

Primary 6 Students' Attitudes towards Mathematical Problem-Solving in a Problem-Based Learning Setting

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Abstract: Research has indicated that student attitudes towards mathematics and mathematics learning can impact on mathematics achievement. While research has also indicated that unconventional pedagogies such as problem-based learning or cooperative learning may promote interest in student learning, the lack of evidence of the effects of these pedagogies in Singapore primary mathematics classrooms fuels the need to understand if these newer learning environments will promote positive attitudes. In this study, 80 Primary 6 students were surveyed to determine their attitudes towards solving mathematical modelling problems in a problem-based learning (PBL) setting after they had solved five modelling tasks. Contrary to a local study that suggests that Grade 7 students were not positive about solving challenging mathematics problems, results from the Attitudes Questionnaire show positive responses in the attributes of *Interest*, *Perseverance*, and *Confidence*, and suggesting that PBL can promote positive attitude in mathematics learning. Mixed-ability students registered higher, but statistically not significant, mean scores in these three areas than high-ability students. Students' open responses suggest that solving problems in a PBL setting is a promising pedagogic approach.

Key Words: Problem-based learning; Mathematical problem-solving; Mathematical modelling; Affective domain; Attitudes

Introduction

The affective domain is an important area to consider in mathematics education as it plays a part in determining students' learning and achievement of mathematics. Unfortunately, according to Goldin (2002), research in this affective domain is limited as the tendency is to focus principally on the cognitive, probably because many see mathematics learning more as an intellectual endeavour than an emotional one. He argued that affect is not merely auxiliary to cognition but is indeed central, that is, affect as a representational system is intertwined with cognitive representation, and it affects student actions (e.g., change of strategy).

Research has shown that student attitude towards mathematics can affect his or her performance and participation in mathematics (Hannula, 2006; McLeod, 1994; Scheonfeld, 1992). Some researchers have found that, for most students who have negative attitudes towards mathematics, they try to avert it because it is a source of frustration, discouragement, and anxiety; they also find mathematics learning a tiresome chore (Ignacio, Nieto, & Barona, 2006). While it appears that attitudes affect mathematics performance, researchers have also claimed that performance in turn affects attitudes suggesting a reciprocal relationship between attitudes and performance (McLeod, 1992; Neale, 1969). Other studies, however, find that such a relationship is not significant (Papanastasiou, 2002), and that high performance in mathematics is not necessarily positively related to attitudes towards mathematics or mathematics learning as in the case of Japanese students (Mullis et al., 2000).

In Singapore, research findings based on the TIMSS 2003 and 2007 data on Singapore students' attitudes towards mathematics learning revealed that Grade 4 and 8 students had positive attitudes and high self-confidence (Mullis et al., 2004, 2008). From a survey of 1215 Singapore Secondary One (Grade 7) students, Fan et al. (2005) also found that these students had generally positive attitudes towards mathematics and mathematics learning. However, they held relatively negative attitudes about working on challenging mathematics problems (31% disliked these problems) and the usefulness of mathematics in adult life. In explaining the findings, Fan et al. (2005) pointed out that the positive attitudes could be attributed to the examination-oriented style of teaching so that students were well-prepared to attempt what had been taught rather than teaching focused on knowledge and skills beyond the classroom. They recommended that school teachers provide students with opportunities to work on non-routine and challenging problems in an authentic way to maximize their higher-order thinking skills. In this way, the students will value the intrinsic essence of mathematics rather than perceive it as representations of rigid processing, routine manipulations, and theoretical operations. Such effort would spur mathematics learning and develop more positive attitudes towards mathematics.

What Fan et al. have proposed provides one view of enacting the Singapore Mathematics Curriculum Framework, which includes *Attitudes* as one of the five core components, apart from *Concepts*, *Skills*, *Processes*, and *Metacognition*, that are to be developed in students for them to become good problem solvers. Recognizing the importance of affect in students' mathematics learning is part of reform efforts in other mathematics curricula as well. For example, the National Council of Teachers of Mathematics (2000) in the United States has highlighted the importance of students' confidence, interest, perseverance, and curiosity in learning mathematics.

Since the findings of Fan et al. (2005) were based on only a survey, this study has taken a different route by investigating the attitudes of Primary 6 students (aged 12) who had solved mathematical modelling problems in a problem-based learning (PBL) setting. This paper will focus on the *interest*, *perseverance*, and *confidence* aspects of this attitude.

