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Estimation of Population Mean Using Known Median and Co-Efficent of Skewness

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Abstract The present paper deals with two modified ratio estimators for estimation of population mean of the study variable using the linear combination of the known population values of the Median and the Co-efficient of Skewness of the auxiliary variable. The biases and the mean squared errors of the proposed estimators are derived and are compared with that of existing modified ratio estimators for certain natural populations. Further we have also derived the conditions for which the proposed estimators perform better than the existing modified ratio estimators. From the empirical study it is also observed that the proposed modified ratio estimators perform better than the existing modified ratio estimators.

Keywords Bias, Mean Squared Error, Modified Ratio Estimators, Simple Random Sampling

1. Introduction

The sampling theory describes a wide variety of techniques for using auxiliary information to obtain more efficient estimators like Ratio, Product and Regression estimators for the estimation of the mean of the study variable Y. Ratio estimators, improves the precision of estimate of the population mean of a study variable by using prior information on auxiliary variable X which is positively correlated with the study variable Y. Over the years the ratio method of estimation has been extensively used because of its intuitive appeal and the computational simplicity. When the population parameters of the auxiliary variable X such as Population Mean, Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, Median are known, a number of modified estimators such as modified ratio estimators, modified product estimators and modified linear regression estimators are proposed in the literature. Before discussing further about the modified ratio estimators and the proposed modified ratio estimators the notations to be used in this paper are described below:

- $N -$ Population size
- $n -$ Sample size
- $f = n/N$, Sampling fraction
- $Y -$ Study variable
- $X -$ Auxiliary variable
- $\bar{X}, \bar{Y} -$ Population means
- $\bar{x}, \bar{y} -$ Sample means
- $S_X, S_y -$ Population standard deviations
- $C_X, C_y -$ Co-efficient of variations
- $\rho -$ Co-efficient of correlation
- $\beta_1 = \frac{N \sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$, Co-efficient of skewness of the auxiliary variable
- $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Co-efficient of kurtosis of the auxiliary variable
- $M_d -$ Median of the auxiliary variable
- $B(.) -$ Bias of the estimator
- $MSE(.) -$ Mean squared error of the estimator
- $\hat{\bar{Y}}_i (\hat{\bar{Y}}_{pi}) -$ Existing (proposed) modified ratio estimator of $\bar{Y}$

The Ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y is defined as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \text{ where } \hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x} \text{ is the estimate of } R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X} \quad (1)$$

The lists of modified ratio estimators together with their biases, mean squared errors and constants available in the literature are classified into two classes namely Class 1, Class 2 and are given respectively in Table 1 and Table 2 in the Appendix.

It is to be noted that "the existing modified ratio estimators" means the list of modified ratio estimators to be considered in this paper unless otherwise stated. It does not mean to the entire list of modified ratio estimators available in the literature. For a more detailed discussion on the ratio estimator and its modifications one may refer to Cochran[1], Kadilar and Cingi[2, 3], Koyuncu and Kadilar[4], Murthy[5], Prasad[6], Rao[7], Singh[9], Singh and Tailor[10,12], Singh et.al[11], Sisodia and Dwivedi[13], Subramani and Kumarapandiyan[14,15], Upadhyaya and Singh[16], Yan and Tian[17] and the references cited there in.

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The modified ratio estimators given in Table 1 and Table 2 are biased but have minimum mean squared errors compared to the classical ratio estimator. The list of estimators given in Table 1 and Table 2 uses the known values of the parameters like $\bar{X}$, C_x, β_1, β_2, ρ, M_d and their linear combinations. Recently Yan and Tian[17] have used Coefficient of Skewness for the estimation of population meanHowever, it seems, no attempt is made to use the linear combination of known values of the Median and Co-efficient of Skewness of the auxiliary variable to improve the ratio estimator. The points discussed above have motivated us to introduce two modified ratio estimators using the linear combination of the known values of Median and Co-efficient of Skewness of the auxiliary variable. When the population median and coefficient of skewness are unknown the proposed estimators can be modified using their respective estimates i.e sample median, and sample coefficient of skewness obtained from the sample. The proposed estimators can be applicable in the following practical situation.

