival times between particles. In this mode, the data rate was limited to 17 kHz. In the second mode of operation, the digital data were fed directly into a pulse-height analyzer, which sampled the output each time a validation pulse was detected from the counters. In this mode, data rates in excess

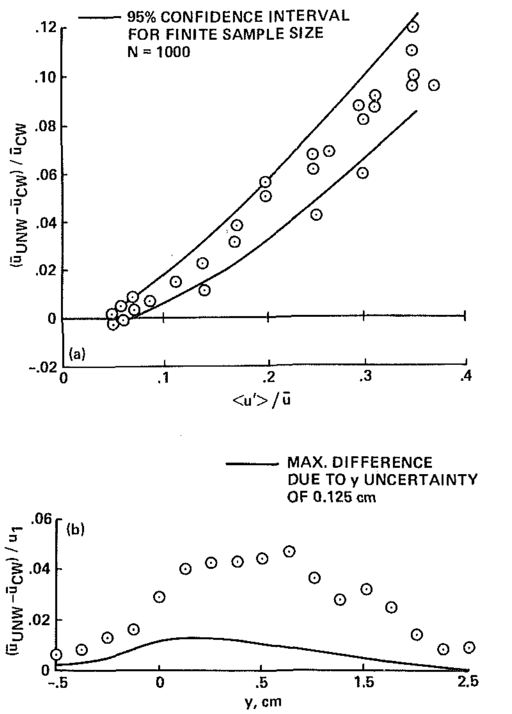

Fig. 4 Differences in continuous-wave and individual-realization (unweighted) mean-velocity measurements:
(a) As a function of turbulence level,
(b) Relative to the maximum shear-layer velocity, u_1

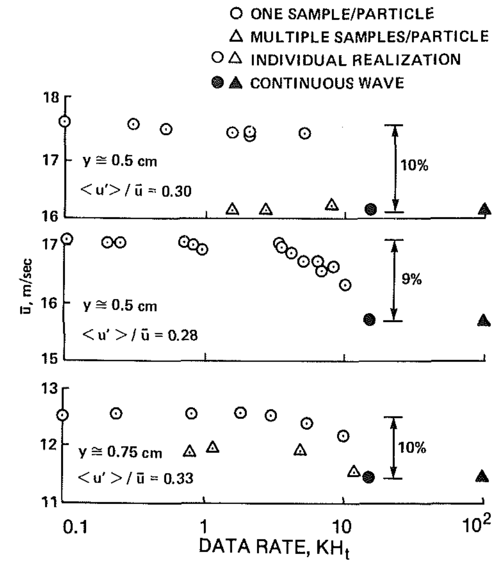

Fig. 5 Measured mean velocity as a function of data rate

of 100 kHz were achieved. The pulse-height analyzer was used to collect data in the CW mode. The primary advantage of the pulse-height analyzer was that it circumvented the problem of insufficient computer memory, allowing long averaging times even at data rates as high as 100 kHz. In this case, up to 500,000 velocity samples were accumulated at each measurement station. The other method of data acquisition via the multiplexer made possible the collection of data in a form which would allow for the correction of sampling bias at low particle concentration levels. In addition, with particle interarrival time measurements available, true time-averaged results could theoretically be obtained. As discussed later, however, the realization of sampling bias-free results at individual-realization conditions using the time-averaging approach is not nearly as straightforward as the approach of collecting data at CW conditions.

The source of air for the flow model was the 120-psig makeup air supply for the NASA Ames Unitary Wind Tunnel Plan. This air is dried to a very low specific humidity to allow supersonic testing without the formation of condensation shocks. Most of the naturally occurring particles are removed in the drying process; as a result, the air is very clean. With out artificially adding particles, data rates of only a few per second could be achieved, an ideal situation for the present experiment. It was easy to vary the concentration of particles from essentially no particles to many particles in the probe volume at any instant in time. To artificially seed the flow, an ultrasonic spray nozzle was used which generated mineral oil droplets with a mean diameter of 0.7 μm.