Theoretical Framework

Descriptions of Attitudes

There are different definitions of attitudes in mathematics education. Researchers have included emotions and beliefs (Garofalo, 1989; Schoenfeld, 1992), anxiety or confidence (Ernest, 1988), liking or disliking mathematics (Ma & Kishor, 1997), positive or negative emotional disposition towards mathematics (Zan & Martino, 2007), and beliefs that mathematics is interesting or uninteresting (McLeod, 1992). In this paper, the meaning of student attitude is adapted from the Singapore Mathematics Curriculum Framework (Ministry of Education, 2007): it refers to having an *interest* in doing mathematics, showing *confidence*, and *persevering* in solving problems. In this paper, *interest* refers to the feeling of liking and enjoyment (Papanastasiou, 2000; Philipp, 2007) in solving mathematical modelling problems in a PBL setting. *Confidence* refers to having the self-belief to be able to solve problems. As modelling tasks are more complex than the typical structured word problems, this study aimed to determine if students who were new to such tasks were confident in working through them. *Perseverance* refers to the spirit of not giving up in the face of difficulties (Kho, Yeo & Lim, 2009). Although solving real-world problems requires students to invest a substantial amount of time in their efforts, Schoenfeld (1989) found that students believed that problems should be solved within one or two minutes, otherwise the problems would not be solvable. Lester and Garofalo (1987) revealed that without confidence and perseverance, students did not have the desire to obtain the correct answer, and the lack of persistence will lead to premature closure in solving the problems.

Mathematical Modelling in a PBL Setting

An earlier section has highlighted a gap between students' attitude towards solving challenging problems and familiar procedural problems. Having students solve modelling problems in a PBL setting provides the opportunity to ascertain their attitudes based on actual experiences.

Mathematical modelling requires students to conceptualize the problem situations and provide mathematical translations to give meaning to the conceptual representations. The focus is not so much the product but the process of formulating

mathematical relationships among problem variables to explain the situations (Doerr & English, 2003; Lesh & Doerr, 2003). Seen in this light, the process itself is seen as the product (Doerr & English, 2003). The process would require students to make mathematizations, that is, their manipulation of data through describing, organizing, analyzing, and interpreting them (Mousoulides, Christou & Sriraman, 2007). From a models-and-modelling perspective, this implies that students are involved in cycles of expressing, testing, and revising their models (English, 2003; Lesh & Doerr, 2003). The challenge of mathematical modelling is to develop students' mathematical thinking through the express-test-revise cycles that go beyond specific mathematical content or skills (Lesh & Zawojewski, 2007).

Mathematical modelling is seen as befitting a PBL instructional setting (Hjalmarsen & Diefes-Dux, 2008). This study dealt with short-cycle PBL. According to Tan (2003), this is the solving of ill-structured authentic problems of a novel nature within a short time-frame rather than PBL involving the resolution of complex and multiple goals. The short-cycle PBL instructional approach was adopted in this study because of time constraints. Two other important tenets of PBL are student collaboration and teacher facilitation. The task-students-teacher interaction makes problem solving meaningful and the learning powerful (Tan, 2003). These constructs are factors that will influence students' attitudes towards learning and problem solving.

Mathematical Modelling in a PBL Setting and Student Attitudes

The modelling tasks used in the PBL setting challenge students to generate multiple perspectives and to reflect the importance of working on real-world situations. If a problem is too theoretical and out of touch with students' experiences and daily lives, the students may not engage in it. The closer the problem is grounded to the students' experiences, the harder they may work on it, and this engagement is tied to the attitudinal aspects of motivation and perseverance (Delisle, 1997; Hiebert et al., 1996; Mayer, 1998). On the other hand, textbook problems that do not allow for serious consideration of possible constraints of the realities of the problem contexts may contribute to students' suspension of sense-making (De Corte, Verschaffel & Greer, 2000).

Students working collaboratively (or cooperatively, used synonymously in this paper) have been shown to develop positive attitudes (Sharan, 1980; Slavin, 1988). Collaborative learning in mathematics classrooms has resulted in better student achievement and positive attitudes towards cooperative learning (Whicker, Bol & Nunnery, 1997). Students were found to develop positive interdependence through showing respect and admiration for the group compared to traditional setting where they tended to be competitive (Johnson, Johnson, & Holubec, 1998). Such a

pedagogic approach has been found to lead to a decrease in student mathematics anxiety (Miller, 2003). In a local study, Ho (1997) found that students encouraged and praised their peers and exhibited persistence in problem solving. The teacher should create a supportive environment (e.g., Lesh & Zawojewski, 2007).