1. A national park is partitioned into N units.
   - Y = the number of animals in the i^{th} unit
   - X = the size of the i^{th} unit
2. A certain city has N bookstores.
   - Y = the sales of a given book title at the i^{th} bookstore
   - X = the size of the i^{th} bookstore
3. A forest that has N trees.
   - Y = the volume of the tree
   - X = the diameter of the tree

2. Proposed Modified Ratio Estimators

In this section, we have suggested two modified ratio estimators using the linear combination of Median and Co-efficient of Skewness of the auxiliary variable. The proposed modified ratio estimators for estimating the population mean $\bar{Y}$ together with the first degree of approximation, the biases and mean squared errors and the constants are given below:

$$\hat{\bar{Y}}_{p1} = \bar{y}\left[\frac{\bar{X}\beta_1 + M_d}{\bar{x}\beta_1 + M_d}\right]$$

$$B\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}\,(\theta_{p1}^2 C_x^2 - \theta_{p1} C_x C_y\,\rho)$$

$$MSE\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\,\bar{Y}^2\left(C_y^2 + \theta_{p1}^2 C_x^2 - 2\theta_{p1} C_x C_y\,\rho\right)$$

$$\text{where } \theta_{p1} = \frac{\bar{X}\beta_1}{\bar{X}\beta_1 + M_d} \qquad (2)$$

$$\hat{\bar{Y}}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + M_d)}(\bar{X}\beta_1 + M_d)$$

$$B\left(\hat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2$$

$$MSE\left(\hat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\left(R_{p2}^2 S_x^2 + S_y^2(1-\rho^2)\right)$$

$$\text{where } R_{p2} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + M_d} \qquad (3)$$

3. Efficiency Comparison

For want of space; for the sake of convenience to the readers and for the ease of comparisons, the modified ratio estimators given in Table 1, Table 2 are represented into two classes as given below. Further it is to be noted that the proposed estimator $\hat{\bar{Y}}_{p1}$ is compared with the modified ratio estimators listed in Class 1 whereas the proposed estimator $\hat{\bar{Y}}_{p2}$ is compared with the modified ratio estimators listed in Class 2.

Class 1: The biases, the mean squared errors and the constants of the modified ratio type estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_9$ listed in the Table 1 are represented in a single class (say, Class 1), which will be very much useful for comparing with that of proposed modified ratio estimators and are given below:

$$B\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\bar{Y}\,(\theta_i^2 C_x^2 - \theta_i C_x C_y\,\rho)$$

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i C_x C_y\,\rho\right); \quad i = 1, 2, 3, \ldots, 9 \qquad (4)$$

where

$$\theta_1 = \frac{\bar{X}}{\bar{X} + C_x}, \theta_2 = \frac{\bar{X}}{\bar{X} + \beta_2},$$

$$\theta_3 = \frac{\bar{X}}{\bar{X} + \beta_1}, \theta_4 = \frac{\bar{X}}{\bar{X} + \rho},$$

$$\theta_5 = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_2}, \theta_6 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + C_x},$$

$$\theta_7 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1 + \beta_2},\ \theta_8 = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_1} \text{ and } \theta_9 = \frac{\bar{X}}{\bar{X} + M_d}$$

Class 2: The biases, the mean squared errors and the constants of the 11 modified ratio estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_{11}$ listed in the Table 2 are represented in a single class (say, Class 2), which will be very much useful for comparing with that of proposed modified ratio estimators and are given below:

$$B\left(\hat{\bar{Y}}_j\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_j^2$$

$$MSE\left(\hat{\bar{Y}}_j\right) = \frac{(1-f)}{n}\left(R_j^2 S_x^2 + S_y^2(1-\rho^2)\right); j = 1, 2, 3, \ldots, 11 \qquad (5)$$

$$\text{where } R_1 = \frac{\bar{Y}}{\bar{X}}, R_2 = \frac{\bar{Y}}{\bar{X} + C_x},$$

$$R_3 = \frac{\bar{Y}}{\bar{X} + \beta_2}, R_4 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + C_x},$$

$$R_5 = \frac{\bar{Y}C_x}{\bar{X}C_x + \beta_2}, R_6 = \frac{\bar{Y}}{\bar{X} + \beta_1}, R_7 = \frac{\bar{Y}}{\bar{X} + \rho}, \quad R_8 = \frac{\bar{Y}C_x}{\bar{X}C_x + \rho},$$

$$R_9 = \frac{\bar{Y}\rho}{\bar{X}\rho + C_x}, R_{10} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho} \text{ and } R_{11} = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2}$$

