QUALITY INTERVAL ACCEPTANCE SINGLE SAMPLING PLAN WITH FUZZY PARAMETER USING POISSON DISTRIBUTION

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ABSTRACT

The purpose of this paper is to present the Quality Interval acceptance single sampling plan when the fraction of nonconforming items is a fuzzy number and being modeled based on the fuzzy Poisson distribution. A new procedure for implementing Fuzzy logic in Quality Interval acceptance sampling plan has also been carried out.

Keywords*: *Statistical quality control, acceptance single sampling plan, fuzzy number.

I. INTRODUCTION

Fuzzy logic is derived from fuzzy set theory dealing with reasoning that is approximate rather than precisely deduced from classical predicate logic. Fuzzy logic was introduced by Lotfi Zadeh(1965) at the University of California, Berkeley. Fuzzy set theory has been used to model systems that are hard to define precisely. As a methodology, fuzzy set theory incorporates imprecision and subjectivity into the model formulation and solution process. Fuzzy set theory represents an attractive tool to aid research in production management when the dynamics of the production environment limit the specification of model objectives, constraints and precise measurement of model parameters.

2. SURVEY ON APPLICATION OF FUZZY SET THEORY

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes. Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. In an effort to gain a better understanding of the use of fuzzy set theory in production management research and to provide a basis for future research, a literature review on fuzzy set theory in production management has been conducted. While similar survey efforts have been undertaken for other topical areas, there is a need in production management for the same. Over the years there have been successful applications and implementations of fuzzy set theory in production management. Fuzzy set theory is being recognized as an important problem modeling and solution technique. A summary of the findings of fuzzy set theory in production management research may benefit researchers in the production management field. Kaufmann and Gupta (1988) report that over 7,000 research papers, reports, monographs, and books on fuzzy set theory and applications have been published since 1965.

3. ACCEPTANCE SAMPLING AND FUZZY SET THEORY

Research on fuzzy quality management is broken down into three areas, acceptance sampling, statistical process control, and general quality management topics.

Ohta and Ichihashi (1988) present a fuzzy design methodology for single stage, two-point attribute sampling plans. An algorithm is presented and example sampling plans are generated when producer and consumer risk are defined by triangular fuzzy numbers. Further it is not addressed how to derive the membership functions for consumer and producer risk.

Chakraborty (1988, 1994a) examines the problem of determining the sample size and critical value of a single sample attribute sampling plan when imprecision exists in the declaration of producer and consumer risk. Earlier, a fuzzy goal programming model and solution procedure are described. Several numerical examples are provided and the sensitivity of the strength of the resulting sampling plans is evaluated. Earlier a paper details how possibility theory and triangular fuzzy numbers are used in the single sample plan design problem. Kanagawa and Ohta (1990) identify two limitations in the sample plan design procedure of Ohta and Ichihashi. First, Ohta and Ichihashi's design procedure does not explicitly minimize the sample size of the sampling plan. Second, the membership functions used, unrealistically model the consumer and producer risk. These deficiencies are corrected through the use of a nonlinear membership function and explicit incorporation of the sample size in fuzzy mathematical programming solution methodology. Chakraborty (1992, 1994b) addresses the problem of designing single stage, Dodge-Romig lot tolerance percent defective (LTPD) sampling plans when the lot tolerance percent defective, consumer's risk and incoming quality level are modeled using triangular fuzzy numbers. In the Dodge-Romig scheme, the design of an optimal

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LTPD sample plan involves solution to a nonlinear integer programming problem. The objective is to minimize average total inspection subject to a constraint based on the lot tolerance percent defective and the level of consumer risk. When fuzzy parameters are introduced, the procedure becomes a possibilistic (fuzzy) programming problem. A solution algorithm employing alpha-cuts is used to design a compromise LTPD plan, and a sensitivity analysis is conducted on the fuzzy parameters used.

4. QUALITY INTERVAL ACCEPTANCE SINGLE SAMPLING PLAN WITH FUZZY PARAMETER USING POISSON DISTRIBUTION

The purpose of this section is to present the Quality Interval Acceptance Single Sampling Plan when the fraction of nonconforming items is a fuzzy number and being modeled based on the fuzzy Poisson distribution.

