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A NEW CHAOTIC ATTRACTOR GENERATED FROM A 3-D AUTONOMOUS SYSTEM WITH ONE EQUILIBRIUM AND ITS FRACTIONAL ORDER FORM

Kishore Bingi[1], Susy Thomas[2]

[1]M.Tech Student, Electrical Engineering Department, National Institute of Technology, Calicut, Kerala, India

[2]Professor & Head, Electrical Engineering Department, National Institute of Technology, Calicut, Kerala, India

ABSTRACT

In this paper, a novel three-dimensional autonomous chaotic system is proposed. The proposed system contains four variational parameters, a cubic nonlinearity term (i.e. product of all the three states) and exhibits a chaotic attractor in numerical simulations. The basic dynamic properties of the system are analyzed by means of equilibrium points, Eigen values and Lyapunov exponents. Finally, the commensurate and non-commensurate fractional order form of the system which exhibits chaotic attractor is also analyzed.

Keywords: Chaos, Chaotic Systems, Chaotic Attractors, Commensurate Order System, Lyapunov Exponents, Non-Commensurate Order System.

1. INTRODUCTION

Chaotic behavior of dynamic systems can be utilized in a variety of disciplines, such as algorithmic trading, biology, computer science, civil engineering, economics, finance, geology, mathematics, microbiology, meteorology, physics, philosophy, and robotics and so on. In 1918, G. Duffing introduced a duffing equation which can be extended to complex domain in order to study strange attractors and chaotic behavior of forced vibrations of industrial machinery [1]. In 1920, Van der Pol introduced a model known as VPO model to study oscillations in vacuum tube circuits. The Van der Pol oscillator (VPO) represents a nonlinear system with an interesting behavior that exhibits naturally in several applications, such as heartbeat, neurons, acoustic models etc. [2]. In 1925, Alfred J. Lotka and Vito Volterra proposed predator-prey equations to describe the dynamics of biological systems in which two species interact on each other, one is a predator and the other is its prey [3]. In

1963, Lorenz found the chaotic attractor in a three-dimensional autonomous system while studying atmospheric convection [4]. In 1976, Otto Rossler proposed Rossler's system with strange attractor which is useful in modeling equilibrium in chemical reactions [5]. In 1981, Newton and Liepnik obtained the set of differential equations from Euler rigid equations which are modified with the addition of a linear feedback. Two strange attractors starting from different initial conditions and same parameter conditions were obtained [6]. In 1985, the chaotic phenomenon in macroeconomics was found. The continuous economical system was described and analyzed by Ma and Chen in 2001 [7]. In 1988, the basic circuit unit of the Cellular Neural Network (CNN) was introduced by L.O.Chua which contains linear and non-linear elements. Such types of CNN are able to show chaotic behavior [8]. In 1999, Chen found a simple three-dimensional autonomous system, which is not topologically equivalent to Lorenz's system and which has a chaotic attractor as well [9]. In 2005, Lu introduced a system which is known as a bridge between the Lorenz system and Chen's system [10]. In 2010, a new system was introduced which contain two variational parameters and exhibits Lorenz like attractor [11]. In the same year a new type of four wing chaotic attractor was generated from a smooth canonical 3-D continuous system [12].

Motivated by such previous work, this paper introduces another simple three-dimensional autonomous system which contains four variational parameters and one cubic nonlinearity term which is a product of all the three states i.e. displacement, velocity and acceleration. Section 2 explains the basic definitions. In section 3 the new system is briefly introduced. In section 4 the dynamic behaviors of the proposed system are discussed. The fractional order form of the system is discussed in section 5. Finally, some concluding comments are given in section 6.

2. BASIC DEFINITIONS

2.1 CHAOS

There is no universally accepted definition for chaos, but the following characteristics are nearly always displayed by the solution of chaotic system.