As derived earlier in section 2, the biases, the mean squared errors and the constants of two proposed modified ratio estimators are given below:

$$B\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}\,(\theta_{p1}^2 C_x^2 - \theta_{p1} C_x C_y\,\rho)$$

$$MSE\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{p1}^2 C_x^2 - 2\theta_{p1} C_x C_y\,\rho\right)$$

$$\text{where } \theta_{p1} = \frac{\bar{X}\beta_1}{\bar{X}\beta_1 + M_d} \qquad (6)$$

$$B\left(\hat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2$$

$$MSE\left(\hat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\left(R_{p2}^2 S_x^2 + S_y^2(1-\rho^2)\right)$$
$$\text{where } R_{p2} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + M_d} \quad (7)$$

From the expressions given in (4) and (6) we have derived the conditions for which the proposed estimator $\hat{\bar{Y}}_{p1}$ is more efficient than the existing modified ratio estimators given in Class 1, $\hat{\bar{Y}}_i$; i = 1, 2, 3, . . . , 9 and are given below.

$$MSE\left(\hat{\bar{Y}}_{p1}\right) < MSE\left(\hat{\bar{Y}}_i\right) \text{ if } \rho < \frac{(\theta_{p1}+\theta_i)}{2}\frac{C_x}{C_y} \; ; i = 1, 2, 3, \ldots, 9 \quad (8)$$

From the expressions given in (5) and (7) we have derived the conditions for which the proposed estimator $\hat{\bar{Y}}_{p2}$ is more efficient than the existing modified ratio estimators given in Class 2, $\hat{\bar{Y}}_j$; j = 1, 2, 3, . . . , 11 and are given below:

$$MSE\left(\hat{\bar{Y}}_{p2}\right) < MSE\left(\hat{\bar{Y}}_j\right) \text{ if } R_{p2} < R_j \; ; j = 1, 2, 3, \ldots, 11 \quad (9)$$

4. Numerical Study

Table 3. Parameters and Constants of the Populations

Parameters	Population 1	Population 2
N	34	80
n	20	20
$\bar{Y}$	856.4117	51.8264
$\bar{X}$	208.8823	2.8513
ρ	0.4491	0.9150
S_y	733.1407	18.3569
C_y	0.8561	0.3542
S_x	150.5059	2.7042
C_x	0.7205	0.9484
β_2	0.0978	1.3005
β_1	0.9782	0.6978
M_d	150.0000	1.4800

Table 4. The constants of the (Class 1) existing and proposed modified ratio estimators

Estimator	Constants θ_i	
	Population 1	Population 2
$\hat{\bar{Y}}_1$ Sisodia and Dwivedi[13]	0.9966	0.7504
$\hat{\bar{Y}}_2$ Singh et.al[11]	0.9995	0.6868
$\hat{\bar{Y}}_3$ Yan and Tian[17]	0.9953	0.8034
$\hat{\bar{Y}}_4$ Singh and Tailor[10]	0.9979	0.7571
$\hat{\bar{Y}}_5$ Upadhyaya and Singh[16]	0.9994	0.6753
$\hat{\bar{Y}}_6$ Upadhyaya and Singh[16]	0.9658	0.7963
$\hat{\bar{Y}}_7$ Yan and Tian[17]	0.9542	0.8416
$\hat{\bar{Y}}_8$ Yan and Tian[17]	0.9935	0.7949
$\hat{\bar{Y}}_9$ Subramani and Kumarapandiyan[15]	0.5820	0.6583
$\hat{\bar{Y}}_{p1}$ (Proposed estimator)*	0.5767*	0.5734*

The performances of the proposed modified ratio estimators are assessed with that of existing modified ratio estimators listed in Table 1 and Table 2 for certain natural populations. In this connection, we have considered two natural populations for the assessment of the performances of the proposed modified ratio estimators with that of existing modified ratio estimators. The population 1 is taken from Singh and Chaudhary[8] given in page 177 and population 2 is taken from Murthy[5] given in page 228. The population parameters and the constants computed from the above populations are given below:

The constants of the existing and proposed modified ratio estimators for the above populations are given in the Table 4 and Table 5:

Table 5. The constants of the (Class 2) existing and proposed modified ratio estimators