Acceptance single sampling is one of the sampling methods for acceptance or rejection which is long with classical attribute quality characteristic. In different acceptance sampling plans the fraction of defective items, is considered as a crisp value, but in practice the fraction of defective items value must be know exactly. Many times these values are estimated or it is provided by experiment. The vagueness present in the value of p from personal judgment, experiment or estimation may be treated formally with the help of fuzzy set theory. As known, fuzzy set theory is powerful mathematical tool for modeling uncertain resulting. In this basis defining the imprecise proportion parameter is as a fuzzy number. With this definition, the number of nonconforming items in the sample has a binomial distribution with fuzzy parameter. However if fuzzy number p is small we can use the fuzzy poison distribution to approximate values of the fuzzy binomial. Classical acceptance sampling plans have been studied by many researchers. They are thoroughly elaborated by Schilling (1982). Single Sampling by attributes with relaxed requirements were discussed by Ohta and Ichihashi (1988) kanagawa and Ohta (1990), Tamaki, Kanagawa and Ohta (1991),and Grzegorzewski(1998,2001b). Grzegrozewski (2000b,2002) also considered sampling plan by variables with fuzzy requirements. Sampling plan by attributes for vague data were considered by Hrniewicz (1992, 1994). Ezzatallah Baloui Jamkhaneh et.al. (2009) have studied acceptance Single Sampling Plan with fuzzy parameter using Poisson distribution. Ezzatallah Baloui Jamkhaneh et.al. (2009) have also studied the preparation important criteria of Rectifying

Inspection for Single Sampling Plan with Fuzzy Paramater. Bahram Sadeghpour- Gildeh et. al.(2008) have studied acceptance Double Sampling Plan with Fuzzy Paramter.

Table 4.1 provides a summary of selected Bibilographies on Fuzzy set theory and applications.

Table 4.1 : Fuzzy Quality Management

Quality Area	Author(s)	Fuzzy Quality Application
Acceptance Sampling	Otha and Ichihashi (1988)	Single-stage, two-point Sampling attribute sampling plan
	Chakraborty (1988, 1994a)	Single sample, attribute sampling plan
	Kanagawa and Ohta (1990)	Extend work of Otha and Ichihashi (1988) to include nonlinear membership function
	Chakraborty (1992, 1994a)	Single-stage Dodge-Romig LTPD sampling plans

Section 4.1 provides some definition and preliminaries of fuzzy sets theory and fuzzy probability. In section 4.2 provides single sampling plan with fuzzy parameter. Section 4.3, deals with quality Interval single sampling plan with fuzzy parameter using Poison distribution with examples.

5. PRELIMINARIES AND DEFINITIONS

Parameter p (probability of a success in each experiment) of the crisp binomial distribution is known exactly, but sometimes we are not able to obtain exact some uncertainty in the value p and is to be estimated from a random sample or from expert opinion. The crisp poison distribution has one parameter, which we also assume is not known exactly.

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Definition 1: the fuzzy subset $\tilde{N}$ of real line IR, with the membership function $\mu_N : IR \rightarrow [0,1]$ is a fuzzy number if and only if (a) $\tilde{N}$ is normal (b) $\tilde{N}$ is fuzzy convex (c) μ_N is upper semi continuous (d) supp ($\tilde{N}$) is bounded.

Definition 2: A triangular fuzzy number $\tilde{N}$ is fuzzy number that membership function defined by three numbers $a_1 < a_2 < a_3$ where the base of the triangle is the interval $[a_1, a_3]$ and vertex is at $x = a_2$ [3].

