

Strategic Design of Distribution System of State-owned Companies : Solution Development

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Abstract: A method is presented to solve location problem of PSO-SOC, which is characterized as Minimum Concave-cost Multi-commodity Flow (MCMF) problem. Core idea of proposed solution procedure is the exploitation of network representation (NR). It is found that despite the solution is approached by an aggregate flow, our network representation could derive disaggregate flows, especially for those which deal with diverse cost functions among each commodity, such as production cost and revenue. In addition, our NR could deal with the situation where total supply is not in balance with total demand. The other characteristic of proposed solution procedure is that location decision of MCMF-NR is not represented by the binary number, as employed on most of location model. Location decision is represented by the flow on links associated to fixed-cost of facility. General heuristic algorithm related to Destination Spanning Tree development is suggested to solve the MCMF-NR, and an illustrative example is also discussed.

Key Words: *Public Service Obligation State-owned Company (PSO-SOC), location model, minimum concave cost multicommodity flow (MCMF) problem, network representation.*

1. INTRODUCTION

One of strategic issues concerns with those who are engaged in the planning and operation of physical distribution system is the determination of the best sites of distribution facilities. The importance of locating facilities has inspired many researchers to elaborate various model formulations and solution approaches ranging widely in terms of mathematical and computational complexity.

This research is a part of series of research on distribution system of State-owned Company (SOC). The focus of primary research is on developing model of determining location of distribution facilities of PSO (Public Service Obligation) - SOC. Preliminary stage of primary research concerns with the identification of the characteristics of distribution system of PSO- SOC. It is found that product differentiation is the distinctive attribute of PSO-SOC's distribution system. Product differentiation means that products are not differentiated merely by type of product, but they are also differentiated based on the type of user of the products. In terms of user, products are categorized as commercial and subsidized (public) product. In the context of demand satisfaction, commercial and subsidized products are treated differently. Subsidized demand must be fully satisfied by the supply from all the possible plants, no matter possible profit the company could take . In contrast, commercial products,

which are more profitable in nature, are satisfied just in case of excess capacity of plants exists.

The other aspect identified in the preliminary stage is the diversity of production cost of each plant due to the lack of raw materials in some plants and various performance of production machines. This problem becomes particular if it is related to demand differentiation stated above. In term of optimization of network distribution, we have to search which combination of production cost and transportation cost that would give the maximum profit regarding the position of the both kind of users, as well as the plants and warehouses. Trade-off between production cost, distribution cost and revenue, becomes important to be considered in the optimization to maximize profit.

Based on such preliminary research, system analysis is done and mathematical formula of PSO-SOC's Location Model is proposed. Its form is a modification of models developed earlier. Production cost and transportation cost, as well as fixed cost of facility and inventory holding cost are optimized simultaneously to attain maximum profit. Transportation cost and production cost follow the principle of economies of scale. Selling price is included as representation of revenue. Three-stage distribution channel is proposed in which it consists of set of plants, consolidation centers, distribution centers and retailers as the end users.

The formula of proposed model is described as follows and its related network is illustrated in figure 1:

$$\begin{aligned} \min Z (X_c, Y_d, \alpha_{p(m)c}, \beta_{cdm}, \gamma_{drm}, \delta_{p(m)d}) = & \\ & \sum_{p \in P} \sum_{c \in C} u_{pc} \cdot \Phi_{pc} \left[\sum_{m \in M | p(m)=p} \alpha_{p(m)c} \right] + \sum_{c \in C} \sum_{d \in D} v_{cd} \cdot \Psi_{cd} \left[\sum_{m \in M} \beta_{cdm} \right] + \sum_{p \in P} \sum_{d \in D} z_{pd} \cdot \xi_{pd} \left[\sum_{m \in M | p(m)=p} \delta_{p(m)d} \right] + \\ & \sum_{d \in D} \sum_{r \in R} w_{dr} \cdot \sum_{m \in M} (\gamma_{drm} \cdot \omega_{drm}) + \sum_{p \in P} \sum_{m \in M} \left(\sum_{c \in C} \alpha_{p(m)c} + \sum_{d \in D} \delta_{p(m)d} \right) \cdot \eta_{p(m)} \left[\sum_{c \in C} \alpha_{p(m)c} + \sum_{d \in D} \delta_{p(m)d} \right] + \\ & \sum_{d \in D} \sum_{m \in M} S_{dm} \cdot IC_{dm} + \sum_{c \in C} X_c \cdot FC_c + \sum_{d \in D} Y_d \cdot FD_d - \sum_{d \in D} \sum_{r \in R} \sum_{m \in M} \gamma_{drm} \cdot \rho_{rm} \end{aligned} \quad (1)$$