Equal particle concentrations in the two streams were confirmed in two ways. First, under conditions of heavy seeding (i.e., many particles in the proble volume at any instant) the dc output of the detectors was confirmed to be the same in both free streams. Assurance that the detectors were operating in the linear range was checked by varying the laser power. In the second case, the seeding level was lowered to insure single-particle occurrences. Oscilloscope sweeps of the unfiltered detector outputs were monitored for a constant threshold setting and confirmed to differ by the velocity ratio of the two streams, as it should for equal concentrations.

This latter check of the uniform seeding concentration was also a partial check on the detectors' frequency response. To confirm that there was little detector biasing, as described in reference [3], the validated signal rate of the burst counters was checked between the two streams and found to vary as the velocity ratio. This check was made with the counter cycle-time delayed to prevent multiple readings from the same particle.

Discussion

Sampling-Bias Results. Mean-velocity profiles obtained for the shear layer in both continuous wave and individual-realization modes are presented in Fig. 3. In the CW mode, the data rate was about 100 kHz, with 500,000 samples taken at each station. The individual-realization results were obtained at a particle concentration level that produced a validated data rate of only about 100 s^{-1}. In this case, between 1,000 and 4,000 samples were acquired at each measurement station. To prevent multiple readings from the same particle in the individual-realization mode, a 60-μs cycle-time between measurements was imposed on the data acquisition system. For both cases, the mean velocities were calculated from the expression

$$\bar{u} = \sum_{i=1}^{N} u_i / N, \tag{1}$$

which assumes unbiased sampling. Under conditions of heavy seeding, the validated data rate remained essentially constant across the shear layer, and the mean-velocity results were independent of the cycle-time imposed between measurements. Both observations are consistent with CW-mode data collection. The differences in the two results follow the trends that would be expectd for sampling bias. The unweighted data indicate larger mean velocities, and the differences are greater where the turbulence levels are higher. The percent difference between the individual-realization and the CW results is shown in Fig. 4(*a*) as a function of turbulence intensity. Included in this figure is the 95 percent confidence interval for $\bar{u}$ when the sample size is limited to 1,000 readings (the minimum sample size for the individual-realization measurements). The difference in the individual-

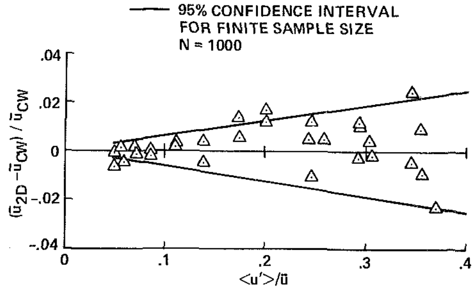

Fig. 6 Differences in continuous-wave and individual-realization (2D weighted) results as a function of turbulence level

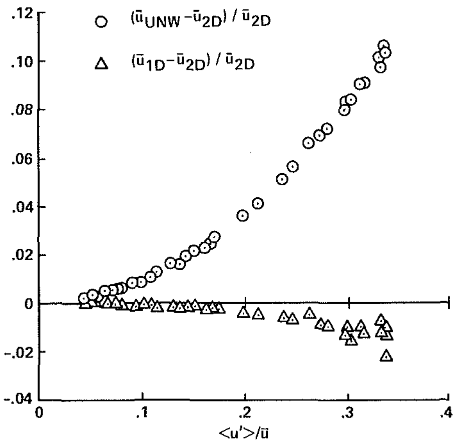

Fig. 7 Comparison of unweighted and 1D weighted mean velocities with 2D weighted mean velocities

realization and CW results are also shown in relation to the maximum velocity, u_1, in Fig. 4(*b*). This figure gives a perspective relative to overall measurement accuracy, because it is really u_1, not the local mean velocity, that dictates the precision of the measurements. The CW results are estimated to be accurate to within ± 1 percent of u_1. Included in Fig. 4(*b*) is the maximum error that could result from an estimated 0.125-mm uncertainty in y location.