The Study

Participating Students

The students were from a neighbourhood primary school in Singapore. The Head-of-Department of the school requested to be involved in this study after attending a PBL mathematics workshop conducted by the researcher. The students were from two Primary 6 classes identified by the Head-of-Department. One was a high-ability class (HA; $n = 39$) and the other a mixed-ability class (MA; $n = 41$). The students had no prior experience in problem-based learning involving mathematical modelling tasks. During the study, the students worked in small groups of four or five, a grouping based on friendship and academic abilities (one high-ability, two mixed abilities, and one lower-ability in a group of four), in particular for the MA class.

The PBL Mathematical Modelling Tasks

The students solved five modelling tasks with increasing complexity and amount of data, situated in a short-cycle PBL setting. Each PBL session lasted almost an hour and was carried out over several weeks.

The five tasks were case-based or data-driven, designed to surface students' mathematical thinking and model construction abilities. Table 1 provides a synopsis of the tasks and the mathematics content.

Table 1
Descriptions of PBL Mathematics Tasks

PBL Session & Mathematics Task	Synopsis of Task	Mathematics Content
Session 1—The Height-Volume Problem	Water flows from a tap into different types of containers (rectangular, hemispherical, and circular). Students have to argue if the height of the water level is directly related to the volume.	Measurement, graph, volume, duration, shape, relationships, estimating, rate, comparing
Session 2—The Biggest Box Problem	Students are involved in a competition. They are provided with a 50 cm by 50 cm vanguard sheet to make the biggest box and convince the judge of their solutions.	Measurement, volume, area, shape, nets, relationships, estimating, graph optimizing, comparing
Session 3—The Floor-Covering Problem	Given three different floor covering materials (carpet, mat and tiles), students are to determine which to use to cover a rectangular floor of a study room. Materials come at different costs and dimensions.	Measurement, relationships, area, estimation, rate optimizing, decimals, costing, comparing
Session 4—The Itinerary Problem	Students act as tour agents to plan an itinerary for tourists heading to Trimbell Island. Data provided for consideration include a map, places of interests, mode of transport and accommodation at varying costs and rates. Constraints are provided as well.	Distance, time, speed, map (scale) reading, rate and proportion, costing, optimizing, systematic listing, comparing
Session 5—The Hiring Problem	Students are assigned a mission to renovate the school. They are to engage the services of cleaners, painters and movers from different companies at different rates. They need to complete the renovation within given constraints.	Combinations of data within and across tables, rates, costing, comparing, systematic listing, optimizing, applying productivity index formula

Instruments and Administration

The items in the Attitude Questionnaire (AQ) were modified from Tapia and Marsh's (2000) Attitude Towards Mathematics Inventory and Chee's (2001) attitudinal inventory on the PBL approach to learning computing mathematics. There were 12 closed-ended items where students responded using a 4-point Likert scale ranging from 1 (strongly disagree) to 4 (strongly agree). The items are given

in Table 3 later. Items 1, 6, 8, and 11 are categorized under *Interest* based on expressive key words, such as “prefer,” “interesting,” “enjoy” and “like” in relation to solving mathematics problems in a PBL setting. Items 2, 5, 9, and 10 are categorized under *Perseverance* based on key words that convey meanings, such as “not giving up,” “kept going,” and “being focused.” Items 3, 4, 7, and 12 are categorized under *Confidence* based on key words that depict better manageability, lower anxiety, and a gain in confidence. The AQ was administered after the students had completed all the five modelling tasks. It was not administered as a pre-test because the students did not have prior experience with mathematical modelling in a PBL setting and would not understand terms like PBL in the AQ items.

In addition to the above 12 items, one open-ended question asked the students to write about “What is the best thing that happened during problem solving?” A limitation of this study was that it did not ask the students to write about the “negative” aspects of this problem-solving approach. After each mathematical modelling session, the groups were encouraged (optional) to write journal entries as reflections of their problem-solving effort. This provides further information about the students’ experiences with this approach.