subject to:

$$\sum_{p \in P} \alpha_{p(m)c} = \sum_{d \in D} \beta_{cdm}, \quad \forall c \in C, \forall m \in M \quad (2)$$

$$\sum_{c \in C} \beta_{cdm} + \sum_{p \in P} \delta_{p(m)d} = \sum_{r \in R} \gamma_{drm}, \quad \forall d \in D, \forall m \in M \quad (3)$$

$$\sum_{d \in D} \gamma_{drm} = \lambda_{rm}, \quad \forall r \in R, \forall m \in M^s \quad (4)$$

$$\sum_{d \in D} \gamma_{drm} \leq \lambda_{rm}, \quad \forall r \in R, \forall m \in M^c \quad (5)$$

$$\gamma_{drm} \leq Y_d \cdot \lambda_{rm}, \quad \forall d \in D, \forall r \in R, \forall m \in M \quad (6)$$

$$\sum_{p \in P} \alpha_{p(m)c} \leq X_c \cdot \sum_{r \in R} \lambda_{rm}, \quad \forall c \in C, \forall m \in M \quad (7)$$

$$\sum_{c \in C} \alpha_{p(m)c} + \sum_{d \in D} \delta_{p(m)d} \leq Cp_{p(m)}, \quad \forall p \in P, \forall m \in M \quad (8)$$

$$\sum_{c \in C} \sum_{m \in M} \beta_{cdm} + \sum_{p \in P} \sum_{m \in M} \delta_{p(m)d} \leq Cd_d, \quad \forall d \in D \quad (9)$$

$$\sum_{p \in P} \sum_{m \in M} \alpha_{p(m)c} \leq Cc_c, \quad \forall c \in C \quad (10)$$

$$\alpha_{p(m)c} \geq 0, \quad \forall p \in P, \forall c \in C, \forall m \in M \quad (11)$$

$$\beta_{cdm} \geq 0, \quad \forall c \in C, \forall d \in D, \forall m \in M \quad (12)$$

$$\gamma_{drm} \geq 0, \quad \forall d \in D, \forall r \in R, \forall m \in M \quad (13)$$

$$\delta_{p(m)d} \geq 0, \quad \forall p \in P, \forall d \in D, \forall m \in M \quad (14)$$

$$S_{dm} \geq 0, \quad \forall d \in D, \forall m \in M \quad (15)$$

$$X_c = [0,1], \quad \forall c \in C \quad (16)$$

$$Y_d = [0,1], \quad \forall d \in D \quad (17)$$

Subscripts:

- p : indicate the Plants
- c : indicate the Consolidation Centers
- d : indicate the Distribution Centers
- r : indicate the Retailers
- m : indicate the Products
- $p(m)$: indicate the plant $p \in P$ that produces product- m

Sets :

- P : Set of plants
- C : Set of consolidation centers
- D : Set of distribution centers
- R : Set of retailers
- M : Set of products
- $M^s \in M$: Set of subsidy (public) products
- $M^c \in M$: Set of commercial products

Decision Variables:

- $X_c = 1$ if Consolidation Center - c is opened , 0 otherwise
- $Y_d = 1$ if Distribution Center - d is opened , 0 otherwise
- $\alpha_{p(m)c}$ is quantity of product- m that flow from Plant $p(m)$ to Consolidation Center- c
- β_{cdm} is quantity of product- m that flow from Consolidation Center- c to Distribution Center- d
- γ_{drm} is quantity of product - m that flow from Distribution Center- d to Retailer- r
- $\delta_{p(m)d}$ is quantity of product - m that flow from Plant $p(m)$ to Distribution Center- d