The results presented in Figs. 3 and 4 clearly demonstrate that for the individual-realization mode of laser velocimetry, sampling bias exists and that it increases approximately with the square of the turbulence intensity. To investigate the dependency of this sampling error on data rate (or mean particle-arrival time), mean-velocity measurements were obtained in the region where the turbulence intensity was a maximum for a wide range of particle concentration levels. The measured mean velocities are presented in Fig. 5 as a function of data rate for three different runs in which the y location was fixed while the seeding concentration was varied. The data rates varied from 100 Hz to 100 kHz. In most of the measurements, the data acquisition cycle-time was set at 60 μs to prevent multiple readings on the same particle. With this cycle-time imposed, the CW data rate was limited to 17 kHz. The other results presented in Fig. 5 were obtained with a cycle-time of less than a few microseconds. In this mode, slower moving particles have a greater chance of being sampled more than once than do faster moving particles, thus resulting in an approximate "signal lifetime" weighting of the data.

Apparent from Fig. 5 is the independence of the sampling bias and data rate when the particle concentrations are sufficiently low for individual-realization measurements. A transition region between about 8 and 17 kHz is evident where the data acquisition system begins to control the sampling rate rather than the particle arrival rate. By allowing multiple readings on the same particle, the effect of sampling bias was totally or at least partially eliminated. This method of correcting for sampling bias can be effective, however, only if (1) the cycle-time of the data acquisition system is very short compared to the lifetime of the signals and (2) the frequency offset is large compared to the frequency shift due to velocity. In many laser velocimeter applications these conditions cannot be met.

Sampling-Bias Correction. To correct the data for sampling bias, each sample should be weighted inversely to the magnitude of the instantaneous velocity $V_i = (u_i^2 + v_i^2 + w_i^2)^{1/2}$ if the sensing volume is spherical [1]. The estimate for the mean velocity in which case is given by

$$\bar{u} = \sum_{i=1}^{N} \omega_i u_i / \sum_{i=1}^{N} \omega_i, \tag{2}$$

where the weighting factor ω_i is given by

$$\omega_i = 1/(u_i^2 + v_i^2 + w_i^2)^{1/2}. \tag{3}$$

When the sensing volume can be considered as a cylinder, the proper weighting factor can easily be shown to be given by

$$\omega_i = 1/\left[(u_i^2 + v_i^2)^{1/2} + \frac{\pi}{4}\,\frac{d}{l}\, w_i \right], \tag{4}$$

where d and l are the diameter and length of the cylinder, respectively.

In the present experiment, the sensing volume can be considered cylindrical with a length-to-diameter ratio of about 30. Thus, the particle arrival rate in the present study should depend little on the cross-stream velocity component w, and weighting the samples according to $1/(u_i^2 + v_i^2)^{1/2}$ (two-dimensional weighting model) should be quite accurate.

Instead of weighting the data with the 2D model, it has been proposed [7, 8] to weight the data according to the lifetime of the signal burst, which can theoretically treat ellipsoidal sensing volumes. The major disadvantages of this approach are that (1) signal lifetime measurements are inherently inaccurate and (2) the signal lifetime varies with particle size and the location the particle crosses the sensing volume. Since variations in signal lifetime as a result of these effects are uncorrelated with velocity, accurate unbiased results can in theory be obtained, but the sample size must be increased to average-out these contributions. To our knowledge there has been no attempt to quantify the degree to which the sample size would have to be increased. If the variances caused by these effects are large, as we suspect, the required sample sizes could be prohibitively large for many applications.

The effectiveness of the 2D weighting model in correcting the biased data is demonstrated in Fig. 6. The weighted data are compared with the CW data as a function of turbulence level. To within the data scatter, the 2D weighted results agree with the CW data. The scatter in the data is primarily a result of the limited sample sizes of the individual-realization results. Figure 7 shows the difference between the unweighted data and the 2D weighted data as a function of turbulence level. In this case, the scatter is reduced since the same data samples are involved. The amount of correction is approximately equal to the square of the intensity of turbulence. Included in Fig. 7 are results using the approximate one-dimensional correction, $\omega_i = 1/u_i$, proposed in reference [1]. For this particular flow, this correction of the mean velocity was quite effective, but obviously, it is inappropriate for turbulent separated flows because of the singularity at $u_i = 0$.