1. Aperiodic (non-periodic) behavior.
2. Bounded structure.
3. Sensitivity to initial conditions.

2.2 FRACTIONAL DERIVATIVE AND INTEGRAL

The continuous integral-differential operator is defined as

$$ {}_aD_t^{\alpha} f(t) = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}}, & \alpha > 0 \\ 1, & \alpha = 0 \\ \int_a^t (d\tau)^{\alpha}, & \alpha < 0 \end{cases} \tag{1} $$

2.3 GRUNWALD-LETNIKOV FRACTIONAL DERIVATIVE

The Grunwald-Letnikov fractional order derivative definition of order α is defined as

$$ {}_aD_t^{\alpha} f(t) = \underset{h \to 0}{lt} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh) \tag{2} $$

For binomial coefficients calculation we can use the relation between Euler's Gamma function, defined as $\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}$ for $\binom{\alpha}{0} = 1$

$\Gamma(\bullet)$Is Euler's Gamma function and a, t are the bounds of operation for ${}_aD_t^\alpha f(t)$.

2.4 RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVE

The Riemann-Liouville fractional order derivative definition of order α is defined as

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{3} $$

$\Gamma(\bullet)$Is Euler's Gamma function and a, t are the bounds of operation for ${}_aD_t^\alpha f(t)$.

2.5 STABILITY OF FRACTIONAL NONLINEAR SYSTEMS

According to stability theorem, the fractional order system $q_1 \neq q_2 \neq .... \neq q_n$ and suppose that m is the LCM of the denominators $u_i's$ of $q_i's$, where $q_i = \frac{v_i}{u_i}$, $v_i, u_i \in Z^+$ for $i = 1, 2, ...n$ and we set $\gamma = \frac{1}{m}$. The fractional order system is asymptotically stable if

$$ |\arg(\lambda)| > \gamma \frac{\pi}{2} $$

For all roots λ of the following equation

$$ \det\left(diag\left(\left[\lambda^{mq_1}\ \lambda^{mq_2}\ ....\lambda^{mq_n}\right]\right)\right) = 0 \tag{4} $$

2.6 CONDITION FOR MINIMUM COMMENSURATE ORDER

Suppose that the unstable Eigen values of scroll focus points are $\lambda_{1,2} = \alpha_{1,2} \pm j\beta_{1,2}$. The necessary condition to exhibit double scroll attractor of fractional order system is the Eigen values $\lambda_{1,2}$ remaining in the unstable region. The condition for commensurate order is

$$ q > \frac{2}{\pi} a \tan\left(\frac{|\beta_i|}{\alpha_i}\right), i = 1, 2 \tag{5} $$

This condition can be used to determine the minimum order for which a nonlinear system can generate a chaos.

3. THE PROPOSED 3-D DYNAMICAL SYSTEM

Consider the following simple 3-D autonomous system:

$$ \left.\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -az - by - cx + dxyz \end{aligned}\right\} \tag{6} $$

Where $[x, y, z]^T \in R^3$ the state vector, and a, b, c and d are positive constant parameters of the system (6).
In the following, some basic properties of system (6) are analyzed.

3.1 EQUILIBRIA

The equilibria of system (6) can be found by solving the following algebraic equations:

$$\left.\begin{aligned} \dot{x} &= y = 0 \\ \dot{y} &= z = 0 \\ \dot{z} &= -az - by - cx + dxyz = 0 \end{aligned}\right\} \tag{7}$$

From the first and second equations of (7),

$y = 0, z = 0$

Substituting this into the third equation of (7),

$x = 0$

Therefore $O(0,0,0)$ is the only equilibrium point of the system (6).

3.2 STABILITY AND EXISTANCE OF ATTRACTOR

By linearzing the system (6), one obtains the Jacobian

$$J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ dyz - c & dxz - b & dxy - a \end{bmatrix}$$

Therefore $J\big|_{O(0,0,0)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c & -b & -a \end{bmatrix}$

So, the Eigen values of the linearized system are obtained as follows:

$\left|\lambda I - J\big|_O\right| = 0 \Rightarrow \lambda^3 + a\lambda^2 + b\lambda + c = 0$

Case 1: If $a = b = c = d = 1$ the Eigen values are $-1, \pm\, j$, the critical case.

Case 2: If $a > 1, b = c = d = 1$ the equilibrium O is stable, ensures that system (6) is not chaotic.

