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The Analysis of The Random System of Polar Move Base on Wavelet

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Abstract: In this paper,we study the energy of random system of Polar Move base on wavelet method,we obtain some statistics properties and density of the system use wavelet transform.

Keywords: Polar; random system; wavelet; density;energy

1 Introduction

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing.In fields ranging from Extragalactic Astronomy to Molecular Spectroscopy to Medical Imaging to computer vision,One must recover a signal,curve,image,spectrum,or density from incomplete,indirect,and noisy data .Wavelets have contributed to this already intensely developed and rapidly advancing field .

Wavelet analysis consists of a versatile collection of tools for the analysis and manipulation of signals such as sound and images as well as more general digital data sets ,such as speech,electrocardiograms ,images .Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing .With hindsight the wavelet transform can be viewed as diverse as mathematics ,physics and electrical engineering .The basic idea is always to use a family of building blocks to represent the object at hand in an efficient and insightful way,the building blocks themselves come in different sizes,and are suitable for describing features with a resolution commensurate with their size .

There are two important aspects to wavelets,which we shall call “mathematical” and “algorithmical” .Numerical algorithms using wavelet bases are similar to other transform methods in that vectors and operators are expanded into a basis and the computations take place in the new system of coordinates .As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain,wavelet based algorithms exhibit a number of new and important properties .Recently some persons have studied wavelet problems of stochastic process or stochastic system.

In paper[1],the random excited process is study for polars' role.Let (ξ_t, η_t)express polars' the Coordinate of (ξ, η) in time tthen polar move equation:

$$\begin{cases} d\xi = -\lambda\xi dt - \omega\eta dt + bd\varphi \\ d\eta = \omega\xi dt - \lambda\eta dt + bd\psi \end{cases} \tag{1}$$

There φ,ψ are random function of time t,$b > 0$ is content.ω is angular velocity,λ is damping. Soppose φ_t,$\psi_t$are two Browain motion on independent each other,and:

$$Ed\varphi = Ed\psi = 0; E(d\varphi)^2 = E(d\psi)^2 = dt \tag{2}$$

Use[1],we knowthe solution of Eq.(1):

$$\xi_t = e^{-\lambda(t-a)}[C_1 \cos\omega(t-a)] - C_2 \sin\omega(t-a) + b\int_a^t e^{-\lambda(t-s)}[\cos\omega(t-s)d\varphi_s - \sin\omega(t-s)d\psi_s]$$

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$$\eta_t = e^{-\lambda(t-a)}[C_1 \sin\omega(t-a)] + C_2\cos\omega(t-a)] + b\int_a^t e^{-\lambda(t-s)}[\sin\omega(t-s)d\varphi_s - \cos\omega(t-s)d\psi_s] \quad (3)$$

We may change (1) into

$$\begin{cases} \dot{\xi} = -\lambda\dot{\xi} - \omega\eta + b\dot{\varphi} \\ \dot{\eta} = \omega\xi - \lambda\eta + b\dot{\psi} \end{cases}$$

We know

$$E\xi_t = e^{-\lambda t}(x_1\cos\omega t - x_2 \sin\omega t) \quad (4)$$

$$E\eta_t = e^{-\lambda t}(x_1\sin\omega t + x_2 \cos\omega t) \quad (5)$$

where x_1, x_2 are constant.

2 Wavelet transform

Recently some persons have studied wavelet problems of stochastic process or stochastic system [2-11].

Definition 1 *Let x_t is a stochastics processesthen its wavelet transform is*

$$w(s,x) = \frac{1}{s}\int_R x(t)\psi(\frac{x-t}{s})dt \quad (6)$$

where, ψ is continue wavelet.

Definition 2 *Let $\psi(x)$is*

$$\psi(x) = \begin{cases} 1, 0 \le x < \frac{1}{2} \\ -1, \frac{1}{2} \le x < 1 \\ 0, other \end{cases} \quad (7)$$

we call it be Haar wavelet.

Use Eq.(6,7),we have

$$w(s,x) = \frac{1}{s}[\int_{x-\frac{s}{2}}^{x} x(t)dt - \int_{x-s}^{x-\frac{s}{2}} x(t)dt] \quad (8)$$

3 Wavelet density of polar move

The Wavelet density of polar move express the energy of polar move.
For ξ_t,use Eq.(8) have

$$w(s,\xi) = \frac{1}{s}[\int_{x-\frac{s}{2}}^{x} \xi_t dt - \int_{x-s}^{x-\frac{s}{2}} \xi_t dt]$$