As in the calculation of the mean velocities, the presence of sampling bias requires that a weighting be applied in the calculation of the higher-order turbulence quantities $\langle u' \rangle$, $\langle v' \rangle$, and $\overline{u'v'}$. The statistical estimators become

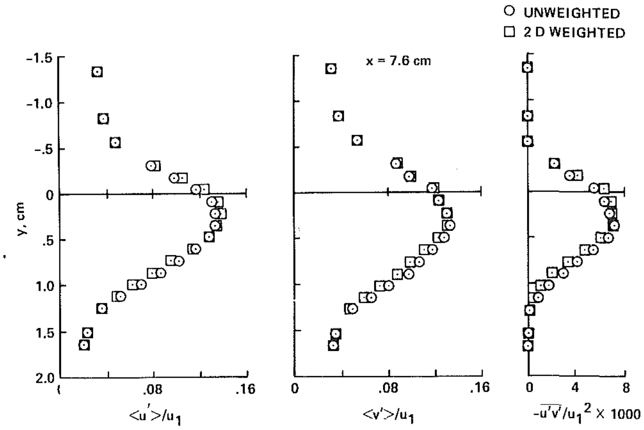

Fig. 8 Unweighted and 2D weighted turbulence intensity and Reynolds shear-stress results

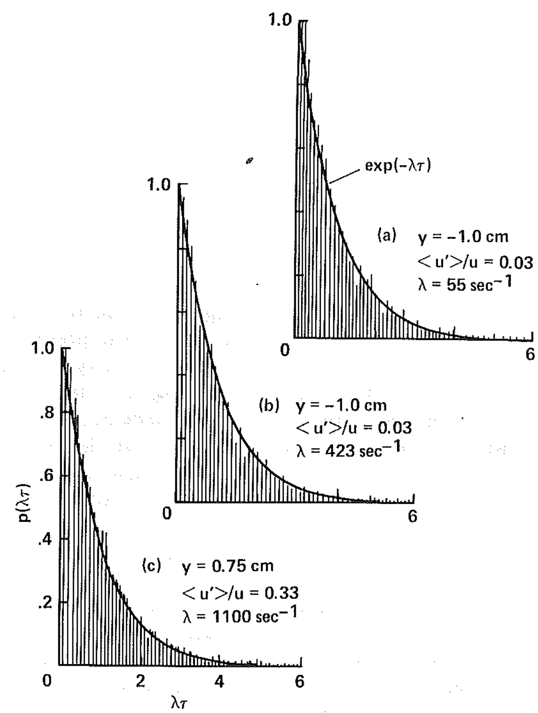

Fig. 9 Particle interarrival time distributions: (*a*) Outside shear layer; (*b*) Outside shear layer; (*c*) Within shear layer

$$\langle u'\rangle = \left[\frac{\sum_{i=1}^{N}\omega_i(u_i-\bar{u})^2}{\sum_{i=1}^{N}\omega_i}\right]^{1/2}, \tag{5}$$

$$\langle v'\rangle = \left[\frac{\sum_{i=1}^{N}\omega_i(v_i-\bar{v})^2}{\sum_{i=1}^{N}\omega_i}\right]^{1/2}, \tag{6}$$

and

$$\overline{u'v'} = \frac{\sum_{i=1}^{N}\omega_i(u_i-\bar{u})(v_i-\bar{v})}{\sum_{i=1}^{N}\omega_i}. \tag{7}$$