Case 3: To ensure that system (6) is chaotic implying that the equilibrium O is saddle point, the condition on the positive constant parameters of system should be considered, i.e. $a < 1, b \le 1.1, c \ge 1$ and $d =$ any value.

4. DYNAMICAL BEHAVIOR OF THE PROPOSED SYSTEM

When $a = 0.45, b = 1.1, c = 1$ and $d = 1$, the Eigen values of the linearized system are $-0.7529, 0.1514 \pm 1.1424\,j$. Therefore, based on the Eigen values we know that equilibrium O is a saddle point.

In this section the fourth and fifth order Range-Kutta integration algorithm was performed to solve the differential equations. Setting the initial condition to $[0.1 0.1 0.1]$, the chaotic attractor is shown in figure 1.

The Lyapunov spectrum of the system (6) versus time is shown in figure 2 with parameters $a = 0.45, b = 1.1, c = 1$ and $d = 1$.

When $a = 0.15, b = 1, c = 1$ and $d = 1$, the chaotic attractor of the system (6) with initial condition [0.10.10.1] is shown in figure 3.

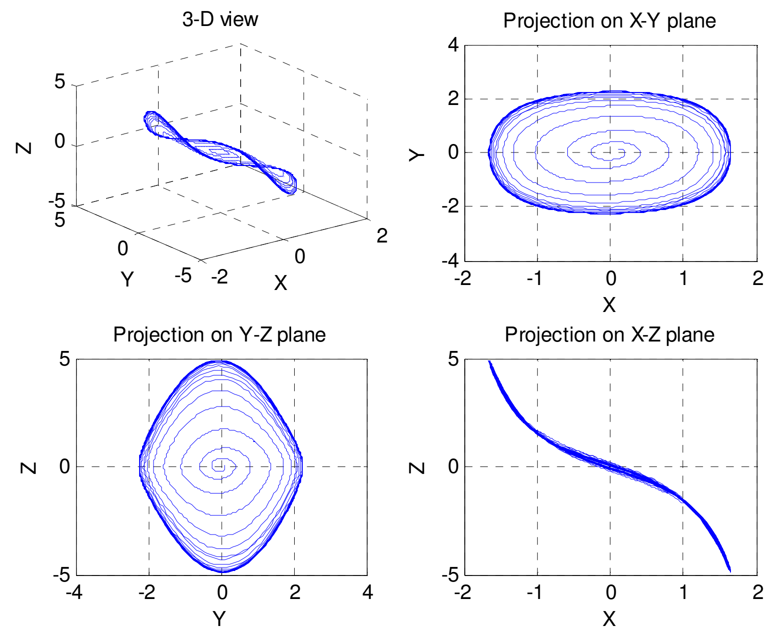

Fig 1: Chaotic attractor the system (6) with parameters $a = 0.45, b = 1.1, c = 1$ and $d = 1$, initial condition [0.10.10.1]