Definition3: The α-cut of a fuzzy number $\tilde{N}$ is a non-fuzzy set defined as $N[\alpha] = \{x \in IR; \mu_N(x) \geq \alpha\}$. Hence $N[\alpha] = [N_\alpha^L, N_\alpha^U]$ where

$$N_\alpha^L = \inf\{x \in IR; \mu_N(x) \geq \alpha\}$$
$$N_\alpha^U = \sup\{x \in IR; \mu_N(x) \geq \alpha\}$$

Definition 4: Due to the uncertainty in the k_i 's values we substitute $\tilde{k}_i$, a fuzzy number, for each k_i and assume that 0< $\tilde{k}_i$ <1 all i . Then X together with the $\tilde{k}_i$ value is a discrete fuzzy probability distribution. We write $\tilde{P}$ for fuzzy P and we have $\tilde{P}(\{x_i\}) = \tilde{k}_i$. Let $A = \{x_1, x_2, ... x_l\}$ be subset of X. Then define:

$$\tilde{P}(A)[\alpha] = \left\{\sum_{i=1}^{l} k_i / s\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.1)$$

For $0 < \alpha < 1$, where stands for the statement "$k_i \in \tilde{k}_i[\alpha], 1 < i < n, \sum_{i=1}^{n} k_i = 1$" this is our restricted fuzzy arithmetic.

Definition 5: let x be a random variable having the Poisson mass function. If $P(x)$ stands for the probability that X= x, then

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \ldots\ldots\ldots\ldots\ldots.. (5.2)$$

for x=0,1,2,….. and parameter $\lambda > 0$.

Now substitute fuzzy number $\tilde{\lambda} > 0$ for λ to produce the fuzzy Poisson probability mass function. Let $P(x)$ to be the fuzzy probability that X= x. Then $\alpha-$cut of this fuzzy number as

$$\widetilde{P}(x)[\alpha] = \left\{ \frac{e^{-\lambda}\lambda^{x}}{x!} \middle| \lambda \in \lambda[\alpha] \right\} \qquad \text{……………(5.3)}$$

For all $\alpha \in [0,1]$.Let X be a random variable having the fuzzy binomial distribution and $\widetilde{p}$ in the definition 4 are small. That is all $p \in \widetilde{p}[\alpha]$ are sufficiently small. Then $\widetilde{P}[a,b][\alpha]$ using the fuzzy poison approximation.

Then $\widetilde{P}[a,b][\alpha] \approx \left\{ \sum_{x=a}^{b} \frac{e^{-\lambda}\lambda^{x}}{x!} \middle| \lambda \in n\widetilde{p}[\alpha] \right\}$

5.1. SINGLE SAMPLING PLAN WITH FUZZY PARAMETER

Suppose that one want to inspect a lot with the large size of N , such that the proportion of damaged items is not known precisely. So represent this parameter with a fuzzy number $\widetilde{p}$ as follows: $\widetilde{p} = (a_1, a_2, a_3),\ p \in \widetilde{p}[1],\ q \in \widetilde{q}[1],\ p + q = 1$.

A single sampling plan with a fuzzy parameter if defined by the sample size n , and acceptance number c , and if the number of observation defective product is less than or equal to c , the lot will be acceptance. If N is a large number, then the number of defective items in this sample (d) has a fuzzy binomial distribution, and if $\widetilde{p}$ is a small, then random variable d has a fuzzy Poisson distribution with parameter $\widetilde{\lambda} = n\widetilde{p}$ .So the fuzzy probability for the number of defective items in a sample size that is exactly equal to d is:

$$\widetilde{P}(d - defective)[\alpha] = [P^{L}[\alpha], P^{U}[\alpha]] \qquad \text{………….. (5.1.1)}$$

$$P^{L}[\alpha] = \min\left\{ \frac{e^{-\lambda}\lambda^{d}}{d!} \middle| \lambda \in n\widetilde{p}[\alpha] \right\} \quad , P^{U}[\alpha] = \max\left\{ \frac{e^{-\lambda}\lambda^{d}}{d!} \middle| \lambda \in n\widetilde{p}[\alpha] \right\}$$

and fuzzy acceptance probability is as follows:

$$\widetilde{P}_{a} = \left\{ \sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^{d}}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha] \right\} \qquad \text{…………… (5.1.2)}$$

$$= \left[P^{L}[\alpha], P^{U}[\alpha]\right]$$

$$P^{L}[\alpha] = \min\left\{ \sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^{d}}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha] \right\}, P^{U}[\alpha] = \max\left\{ \sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^{d}}{d!} \middle| \lambda \in \widetilde{\lambda}[\alpha] \right\}$$