Estimator	Constants $\mathbf{R_j}$	
	Population 1	Population 2
$\hat{\bar{Y}}_1$ Kadilar and Cingi[2]	4.1000	18.1764
$\hat{\bar{Y}}_2$ Kadilar and Cingi[2]	4.0859	13.6396
$\hat{\bar{Y}}_3$ Kadilar and Cingi[2]	4.0981	12.4829
$\hat{\bar{Y}}_4$ Kadilar and Cingi[2]	3.9598	14.4744
$\hat{\bar{Y}}_5$ Kadilar and Cingi[2]	4.0973	12.2737
$\hat{\bar{Y}}_6$ Yan and Tian[17]	4.0809	14.6027
$\hat{\bar{Y}}_7$ Kadilar and Cingi[3]	4.0912	13.7606
$\hat{\bar{Y}}_8$ Kadilar and Cingi[3]	4.0878	13.5810
$\hat{\bar{Y}}_9$ Kadilar and Cingi[3]	4.0687	13.3305
$\hat{\bar{Y}}_{10}$ Kadilar and Cingi[3]	4.0115	14.5790
$\hat{\bar{Y}}_{11}$ Kadilar and Cingi[3]	4.0957	12.1299
$\hat{\bar{Y}}_{p2}$ (Proposed estimator)*	2.3643*	10.4231*

The biases of the existing and proposed modified ratio estimators for the above populations are given in the Table 6 and Table 7:

Table 6. The biases of the (Class 1) existing and proposed modified ratio estimators

Estimator	Bias **B**(.)	
	Population 1	Population 2
$\hat{\bar{Y}}_1$ Sisodia and Dwivedi[13]	4.2233	0.5361
$\hat{\bar{Y}}_2$ Singh et.al[11]	4.2631	0.4142
$\hat{\bar{Y}}_3$ Yan and Tian[17]	4.2070	0.6484
$\hat{\bar{Y}}_4$ Singh and Tailor[10]	4.2406	0.5497
$\hat{\bar{Y}}_5$ Upadhyaya and Singh[16]	4.2607	0.3937
$\hat{\bar{Y}}_6$ Upadhyaya and Singh[16]	3.8212	0.6328
$\hat{\bar{Y}}_7$ Yan and Tian[17]	3.6732	0.7355
$\hat{\bar{Y}}_8$ Yan and Tian[17]	4.1831	0.6297
$\hat{\bar{Y}}_9$ Subramani and Kumarapandiyan[15]	0.2581	0.3643
$\hat{\bar{Y}}_{p1}$ (Proposed estimator)*	0.2273*	0.2323*

Table 7. The biases of the (Class 2) existing and proposed modified ratio estimators

Estimator	Bias $\mathbf{B}(.)$	
	Population 1	Population 2
$\hat{\bar{Y}}_1$ Kadilar and Cingi[2]	9.1539	1.7481
$\hat{\bar{Y}}_2$ Kadilar and Cingi[2]	9.0911	0.9844
$\hat{\bar{Y}}_3$ Kadilar and Cingi[2]	9.1454	0.8245
$\hat{\bar{Y}}_4$ Kadilar and Cingi[2]	8.5387	1.1086
$\hat{\bar{Y}}_5$ Kadilar and Cingi[2]	9.1420	0.7971
$\hat{\bar{Y}}_6$ Yan and Tian[17]	9.0688	1.1283
$\hat{\bar{Y}}_7$ Kadilar and Cingi[3]	9.1147	1.0019
$\hat{\bar{Y}}_8$ Kadilar and Cingi[3]	9.0995	0.9759
$\hat{\bar{Y}}_9$ Kadilar and Cingi[3]	9.0149	0.9403
$\hat{\bar{Y}}_{10}$ Kadilar and Cingi[3]	8.7630	1.1246
$\hat{\bar{Y}}_{11}$ Kadilar and Cingi[3]	9.1349	0.7785
$\hat{\bar{Y}}_{p2}$ (Proposed estimator)*	5.2731*	0.6691*

The mean squared errors of the existing and proposed modified ratio estimators for the above populations are given in the Table 8 and Table 9:

Table 8. The mean squared errors of the (Class 1) existing and proposed modified ratio estimators