In Fig. 8, measurements of $\langle u'\rangle$, $\langle v'\rangle$, and $\overline{u'v'}$, obtained under individual-realization conditions, are presented for $\omega_i = 1$ (unweighted), and $1/(u_i^2 + v_i^2)^{1/2}$. As evident from this figure, the differences in the weighted and unweighted results are not large. Results for $\omega_i = 1/u_i$ were also obtained although not shown in this figure. These 1D model results were nearly identical to those for the 2D model. The effect of sampling bias for this flow is primarily a shift in the probability density function toward lower velocities, with the variances remaining nearly unchanged. It would be extremely difficult with these small differences to verify the effects of sampling bias on turbulence intensity and Reynolds shear-stress measurements. Moreover, theere is no standard available. Uncertainties in the measurements of these quantities in the CW mode arise from Doppler ambiguity effects [9] and spatial averaging if the sensing volume is relatively long. This latter effect in the present experiment resulted in $\overline{u'v'}$ dropping by a factor of 2 under CW conditions. Improvement of the cross-stream resolution by the insertion of a 0.3-mm pinhole in the collection optics raised the shear stress to about 80 percent of that obtained under individual-realization conditions.

Although the turbulence intensities and shear stress for the present flow were not markedly influenced by sampling bias, this is not necessarily the case for flows with a more extreme turbulence, as illustrated by the complex flow examples presented in a later section.

Interarrival Time Statistics. In the early phases of this work, the plan was to verify the existence of sampling bias by comparing mean velocities calculated by the two expressions

$$\bar{u} = \sum_{i=1}^{N} u_i/N \tag{8}$$

and

$$\bar{u} = \sum_{i=1}^{N} u_i\tau_i/\sum_{i=1}^{N}\tau_i \tag{9}$$

at individual-realization conditions but at very high sample rates. In equation (9), τ_i is the interarrival time between velocity samples. When τ_i is always sufficiently less than the integral time-scale of the turbulence, τ_t, equation (9) should be a good estimate of the desired time-averaged mean velocity [10]. This approach was generally unsuccessful, however, because in most cases either the data rate was too low for $\tau_i << \tau_t$ to be satisfied or the data rate was so high that τ_i was being controlled by the cycle-time of the data acquisition system ($\approx 60\ \mu s$) rather than by the particle arrival rate.

In either of these situations, both expressions will give the same answer. At the lower data rate, both are affected by sampling bias and at the high data rate neither is affected. A positive result of this exercise, however, was the determination of particle interarrival time distributions. These distributions at first did not agree with our preconceived notions but were later shown to be consistent with Poisson statistics. It was expected that the most likely interarrival time would correspond to the average interarrival time (i.e., the reciprocal of the mean data rate); that this was not the case is shown in Fig. 9, where measured interarrival time distributions are presented. Instead, the likelihood monotonically increases as τ approaches zero; however, this behavior is what should be expected for a Poisson process, as shown in the next paragraph.

The probability of no particle occurrences, say in the time interval τ_a, is given by

$$P(0,\lambda\tau_a) = \int_{\lambda\tau_a}^{\infty} p(\lambda\tau)d\lambda\tau, \tag{10}$$