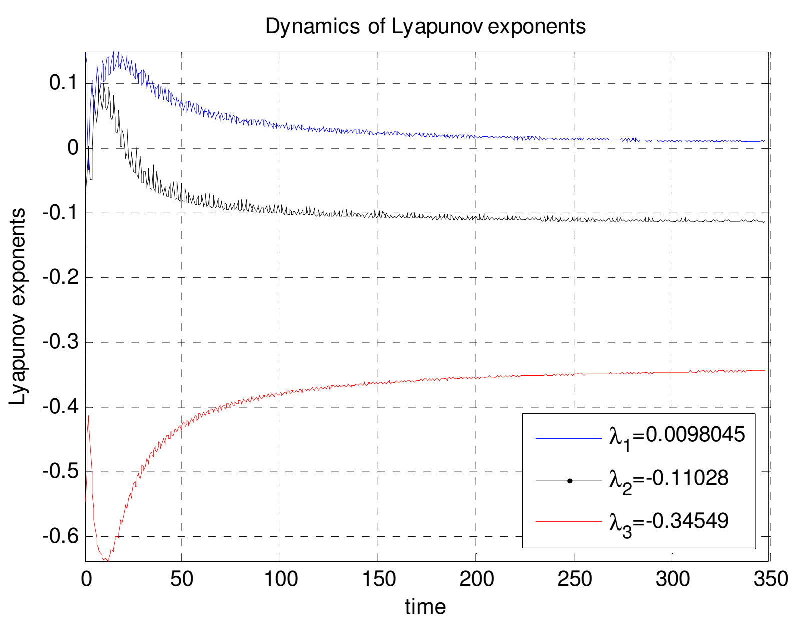

Fig 2: Lyapunov spectrum of the system (6) with parameters $a = 0.45, b = 1.1, c = 1$ and $d = 1$, initial condition [0.10.10.1]

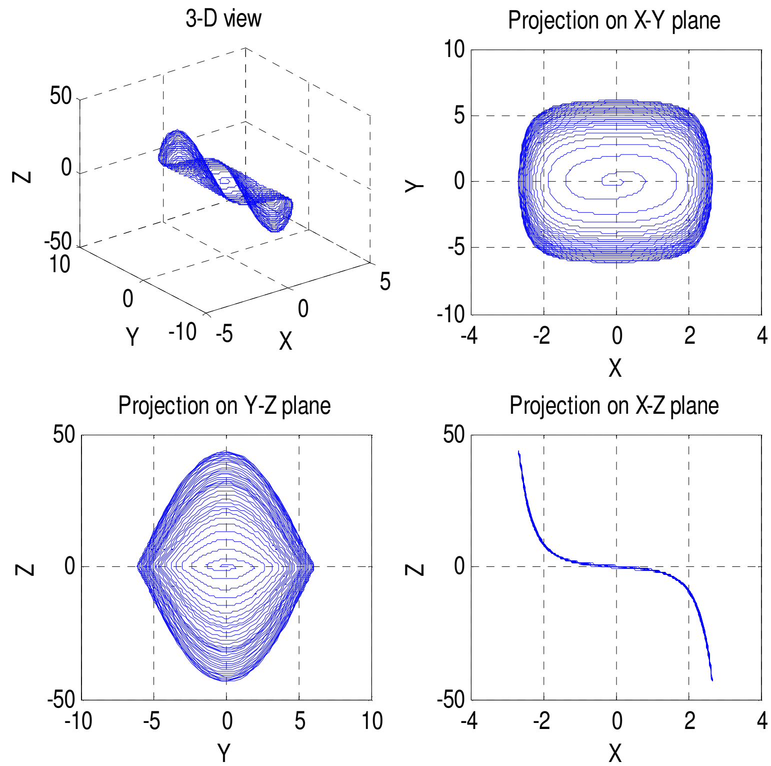

Fig 3: Chaotic attractor the system (6) with parameters $a = 0.15, b = 1, c = 1$ and $d = 1$, initial condition [0.10.10.1]

5. FRACTIONAL ORDER FORM OF THE PROPOSED SYSTEM

The fractional order form of the system (6) is defined as follows

$$\left.\begin{aligned} \frac{d^{q_1}x}{dt^{q_1}} &= y \\ \frac{d^{q_2}y}{dt^{q_2}} &= z \\ \frac{d^{q_3}z}{dt^{q_3}} &= -az - by - cx + dxyz \end{aligned}\right\} \tag{8}$$

Where q_1, q_2 and q_3 are the derivative orders.

For numerical simulation of the fractional order system (8), we have considered the two cases: first, commensurate order system and second, non-commensurate order system.

Case 1: Commensurate order system

From equation (5) the commensurate order of the system is given by $q > \frac{2}{\pi} a\tan\left(\frac{1.1424}{0.1514}\right) \approx 0.9161$

In figure 4 is depicted the chaotic attractor of the commensurate fractional order (8) with parameters $a = 0.45, b = 1.1, c = 1, d = 1$, derivative orders $q_1 = q_2 = q_3 = 0.97$ with initial condition $[0.1 0.1 0.1]$ for simulation time $T_{sim} = 500\,s$ and step time $h = 0.05$.