Estimator	Mean Squared Error $\mathbf{MSE}(.)$	
	Population 1	Population 2
$\hat{\bar{Y}}_1$ Sisodia and Dwivedi[13]	10514.2250	17.1881
$\hat{\bar{Y}}_2$ Singh et.al[11]	10535.8620	12.8426
$\hat{\bar{Y}}_3$ Yan and Tian[17]	10505.3563	21.3660
$\hat{\bar{Y}}_4$ Singh and Tailor[10]	10523.6171	17.6849
$\hat{\bar{Y}}_5$ Upadhyaya and Singh[16]	10534.5417	12.1351
$\hat{\bar{Y}}_6$ Upadhyaya and Singh[16]	10298.4432	20.7801
$\hat{\bar{Y}}_7$ Yan and Tian[17]	10220.4736	24.6969
$\hat{\bar{Y}}_8$ Yan and Tian[17]	10492.3779	20.6613
$\hat{\bar{Y}}_9$ Subramani and Kumarapandiyan[15]	8852.3417	11.1366
$\hat{\bar{Y}}_{p1}$ (Proposed estimator)*	8848.4821*	6.9213*

From the values of Table 6 and Table 7, it is observed that the bias of the proposed modified ratio estimator $\hat{\bar{Y}}_{p1}$ is less than the biases of the existing modified ratio estimators $\hat{\bar{Y}}_i\,; i = 1, 2, 3, \ldots, 9$ given in Class 1 and the bias of the proposed modified ratio estimator $\hat{\bar{Y}}_{p2}$ is less than the biases of the existing modified ratio estimators $\hat{\bar{Y}}_j\,; j = 1, 2, 3, \ldots, 11$ given in Class 2. Similarly from the values of Table 8 and Table 9, it is observed that the mean squared error of the proposed modified ratio estimator $\hat{\bar{Y}}_{p1}$ is less than the mean squared errors of the existing modified ratio estimators $\hat{\bar{Y}}_i\,; i = 1, 2, 3, \ldots, 9$ given in Class 1 and the mean squared error of the proposed modified ratio estimator $\hat{\bar{Y}}_{p2}$ is less than the mean squared errors of the existing modified ratio estimators $\hat{\bar{Y}}_j\,; j = 1, 2, 3, \ldots, 11$ given in Class 2.

Table 9. The mean squared errors of the (Class 2) existing and proposed modified ratio estimators

Estimator	Mean Squared Error $\mathbf{MSE}(.)$	
	Population 1	Population 2
$\hat{\bar{Y}}_1$ Kadilar and Cingi[2]	16673.4489	92.6563
$\hat{\bar{Y}}_2$ Kadilar and Cingi[2]	16619.6435	53.0736
$\hat{\bar{Y}}_3$ Kadilar and Cingi[2]	16666.1389	44.7874
$\hat{\bar{Y}}_4$ Kadilar and Cingi[2]	16146.6142	59.5095
$\hat{\bar{Y}}_5$ Kadilar and Cingi[2]	16663.3064	43.3674
$\hat{\bar{Y}}_6$ Yan and Tian[17]	16600.5393	60.5325
$\hat{\bar{Y}}_7$ Kadilar and Cingi[3]	16639.8457	53.9825
$\hat{\bar{Y}}_8$ Kadilar and Cingi[3]	16626.8702	52.6365
$\hat{\bar{Y}}_9$ Kadilar and Cingi[3]	16554.4002	50.7876
$\hat{\bar{Y}}_{10}$ Kadilar and Cingi[3]	16338.6465	60.3426
$\hat{\bar{Y}}_{11}$ Kadilar and Cingi[3]	16657.1867	42.4051
$\hat{\bar{Y}}_{p2}$ (Proposed estimator)*	11440.8220*	31.8493*

5. Conclusions

In this paper we have proposed two modified ratio estimators using linear combination of Median and Co-efficient of Skewness of the auxiliary variable. The biases and mean squared errors of the proposed estimators are obtained and compared with that of existing modified ratio estimators. Further we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators. We have also assessed the performances of the proposed estimators for some known populations and observed that the biases and mean squared errors of the proposed estimators are less than the biases and mean squared errors of the existing modified ratio estimators.

Hence we strongly recommend the proposed modified estimators over the existing modified ratio estimators for the use of practical applications.