Findings

The findings from the Attitudes Questionnaire are presented according to the clustering of items by categories, by individual items, and by ability-class types. Open-ended responses are categorized into six common themes and discussed to reflect those themes.

Attitudes by Categories

Given four items in each category of 4-point Likert scale, the maximum point for each category was 16. A favourable response for a category would have a mean of at least 8 points. Table 2 shows the mean for each category and the overall mean for all the items.

Table 2
Attitudinal perception items clustered according to categories (n = 80)

Category	Mean	SD	Maximum	Minimum	Cronbach’s alpha
Interest	13.49	2.88	16	6	.75
Perseverance	13.18	2.50	16	8	.73
Confidence	13.18	2.73	16	6	.70
Overall	39.77	8.12	48	20	.89

Cronbach's alphas for the individual categories and overall scale ranged from 0.70 to 0.89, and this indicates strong internal consistency. The means for the categories were about 13, and these values suggest that the students were evenly positive about their experiences with the PBL approach.

Attitudes by Items

Table 3 shows the means and standard deviations by items under each category. The means suggest that the students were generally positive about the *interest* (means between 3.28 and 3.45), *perseverance* (means between 3.20 and 3.45), and *confidence* (means between 3.16 and 3.42) aspects. The highest mean of 3.45 was found for item 5 (The challenge of solving the problem task kept me going and thinking) and item 6 (This PBL approach makes mathematics problem solving more interesting and challenging). Higher confidence in problem solving in the PBL setting was found when the students worked in groups (see items 3 and 7). The two items with lowest means (4 and 12) suggest that the students were not more confident in solving problems than before even after the PBL experience.

Attitudes Findings by Ability-Class Types

Table 4 compares the means between the HA and the MA students using two independent samples t-tests. The differences were statistically not significant at the 5% level, even though the MA group had slightly higher means than the HA group on all the three categories. This seems to differ from the perception that PBL favours students who are academically more inclined (Carriga-Lo, Richards, Hollingsworth, & Camps, 1996).

Problem-Solving Perceptions from Open-Ended Responses

The responses to the open-ended item, "What is the best thing that happened during problem solving?" were grouped into the following six categories:

- (1) The *Problem-Solving Process*—students described how their problem-solving process was like;
- (2) The *Interest-Motivation Aspect*—students described their motivation and interest in the problem-solving endeavour;
- (3) The *Task Aspect*—students described the impact of the tasks;
- (4) *Teamwork-Motivation Aspect*—students described their motivation through working together;
- (5) The *Achievement-Motivation Aspect*—students described their desire to achieve their goal;
- (6) *Teacher-Involvement*—students described the impact of the teacher.

Table 3
Mean score of each attitude item ($n = 80$)

Category	Items	Mean	SD
Interest	1. I would prefer this approach (PBL) of solving mathematics problems than solving textbook problems.	3.31	0.76
	6. This PBL approach makes mathematics problem solving more interesting and challenging.	3.45	0.65
	8. I enjoyed working with my friends to solve Mathematics problems.	3.43	0.69
	11. I like this PBL approach as I learn better.	3.28	0.78
Perseverance	2. The problem solving session was long but I did not give up.	3.20	0.60
	5. The challenge of solving the problem task kept me going and thinking.	3.45	0.57
	9. I was focused on finding the solutions to the problems.	3.29	0.64
	10. When I got stuck, I did not stop. I discussed with my friends or teacher on possible steps.	3.23	0.68
Confidence	3. Solving problems with a group of friends made the problem solving process easier to manage.	3.41	0.66
	4. I feel I am better at solving more difficult problems now than before.	3.16	0.68
	7. I was not afraid of working in groups to solve mathematics problems.	3.42	0.67
	12. I gained more confidence in solving mathematics problems through a PBL approach because of help from friends and the teacher.	3.18	0.70

Table 4
Comparison of attitudes between ability classes

	Interest				Perseverance				Confidence			
	<i>N</i>	Mean	SD	<i>p</i>	<i>N</i>	Mean	SD	<i>p</i>	<i>N</i>	Mean	SD	<i>p</i>
HA	39	3.31	.81	.06	39	3.30	.69	.45	39	3.26	.76	.09
MA	41	3.44	.62		41	3.31	.57		41	3.36	.60	

Table 5 shows the percentages of these responses by ability-class types. The *Others* category was from the only student who wrote about being able to decorate the answer sheet (aesthetic reason). Examples of the other responses are discussed below.