Case 2: Non-commensurate order system

We consider non-commensurate order system with parameters $a = 0.45, b = 1.1, c = 1, d = 1$, derivative orders $q_1 = 0.97 = \frac{97}{100}, q_2 = 0.98 = \frac{98}{100}$ and $q_3 = 0.99 = \frac{99}{100}$. Therefore $\gamma = \frac{1}{m} = \frac{1}{100}$

From equation (4) the characteristic equation of the linearized system is

$$\lambda^{294} + 0.45\lambda^{195} + 1.1\lambda^{97} + 1 = 0$$

The unstable roots are $\lambda_{1,2} = 1.001389 \pm 0.014673j$, because $\left|\arg(\lambda_{1,2})\right| \approx 0.01465 < \gamma\frac{\pi}{2}$

In figure 5 is depicted the chaotic attractor of the non-commensurate fractional order (8) with parameters $a = 0.45, b = 1.1, c = 1, d = 1$, derivative orders $q_1 = 0.97, q_2 = 0.98, q_3 = 0.99$ with initial condition $[0.1 0.1 0.1]$ for simulation time $T_{sim} = 500\,s$ and step time $h = 0.05$.

In figure 6 is depicted the chaotic attractor of the non-commensurate fractional order (8) with parameters $a = 0.45, b = 1.1, c = 1, d = 1$, derivative orders $q_1 = 1.5, q_2 = 1.0, q_3 = 1.3$ with initial condition $[0.1 0.1 0.1]$ for simulation time $T_{sim} = 500\,s$ and step time $h = 0.05$.

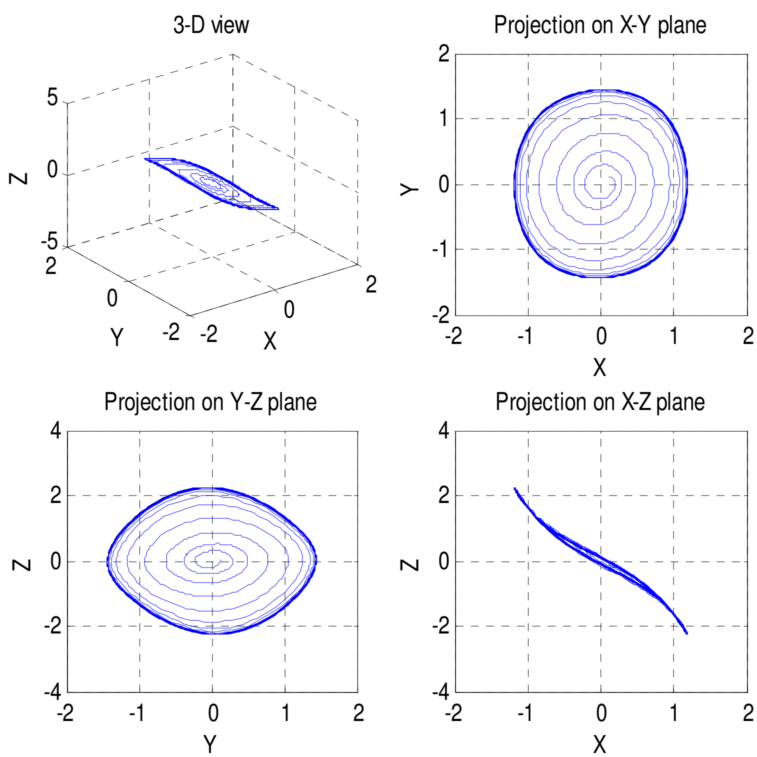

Fig 4: Chaotic attractor of the commensurate fractional order (8) with parameters $a = 0.45, b = 1.1, c = 1, d = 1$, derivative orders $q_1 = q_2 = q_3 = 0.97$ with initial condition $[0.1 0.1 0.1]$