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APPENDIX

Table 1. Existing modified ratio estimators (Class 1) with their biases, mean squared errors and their constants

Estimator	Bias - $\mathbf{B}(.)$	Mean squared error $\mathbf{MSE}(.)$	Constant $\boldsymbol{\theta}_\mathbf{i}$
$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X} + C_x}{\bar{x} + C_x}\right]$ Sisodia and Dwivedi[13]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho)$	$\theta_1 = \frac{\bar{X}}{\bar{X} + C_x}$
$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right]$ Singh et.al[11]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho)$	$\theta_2 = \frac{\bar{X}}{\bar{X} + \beta_2}$
$\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right]$ Yan and Tian[15]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_3^2 C_x^2 - \theta_3 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho)$	$\theta_3 = \frac{\bar{X}}{\bar{X} + \beta_1}$
$\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X} + \rho}{\bar{x} + \rho}\right]$ Singh and Tailor[10]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_4^2 C_x^2 - \theta_4 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho)$	$\theta_4 = \frac{\bar{X}}{\bar{X} + \rho}$
$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{\bar{X} C_x + \beta_2}{\bar{x} C_x + \beta_2}\right]$ Upadhyaya and Singh[16]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_5^2 C_x^2 - \theta_5 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_x C_y \rho)$	$\theta_5 = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_2}$
$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\bar{X} \beta_2 + C_x}{\bar{x} \beta_2 + C_x}\right]$ Upadhyaya and Singh[16]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_6^2 C_x^2 - \theta_6 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_x C_y \rho)$	$\theta_6 = \frac{\bar{X} \beta_2}{\bar{X} \beta_2 + C_x}$
$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{\bar{X} \beta_1 + \beta_2}{\bar{x} \beta_1 + \beta_2}\right]$ Yan and Tian[17]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_7^2 C_x^2 - \theta_8 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_7^2 C_x^2 - 2\theta_8 C_x C_y \rho)$	$\theta_7 = \frac{\bar{X} \beta_1}{\bar{X} \beta_1 + \beta_2}$
$\hat{\bar{Y}}_8 = \bar{y}\left[\frac{\bar{X} C_x + \beta_1}{\bar{x} C_x + \beta_1}\right]$ Yan and Tian[17]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_8^2 C_x^2 - \theta_8 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_8^2 C_x^2 - 2\theta_8 C_x C_y \rho)$	$\theta_8 = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_1}$
$\hat{\bar{Y}}_9 = \bar{y}\left[\frac{\bar{X} + M_d}{\bar{x} + M_d}\right]$ Subramani and Kumarapandiyan[15]	$\frac{(1-f)}{n}\,\bar{Y}\,(\theta_9^2 C_x^2 - \theta_9 C_x C_y \rho)$	$\frac{(1-f)}{n}\,\bar{Y}^2 (C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 C_x C_y \rho)$	$\theta_9 = \frac{\bar{X}}{\bar{X} + M_d}$

Table 2. Existing modified ratio estimators (Class 2) with their biases, mean squared errors and their constants

Estimator	Bias-$\mathbf{B}(.)$	Mean squared error $\mathbf{MSE}(.)$	Constant $\mathbf{R_i}$
$\hat{\bar{Y}}_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2$	$\frac{(1-f)}{n}\left(R_1^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_1=\frac{\bar{Y}}{\bar{X}}$
$\hat{\bar{Y}}_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_2^2$	$\frac{(1-f)}{n}\left(R_2^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_2=\frac{\bar{Y}}{\bar{X}+C_x}$
$\hat{\bar{Y}}_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_3^2$	$\frac{(1-f)}{n}\left(R_3^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_3=\frac{\bar{Y}}{\bar{X}+\beta_2}$
$\hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_4^2$	$\frac{(1-f)}{n}\left(R_4^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_4=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}$
$\hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2$	$\frac{(1-f)}{n}\left(R_5^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_5=\frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$
$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1)}(\bar{X}+\beta_1)$ Yan and Tian[17]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2$	$\frac{(1-f)}{n}\left(R_6^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_6=\frac{\bar{Y}}{\bar{X}+\beta_1}$
$\hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2$	$\frac{(1-f)}{n}\left(R_7^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_7=\frac{\bar{Y}}{\bar{X}+\rho}$
$\hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2$	$\frac{(1-f)}{n}\left(R_8^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_8=\frac{\bar{Y}C_x}{\bar{X}C_x+\rho}$
$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2$	$\frac{(1-f)}{n}\left(R_9^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_9=\frac{\bar{Y}\rho}{\bar{X}\rho+C_x}$
$\hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2$	$\frac{(1-f)}{n}\left(R_{10}^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_{10}=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$
$\hat{\bar{Y}}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2$	$\frac{(1-f)}{n}\left(R_{11}^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_{11}=\frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$

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