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5.2. QUALITY INTERVAL SINGLE SAMPLING PLAN WITH FUZZY PARAMETER

In this section Fuzzy Quality Decision Region (FQDR) and Fuzzy Probabilistic Quality Region (FPQR) are defined as follows:

FUZZY QUALITY DECISION REGION (FQDR)

Fuzzy QDR is as follows :

$$\tilde{d}_1[\alpha] = [d_1^L[\alpha], d_1^U[\alpha]] \qquad \ldots\ldots\ldots\ldots\ldots (5.2.1)$$

$$d_1^L[\alpha] = \min\{(p_* - p_1) \mid \lambda \in n\tilde{p}[\alpha]\},\ d_1^U[\alpha] = \max\{(p_* - p_1) \mid \lambda \in n\tilde{p}[\alpha]\}$$

and Fuzzy QDR is derived from fuzzy probability of acceptance is as follows:

$$\tilde{P}(p_1 < p < p_*) = \left\{ \sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \middle| \lambda \in \tilde{\lambda}[\alpha] \right\} \quad for\ p_1 < p < p_*$$

$$= [P^L(\alpha), P^U(\alpha)]$$

Example:

A company fixes MAPD as 6 % and varying AQL from 1 to 5%. Therefore Fuzzy Quality Decision Region is described as follows:

$\tilde{d}_1$ values obtained are 0.05,0.04,0.03,0.02,0.01.