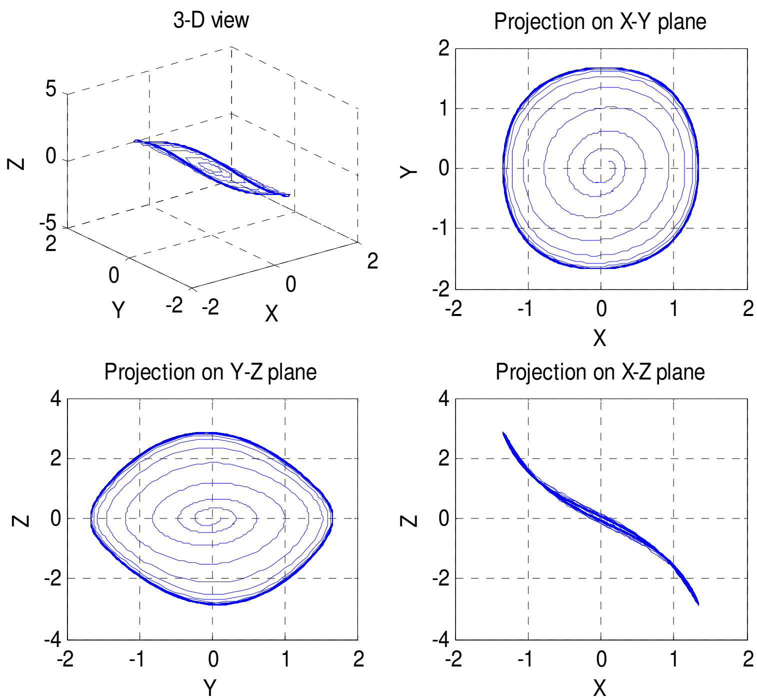

Fig 5: Chaotic attractor of the non-commensurate fractional order (8) with parameters $a = 0.45, b = 1.1, c = 1, d = 1$, derivative orders $q_1 = 0.97, q_2 = 0.98, q_3 = 0.99$ with initial condition [0.10.10.1]

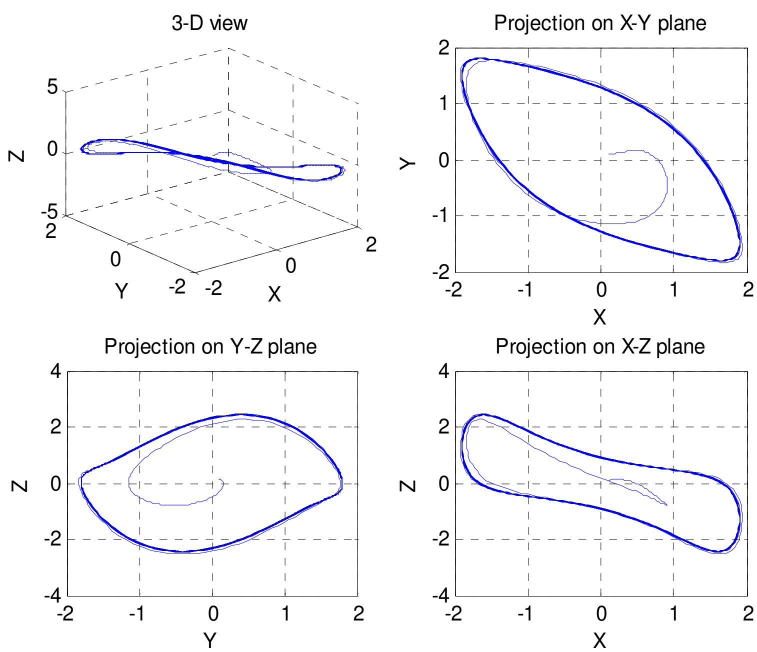

Fig 6: Chaotic attractor of the non-commensurate fractional order (8) with parameters $a = 0.45, b = 1.1, c = 1, d = 1$, derivative orders $q_1 = 1.5, q_2 = 1.0, q_3 = 1.3$ with initial condition [0.10.10.1]

6. CONCLUSION

In this paper, a new novel three-dimensional chaotic system is proposed. The proposed system has only one equilibrium point for any arbitrary set of parameters and also some dynamic properties of the system have been investigated. The dynamics of the proposed system with commensurate and non-commensurate fractional order forms was studied based on stability theorems. On the other hand chaos control and synchronization of this system are interesting problems to be investigated and should also be considered in the future work.

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