Table 5
Percentages of open-ended responses in the six categories

Open-Ended Responses	HA class n=39	MA class n=41	Total N=80
1 The Problem-Solving Process	10.2% (4)	22.0% (9)	16.3% (13)
2 The Interest-Motivation Aspect	12.8% (5)	9.7% (4)	11.2% (9)
3 The Task Aspect	5.1% (2)	4.9% (2)	5.0% (4)
4 Teamwork- Motivation Aspect	43.6% (17)	48.8% (20)	46.3% (37)
5 The Achievement-Motivation Aspect	23.1% (9)	12.2% (5)	17.5% (14)
6 Teacher-Involvement	2.6% (1)	2.4% (1)	2.5% (2)
Others – Aesthetic reason	2.6% (1)	0.00% (0)	1.2% (1)

- (1) *The Problem-Solving Process*. About 16.3% of the students had penned words like “*planning*,” “*knowing the problem*,” “*calculating*,” “*elaborating*,” “*finding ways to solve...*,” and “*...enjoyed the entire problem solving process*” as descriptions of what they perceived as the best thing about the PBL sessions. These responses captured in essence what a mathematical problem-solving process should entail.
- (2) *The Interest-Motivation Aspect*. Typical responses under this category were “*It is fun*.” “*It is interesting*” or “*exciting*.” One student from the MA class wrote, “*The PBL was very exciting. It really rocks!!! Normal math class is BORING*.” The *Interest* category had the highest overall mean in the AQ; about 11% of the responses mentioned the fun and excitement element as the best part of the PBL sessions.
- (3) *The Task Aspect*. This category did not garner a high percentage of responses (5%). These students perceived solving mathematics problems in this problem-based approach to be “*challenging*,” where the engagement of the tasks enabled “*many ways to solve the problem*,” and being “*real-life*.”
- (4) The *Teamwork-Motivation* aspect. Students wrote about learning through teamwork. Some examples include “*We learned about teamwork and we had lots of fun*,” “*We learned how to work as a team and cooperate while solving the problem*,” and “*We cracked our heads but to no avail and we broke our friendship but we have fun as it makes us think a lot*.” This finding had the highest percentage of responses (46.3%), suggesting that the students valued working together in solving problems. The finding is consistent with the sole study conducted in a local polytechnic (Chee, 2001) that showed at least 75% of the students favoured solving mathematics problems in the PBL setting due to the working in group factor.
- (5) *The Achievement-Motivation Aspect* also covered reasons why students liked solving mathematics problems in PBL settings. This category accounted for

17.5% of the responses, and the students wrote about their sense of achievement or satisfaction in having solved the problems. Typical responses include “*getting the correct answers*,” “*getting the best answers*,” “*completed the mission*,” “*having solved the problem*,” and “*getting a sense of satisfaction in solving it*.” This is also consistent with Liu’s (2005) findings, when she found that students gained a sense of satisfaction after having worked hard and being able to solve the problem and feeling proud of it. Liu claimed that being mindful of goals had kept the students cognitively occupied, task-oriented, and with heightened metacognitive thinking.

- (6) *Teacher-Involvement*. Teacher-involvement is one of the key facets in a PBL setting. However, only two responses had expressed an appreciation of the teacher as a scaffolding agent as the best thing in the problem-solving experience: “*can discuss with the teacher*,” and “*the teacher helped us*.”

Difficulties and Issues Faced by Students During Problem Solving

The students had expressed in their group journals their difficulties and issues about the sessions. Their difficulties were grouped as *task-specific* reason, *peer-specific* reasons, and *resource constraint* reasons, as exemplified below. Most of the reasons were task-specific, with a few peer-specific, and even fewer resource-constraint specific. As the writing of group journals was optional, this cannot be taken as reflective of the difficulties encountered by all the students.