Input Parameters:

- ρ_m is the selling price of the product- m at retailer - r
- u_{pc} is the distance from Plant- p to Consolidation Center- c
- v_{cd} is the distance Consolidation Center- c to Distribution Center- d
- w_{dr} is the distance from Distribution Center- d to Retailer- r
- z_{pd} is the distance from Plant $p(m)$ to Distribution Center- d
- S_{dm} is stock level in Distribution Center- d
- IC_{dm} is unit inventory holding cost of product- m in Distribution Center - d
- FC_c is fixed cost of facility of Consolidation Center - c
- FD_d is fixed cost of facility of Distributor Center - d
- ω_{drm} is per-mile cost to ship a unit of product- m from Distribution Center- d to Retailer- r
- λ_m is demand of product- m in Retailer- r
- $Cp_{p(m)}$ is the capacity of plant- p to produce product- m

- Cd_d is the capacity of Distribution Center- d
- Cc_c is the capacity of Consolidation Center- c

Input Functions:

- $\Phi_{pc}[\cdot], \forall p \in P, \forall c \in C$: is the cost per mile for transporting product- m from Plan- p to Consolidation Center- c (a concave function of total volume)
- $\Psi_{cd}[\cdot], \forall c \in C, \forall d \in D$: is the cost per mile for transporting product- m from Consolidation Center- c to Distribution Center- d (a concave function of total volume)
- $\xi_{pd}[\cdot], \forall p \in P, \forall d \in D$: is the cost per mile for transporting product- m from Plan- p to Distribution Center- c (a concave function of total volume)
- $\eta_{p(m)}[\cdot], \forall p \in P$: is the cost per ton for producing product- m in plant- p

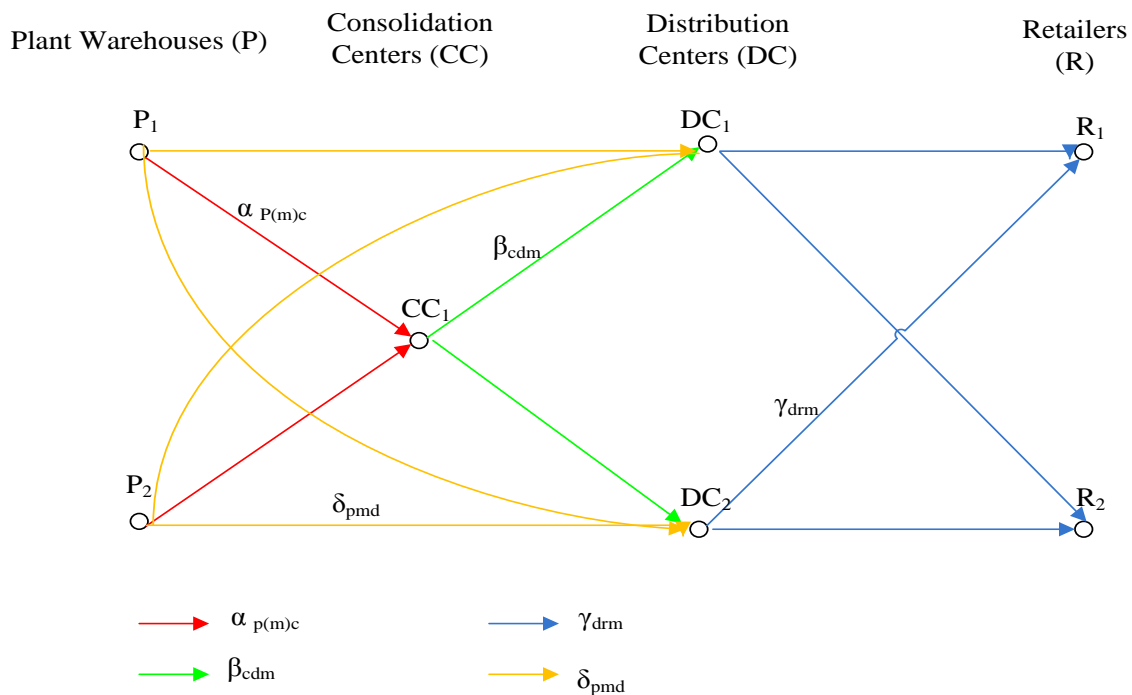


Figure 1 Example of Proposed (Original) Distribution Network

2. PROPERTIES OF PROPOSED MODEL

In order to develop solution procedure of proposed model, we focus on properties of the proposed model. It can be explained as following:

- Due to consideration of the economies of scale on transportation cost as well as production cost, cost per unit of flow decreased as the quantity of product delivered (or produced) increased. Consequently, it leads to the concave total cost function (Zangwill, 1968). Economies of scale in transportation cost could utilize the existence of consolidation centers (CC's), in which products from some plants could be consolidated in CC and sent together to the distribution centers (DC's) by certain vehicle in a more economic way. Surely, we have to compare such cost in alternative way, that is transporting products directly from plant to distribution centers, whereas in our proposed

model there is no consolidation activities between DC's and retailers. In addition, small fully occupied vehicles are usually used between those nodes. Hence, there is no economies of scale between DC's and retailers, and it is assumed that the transportation cost between those nodes to be linear. Furthermore, fixed cost of facility, that is the fixed cost to open both consolidation centers and distribution centers, is a constant. It does not depend on the amount of flow passing through it itself. Mathematically, such cost depends only on binary variable $[0,1]$ which shows whether associated warehouse is opened or not. The cost which deals with the variable component of cost of facility, such as handling cost, is assumed to be included in transportation cost. In addition, for the simplicity, inventory holding cost is designed as a linear function of amount of product handled in each warehouse. All the assumptions associated to the form of cost function of the proposed model are not rigid. It is possible to replace linear cost function by the concave one, and conversely. Moreover, revenue is characterized with linear function. Revenue depends linearly on the amount of products sold and its associated selling price. Based on the characteristics of its components, the proposed objective function simply could be presumed as a concave cost function.

- The other essential attribute of the proposed model is that the model is set to be utilized to handle multi-commodities. As described above, the term 'multi-commodity' refers to both types of products and types of users of the products. In the context of user satisfaction, both types of user, namely commercial and public users, are treated differently. This consideration is represented by flow requirement denoted in equation (4) and (5) as equality and inequality function for public demand and commercial one respectively.
- The total amount of products supplied by each plant and total amount of flow held by either distribution center or consolidation center are limited to its each capacity. Unlike the nodes, it is assumed that all links are not restrained by their capacity.
- Finally, as its properties described above, our proposed model could be classified as Minimum Concave-cost Multi-commodity Flow (MCMF) problem.

3. PRIOR RELATED STUDIES ON SOLUTION DEVELOPMENT

In this session some papers are reviewed to map out the preceding research works related to the concave cost function. Some reviews focus on the solution methods of the minimum concave cost minimization problem, while the others focus on those of location problems, particularly associated with the attribute of concave cost function.

Zangwill (1968) characterized key mathematical difficulty in analyzing concave cost minimization problem comes from the existence of the enormous number of local optima in the search space. Optimum point x^* may be a relative minimum and not a global minimum. The value of x^* may be minimized the objective function over the intersection of the feasible region and a neighborhood of x^* , but x^* may not minimize the objective function over the entire feasible region. He proposes some theorems that explicitly characterize the extreme points for particular single commodity network. Although concave cost function can be minimized through exhaustive search of all the extreme points of the convex feasible region, such an approach is impractical for all but the simplest of problems.

Gallo, et al (1979) found that due to the concavity of the cost function and the network complexity, finding optimal solutions is often an exceedingly hard task. For this reason, it is necessary for the existing methods to make use heuristic procedures for finding or improving local optima. They propose a Branch and Bound method to solve minimum cost flow problem on an un-capacitated single source single commodity multiple destination network.

Most approaches in solving concave cost minimization problem are based on traditional mathematical programming techniques (such as linear approximation, lagrangean relaxation, the sub-gradient method, the branch-and-bound method and dynamic programming), included such research works described above. Some algorithms have been developed to improve the solution efficiency, particularly to overcome the problem on searching enormous number of local optima to find global one. Several meta-heuristic algorithms have recently been developed, with a traditional local search as the core, combined with high-level meta-strategies for jumping out of the local optima found in the neighborhood searches, thus finding a better solution (Osman et al, 1996). Some examples of recent meta-heuristics include the Simulated Annealing (SA), the Genetic Algorithm (GA), Tabu Search (TS), the Great Deluge Algorithm (GDA) and Threshold Accepting (TA). The following description explains some research works which based on such approaches.