where λ is the mean particle occurrence rate and $p(\lambda\tau)$ is the

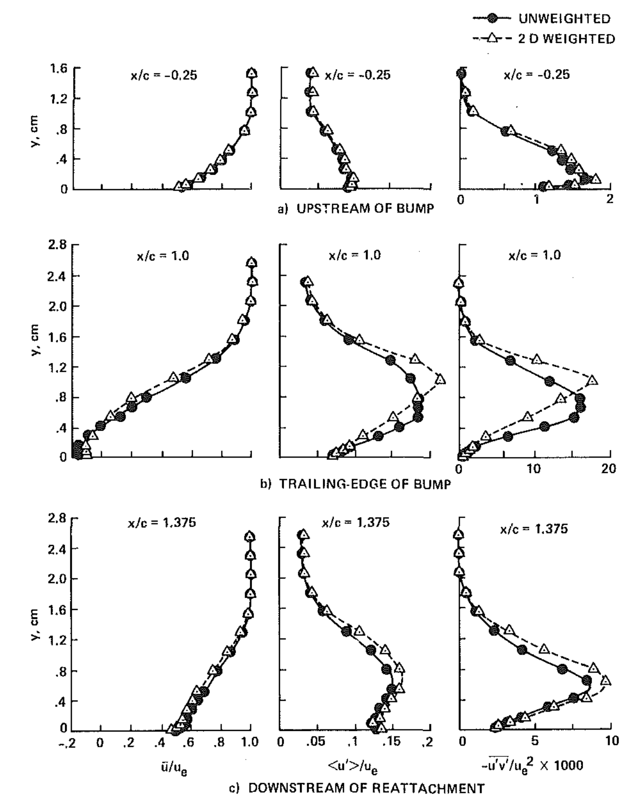

Fig. 10 Uncorrected and bias-corrected results; axisymmetric bump model ($M_\infty = 0.875$)

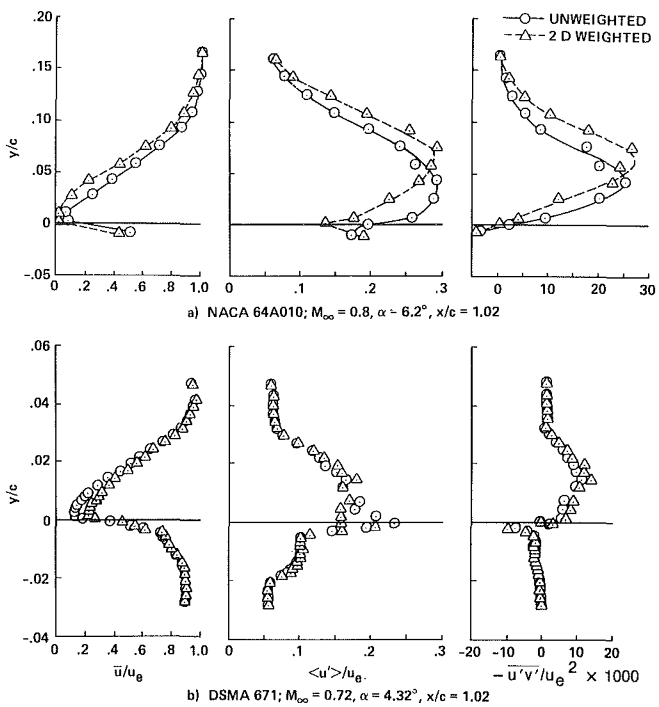

Fig. 11 Uncorrected and bias-corrected results; airfoil models

probability density function of the interarrival times [it is $p(\lambda\tau)$ which is presented in Fig. 9]. Equation (10) states that the probability of no particle in the time interval, τ_a, is given by the probability that the interarrival time τ_i is greater than or equal to τ_a. But if the particle arrivals obey Poisson statics, then the following must be true if the local velocity is invariant (e.g., steady laminar flow):

$$P(0,\lambda\tau_a) = \exp(-\lambda\tau_a). \tag{11}$$

It follows from equations (10) and (11) that $p(\lambda\tau)$ must be given by the following:

$$p(\lambda\tau) = \exp(-\lambda\tau). \tag{12}$$

Equation (12) is plotted in Fig. 9 and describes the measurements almost exactly. It is evident from Fig. 9 that for very small τ the probability of a particle occurrence is given by $\lambda\tau$, which must be true in a Poisson process.

Although equation (12) is strictly valid only when the local velocity is invariant, it was found to describe $p(\lambda\tau)$ quite well even where the turbulence intensity was a maximum, as seen in Fig. 9(c). This observation that the distribution of $\lambda\tau$ for even moderately high turbulence levels is primarily dependent on the spatial distribution of the particles rather than the local turbulence level can be theoretically shown. However, such an analysis would be beyond the scope of the present paper.

Complex Flows. The real concern of sampling-bias effects is not for the simple flow of the present experiment but for complex flows in which the turbulence levels are more extreme. For these flows, the effects of sampling bias can be considerably larger. In this section, uncorrected and sample-bias-corrected data are presented for several separated flow cases previously investigated [6, 11-13]. These data are presented to illustrate the levels of error that can result if no account of sampling bias is taken.