- (1) *Task-Specific Reasons*. These reasons include not understanding the task context or not knowing where to start when the task did not provide figures for them to work on (the first and second tasks). Some examples are “*Not being able to cover all the places of interest*,” “*Finding the cost. Think and think and think*,” and “*Solving the question without numbers*.” The reasons were not unexpected as the tasks had considered optimization, economic objectives, and novelties to serve the problem-solving agenda.
- (2) *Peer-Specific Reasons*. Although the teamwork and collaboration aspects were positively perceived based on the findings from the AQ, working with peers to solve problems was most difficult as expressed by some groups. Communication problems and distractions were amongst the main reasons included in this category, and one group mentioned gender differences. Responses include “*We could not communicate well with one another*,” “*When some people were causing distractions. We asked them to focus on their work*,” “*...though we argued quite a bit, after all, we still came to a decision. We thought and tried our best to communicate*,” and “*The boys do not even discuss with us*.” These reasons are not unexpected because under PBL, task complexity not only drives discussion and collaboration, but also causes

cognitive conflicts, leading to disagreements for argumentation to take place. While this interaction was encouraged, some students were observed to guard their own solutions, and this resulted in devaluing the contributions of other members. This is where peer monitoring from level-headed members or the alert teacher should step in to keep the group on task, diffuse social conflicts, or heighten the students' thinking.

- (3) *Resource Constraint Reasons.* It would have been ideal to have the students spend more time on their problem-based experiences but the contractual understanding with the school for data collection (video-recording for the main study) was to last only one-hour per session. A few groups had cited time as the most difficult part of their problem solving experience. Two groups wrote “*Time management to complete our work. Talk less, do more,*” and “*Finishing before the time runs out*” respectively. This study was unable to ascertain whether more time would improve performance and attitudes in problem solving. The time factor, however, could also pose a sense of realism, where students have to work within constraints in solving problems as in any real life situation.

Discussion and Implications

In this study, findings from the quantitative analyses and open-ended responses indicate that the Primary 6 students were generally positive about solving mathematical modelling problems in a PBL setting. In general, the students had enjoyed the mathematical problem-solving experiences because the activities could generate interest, engage collaboration, and provide a sense of challenge. This overall finding is contrary to the finding that Grade 7 students were not positive about being engaged in challenging problems as reported by Fan et al. (2005) using student perceptions. Mixed-ability students were found to have slightly more positive attitudes than the high-ability students, although this difference was not statistically significant. This concurs with the findings reported by Cotic and Zuljan's (2009) that of 179 nine-year old students involved in solving challenging problems in problem-based mathematics learning, their motivation did not decrease, especially the academically weaker ones. If students were not interested in solving challenging structured types of mathematics problems, the introduction of modelling problems in a problem-based approach could instead stir greater interest and engagement. Moreover, such engagement allows for flexibility in the way students solve problems as they move through the stages of the modelling cycles. Some researchers see in this process how modeling might affect student attitudes (Lesh & Doerr, 2003).

Working collaboratively with peers was highly valued by students in this study. The need to express, test, modify, and justify methods implies use of mathematical reasoning and communication, which are elicited through meaningful student collaboration. In this respect, students learn from one another by exploring mathematical ideas related to the real world (Zbeik & Conner, 2006). This points to the need to support students' collaborative discourse through a pedagogic setting that is different from a dominantly teacher-centered one. Since the learning environment has an effect on the way students learn (Prosser & Trigwell, 1999), solving mathematics problems in a PBL setting possibly could have resulted in the students' positive attitudes.

The difficulties highlighted by the students as *task-specific* and *peer-specific* reasons are in a sense problems to be welcomed. These difficulties are precisely the essence of what should entail in a problem-based socio-learning environment. They enable students to wrangle in the complexities of working with one another and the modelling task towards goal resolution, and this resolution provides the challenge and sense of achievement. What is important is for the teacher to know how to manage the students and not let the difficulties lead to unproductive personal confrontations among the students. By supporting the students in their problem-solving endeavour, teachers can also strengthen the students' sense of commitment and perseverance. It is likely that through building better rapport with students, making daily lessons more inquiry-based, and empowering students to share ideas more frequently, there will evolve a better learning community, which will influence students' attitudes in learning. Research should be carried out to determine if such practices can sustain positive attitudes in students as they get used to a problem-based learning instructional approach over time.

Conclusion

This study has found that a small sample of Primary 6 students had positive attitudes towards solving challenging mathematical modelling problems in a PBL setting. It calls for the need to design suitable learning environments for shaping desirable learning behaviours and actions, to include more students in larger study, and to look into how the roles of the teacher, the quality of their facilitation, and group dynamics can affect students' attitudes and problem-solving in a PBL setting.

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