$$\tilde{d}_1[\alpha] = [d_1^L[\alpha], d_1^U[\alpha]]$$

$$d_1^L[\alpha] = \min\{(p_* - p_1) \mid \lambda \in n\tilde{p}[\alpha]\}$$

$$= 0.01$$

$$d_1^U[\alpha] = \max\{(p_* - p_1) \mid \lambda \in n\tilde{p}[\alpha]\}$$

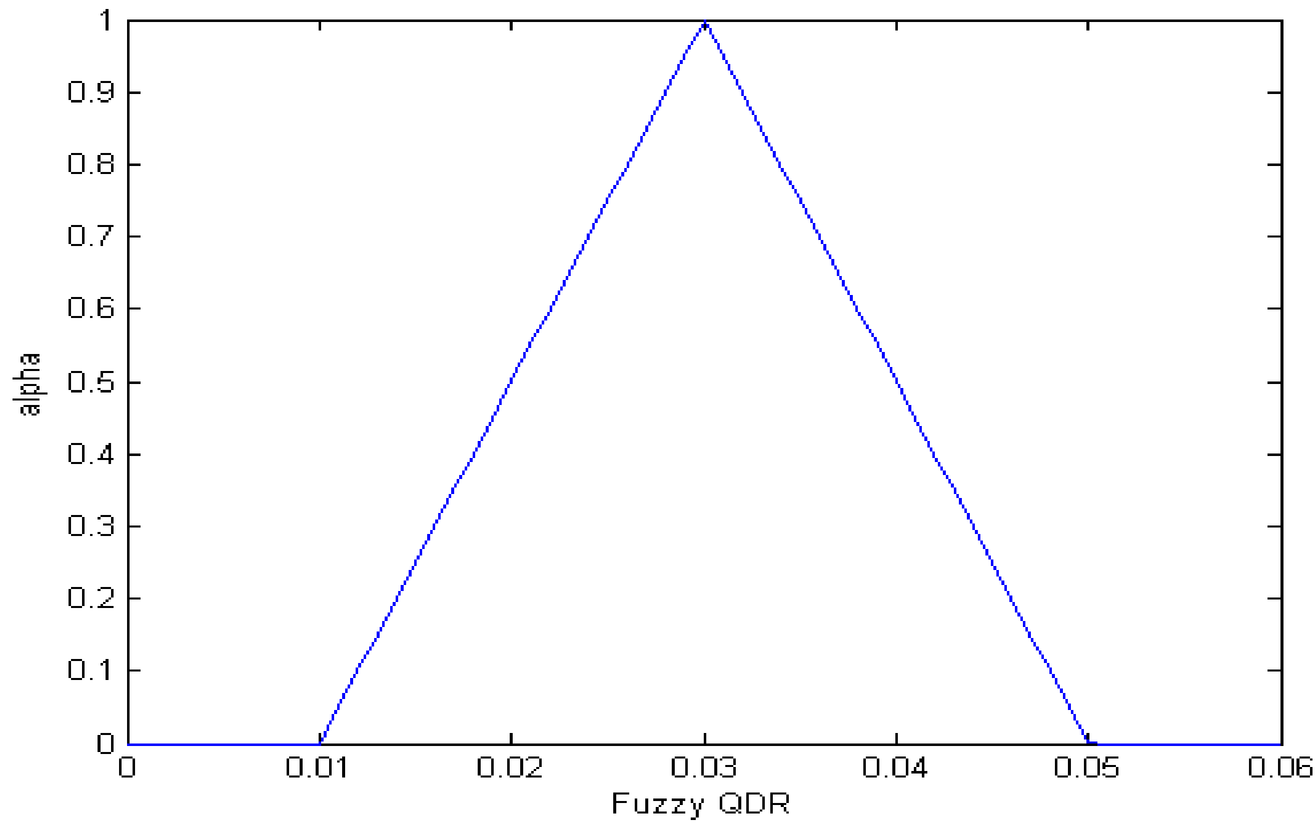
$$= 0.05$$

Figure 5.2.1 is drawn using the MATLAB Software and the program is described below.

Program for Fuzzy QDR:

```
>> x = (0: 0.001: 0.06);
>> y = trimf(x, [0.01, 0.03, 0.05]);
>> plot (x, y);
>> x label (‘Fuzzy QDR’);
>> y label  (‘alpha’);
```


Figure 5.2.1. Fuzzy Quality Decision Region (FQDR) with $\tilde{d}_1 = [0.01, 0.03, 0.05]$



FUZZY PROBABILISTIC QUALITY REGION (FPQR)

Fuzzy PQR is as follows :

$$\tilde{d}_2[\alpha] = [d_2^{\,L}[\alpha], d_2^{\,U}[\alpha]] \qquad \ldots\ldots\ldots\ldots\ldots\; (.5.2.2)$$

$$d_2^{\,L}[\alpha] = \min\{(p_2 - p_1) \mid \lambda \in n\tilde{p}[\alpha]\},\; d_2^{\,U}[\alpha] = \max\{(p_2 - p_1) \mid \lambda \in n\tilde{p}[\alpha]\}$$

and Fuzzy QDR is derived from fuzzy probability of acceptance is as follows:

$$\tilde{P}(p_1 < p < p_2) = \left\{ \sum_{d=0}^{c} \frac{e^{-\lambda}\lambda^d}{d!} \,\middle|\, \lambda \in \tilde{\lambda}[\alpha] \right\} \quad for\ p_1 < p < p_2$$
$$= [P^L(\alpha), P^U(\alpha)]$$

Example:

A company fixes LQL as 10% and varying AQL from 4 to 8%. Therefore Fuzzy Probabilistic Quality Region is described as follows:

$\tilde{d}_2$ values obtained are 0.06,0.05,0.04,0.03,0.02.