The first flow example is that generated on an axisymmetric model designed for study of interactions between transonic shock waves and turbulent boundary layers. The model consists of an axisymmetric circular bump affixed to a hollow 15-cm-diam cylinder aligned with the oncoming flow. The profile data shown in Fig. 10 were obtained at a free-stream Mach number of 0.875, which was sufficiently high to cause boundary-layer separation just downstream of the shock wave ($x/c = 0.7$). Data at three measurement stations are presented. The first station, $x/c = -0.25$, is just upstream of the bump, where a relatively mild adverse pressure gradient is present. The second station, $x/c = 1.0$, is at the trailing edge of the bump, where the separation bubble is the thickest. The remaining station, $x/c = 1.375$, was the farthermost downstream measurement station. Reattachment occurred at $x/c = 1.1$. The uncorrected and bias-corrected turbulent shear-stress distributions at $x/c = 1.0$ show a trend similar to that observed for the simple shear flow of the previous section (Fig. 8). At this streamwise station, the flow is basically a detached shear layer.

For separated flows, the situation where u_i and v_i are identically zero can arise, for which the two-dimensional weighting model is singular. In practice, however, these occurrences are so rare that they can be ignored without significantly affecting the statistical estimates. The condition where $u_i = v_i = 0$ can only occur if (1) the particle comes to rest in the sensing volume or (2) the particle enters the sensing volume from the side ($w_i \neq 0$). The likelihood of the former ($u_i = v_i = w_i = 0$) is very low, and, if the sensing volume is cylindrical with $l/d >> 1$, the latter also has a very low likelihood. For the data presented in this section, the number of samples with $u_i = v_i = 0$ never exceeded 0.3 percent of N, and most of these occurrences could have been the result of the finite clock frequency of the counters, which limited resolution of u_i and v_i to approximately $0.02u_e$.

The other two flow examples are for a NACA 64A010 and a supercritical (DSMA 671) airfoil section at transonic conditions. Profile data obtained just downstream of the trailing edges of these two airfoils are presented in Fig. 11. The data for the 64A010 section were obtained under conditions of shock-induced separation, whereas the supercritical section data were obtained at near-cruise conditions (only a very small separation bubble was present at the trailing edge).

To some readers, the differences in the uncorrected and bias-corrected results in Figs. 10 and 11 may seem large. Others may consider them small in light of the difficulty of the measurements. The obvious question, and one that remains to be answered, is whether the differences are sufficiently large to cause erroneous conclusions pertaining to turbulence model formulations.

Concluding Remarks

The presence of sampling bias effects in individual-realization laser velocimeter mean-velocity measurements has been demonstrated in a free-shear layer flow by using laser-velocimeter results obtained under continuous-wave conditions as the *standard* for comparison. It has also been demonstrated that these bias effects are independent of sampling rate provided the seeding concentration is sufficiently low to insure individual-realization measurements. A two-dimensional weighting of the velocity samples was shown to be effective in correcting the individual-realization measurements for the sampling bias. This correction is valid provided the length of the laser velocimeter sensing volume is reasonably long in comparison to its cross section, as was the case in the present experiment (this generally is true for most laser-velocimeter systems). Although a confirmation of sampling-bias effects on higher-order turbulence quantities, such as the turbulence intensities and Reynolds shear-stress, could not be made since no standard is available in this case, it follows that the statistical estimators for these quantities must also include appropriate weighting for sampling bias.

Only at extreme levels of turbulence ($\langle u' \rangle / \bar{u} > 0.2$, approximately) do sampling-bias effects become important. At lower turbulence levels, the effects of sampling bias are generally less than the overall experimental uncertainty. However, in the case of turbulent separated flows, the effects can be significant, as illustrated in the transonic-flow cases presented in this paper, and the possibility of making erroneous conclusions regarding the physical aspects of a flow as a result of ignoring these effects cannot be ruled out.

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