Then

$$\begin{aligned} Ew(s,\xi) &= \frac{1}{s}[\int_{x-\frac{s}{2}}^{x} E\xi_t dt - \int_{x-\frac{s}{2}}^{x-\frac{s}{2}} E\xi_t dt] \\ &= \frac{1}{s}[\int_{x-\frac{s}{2}}^{x} e^{-\lambda t}(x_1\cos\omega t - x_2\sin\omega t)dt - \int_{x-s}^{x-\frac{s}{2}} e^{-\lambda t}(x_1\cos\omega t - x_2\sin\omega t)dt] \\ &= \frac{1}{s}[x_1\frac{\omega\sin\omega x - \lambda\cos\omega x}{\lambda^2+\omega^2} - x_1\frac{\omega\sin\omega(x-\frac{s}{2}) - \lambda\cos\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} \\ &- x_2\frac{-\lambda\sin\omega x - \omega\cos\omega x}{\lambda^2+\omega^2} + x_2\frac{-\lambda\sin\omega(x-\frac{s}{2}) - \omega\cos\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} \\ &- x_1\frac{\omega\sin\omega(x-\frac{s}{2}) - \lambda\cos\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} + x_1\frac{\omega\sin\omega(x-s) - \lambda\cos\omega(x-s)}{\lambda^2+\omega^2} \\ &+ x_2\frac{-\lambda\sin\omega(x-\frac{s}{2}) - \omega\cos\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} - x_2\frac{-\lambda\sin\omega(x-s) - \omega\cos\omega(x-s)}{\lambda^2+\omega^2}] \end{aligned}$$

$$
\begin{aligned}
Ew(s,\eta) &= \frac{1}{s}[\int_{x-\frac{s}{2}}^{x} E\eta_t dt - \int_{x-s}^{x-\frac{s}{2}} E\eta_t dt] \\
&= \frac{1}{s}[\int_{x-\frac{s}{2}}^{x} e^{-\lambda t}(x_1 \sin\omega t + x_2 \cos\omega t)dt - \int_{x-s}^{x-\frac{s}{2}} e^{-\lambda t}(x_1 \sin\omega t + x_2 \cos\omega t)dt] \\
&= \frac{1}{s}[x_1 \frac{\omega\cos\omega x - \lambda \sin\omega x}{\lambda^2+\omega^2} - x_1 \frac{\omega\cos\omega(x-\frac{s}{2}) - \lambda\sin\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} \\
&- x_2 \frac{-\lambda\cos\omega x - \omega\sin\omega x}{\lambda^2+\omega^2} + x_2 \frac{-\lambda\cos\omega(x-\frac{s}{2}) - \omega\sin\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} \\
&- x_1 \frac{\omega\cos\omega(x-\frac{s}{2}) - \lambda\sin\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} + x_1 \frac{\omega\cos\omega(x-s) - \lambda\sin\omega(x-s)}{\lambda^2+\omega^2} \\
&+ x_2 \frac{-\lambda\cos\omega(x-\frac{s}{2}) - \omega\sin\omega(x-\frac{s}{2})}{\lambda^2+\omega^2} - x_2 \frac{-\lambda\cos\omega(x-s) - \omega\sin\omega(x-s)}{\lambda^2+\omega^2}]
\end{aligned}
$$

Suppose Rτ=E$ws,\xi+\tau)w(s,\xi)$ then

$$
\begin{aligned}
R(\tau) &= E\left\{\frac{1}{s}[\int_{x-\frac{s}{2}}^{x} \xi_{t+\tau}dt - \int_{x-s}^{x-\frac{s}{2}} \xi_{t+\tau}dt] \cdot \frac{1}{s}[\int_{x-\frac{s}{2}}^{x} \xi_t dt - \int_{x-s}^{x-\frac{s}{2}} \xi_t dt]\right\} \\
&= \frac{1}{s^2}\left\{[\int_{x-\frac{s}{2}}^{x}\int_{x--\frac{s}{2}}^{x} E(\xi_t\xi_{u+\tau})dudt - \int_{x-\frac{s}{2}}^{x}\int_{x-s}^{x-\frac{s}{2}} E(\xi_t\xi_{u+\tau})dudt - \int_{x-s}^{x-\frac{s}{2}}\int_{x-\frac{s}{2}}^{x} E(\xi_{u+\tau}\xi_t)dudt \right. \\
&\left. + \int_{x-s}^{x-\frac{s}{2}}\int_{x-S}^{x-\frac{s}{2}} E(\xi_{u+\tau}\xi_t)dudt]\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
E(\xi_t\xi_{u+\tau}) &= e^{-\lambda t}(x_1\cos\omega t - x_2\sin\omega t)e^{-\lambda(u+\tau)}(x_1\cos\omega(u+\tau) - x_2\sin\omega(u+\tau) \\
&= e^{-\lambda(t+u+\tau)}(x_1\cos\omega t - x_2\sin\omega t)(x_1\cos\omega(u+\tau) - x_2\sin\omega(u+\tau)
\end{aligned}
$$

Then we can obtain density

$$
\bar{d}s = \sqrt{\left|\frac{R^{(4)}(0)}{\pi^2 R^{(2)}(0)}\right|}
$$

Use the same method ,we can obtain the energy of η_t,then we obtain the energy of (ξ_t,η_t).

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