$$\tilde{d}_2[\alpha] = [d_2^{\,L}[\alpha], d_2^{\,U}[\alpha]]$$

$$d_2^{\,L}[\alpha] = \min \{(p_2 - p_1) \mid \lambda \in n\tilde{p}[\alpha]\}$$

$$= 0.02$$

$$d_2^{\,U}[\alpha] = \max \{(p_2 - p_1) \mid \lambda \in n\tilde{p}[\alpha]\}$$

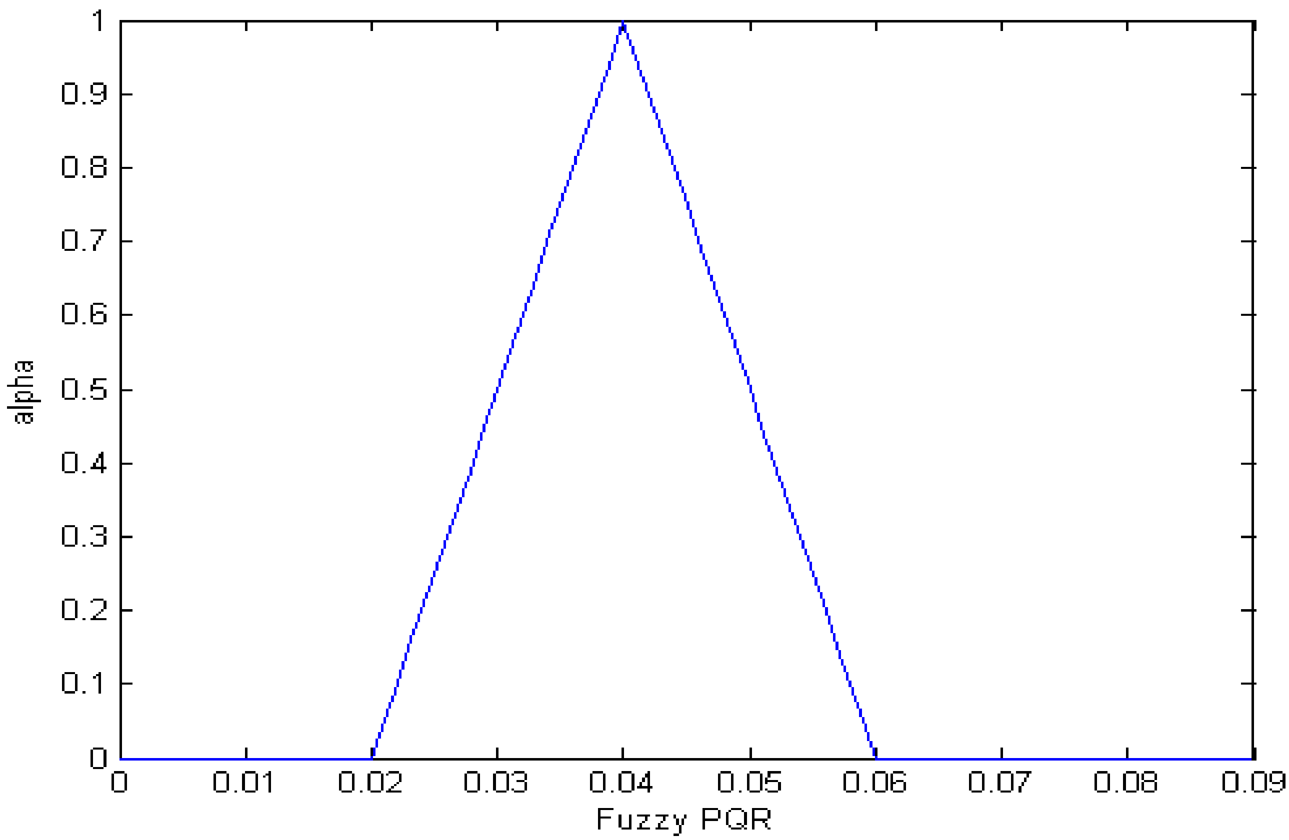
$$= 0.06$$

Figure .5.2.2 is drawn using the MATLAB Software and the program is described below.

Program for Fuzzy PQR:

```
>> x = (0: 0.001: 0.09);
>> y = trimf(x, [0.02, 0.04, 0.06]);
>> plot (x, y);
>> x label ('Fuzzy PQR');
>> y label  ('alpha');
```

Figure 5.2.2. Fuzzy Probabilistic Quality Region (FPQR) with $\tilde{d}_2 = [0.02, 0.04, 0.06]$



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