Yan, et. al. (1999) argue that simplification of concave cost function into linear one in order to facilitate problem solving may not reflect actual operations, which generally results in decreased operational performance. They employ the technique of simulated annealing and threshold accepting to develop several heuristics that would efficiently solve concave cost transportation network problems.

Yan, et. al. (2005) propose global search algorithm for solving concave cost transshipment problems. They employ TA, GDA and TS to develop four efficient local search algorithms, which can be compared with the proposed global search algorithm. Efficient methods for encoding, generating initial population, selection, crossover and mutation are proposed, according to the problem characteristics.

Fontes & Goncalves (2007) found that there is no simple criterion for deciding whether a local minimum is also a global minimum. He proposes a hybrid approach on single source un-capacitated network which combine Genetic Algorithm with a local search.

The following research works are particularly related to the location problem in which their objective functions take form of concave function.

Lin (2002) carries out a research in which its goals of developing a location model for multi-product and multi-echelon distribution systems where there are significant economies of scale in the transportation movements, particularly between the plants, consolidation centers and the distribution centers. The objective function of his model is to minimize transportation cost, facility fixed-cost as well as the inventory cost and penalty cost. The solution procedure developed is a greedy heuristic. The greedy heuristic iterates between locating DC's given a collection of CC's and locating CC's given a fixed of DC's.

Dupont, L. (2008) introduces a new type of facility location model, in which the global cost incurred for each established facility is a concave function of the quantity delivered by this facility. He introduces some properties of an optimal solution and derive heuristic algorithms

and a branch and bound method from these properties. He proposes un-capacitated facility location problem (UFLP) which characterized by single commodity and single stage network.

4. NETWORK REPRESENTATION AS AN APPROACH OF SOLUTION

As it is commonly known that minimum concave cost flow problems are categorized as NP-hard (Larsson et al, 1994), we propose heuristic approach as the solution procedure. The theorems, as well as algorithm developed by Gallo *et. al.* (1979) are utilized as core ideas of our heuristics approach. We adopt Gallo's procedure due to its simplicity, particularly when it is compared to the other techniques, such as Genetic Algorithm, Simulated Annealing and Tabu Search. Such techniques need to derive intricate or empirical probability value when it deals with combinatorial problems, whereas Gallo's makes use of branch and bound method by introducing simple parameter which is called penalty. Furthermore, our proposed method elaborate the inherent network characteristics in distribution as well as location problems.

Gallo proposes that in solving minimum concave cost flow problems (P), we may restrict ourselves to extreme flows of X , whereas X are defined as feasible set of solution of P and their vertices are to be focused further for the optimal solution of P . This approach is coming from the definition of P , that is P is defined as a concave minimization problem bounded with low-bound constraints. Furthermore, he proposes that there is a one-to-one correspondence between the set of extreme flows and all trees of the network under consideration of P . In other words, the solution of P corresponds to destination spanning tree (DST) of associated graph of distribution network of P .

In term of multi-commodity, we refer our solution procedure to the theorem of Zangwill (1968). He proposes that, in fact, solving the aggregate model of minimum concave cost multi-commodity flow (MCMF) problem solves the disaggregate model of such problem.

Since destination spanning tree, as our solution approach, substantially deals with network structure, then we interpret our model as network representation based problem. We propose network representation, of which nodes and links represent any attributes of cost, revenue, quantity of flows and any other representations of the complex attribute denoted by equation (1) ~ (17). By this approach, it is expected that the solution of our model could adopt and exploit the Gallo's algorithm concerning of minimum concave cost flow problem, particularly of destination spanning tree problem.

Considering the formulas denoted in equation (1) ~ (17), the following description of stepwise approach is then proposed to extract the properties of model and lead to solution of the model.

1) Development of Network Representation

Based on the proposed physical network, as denoted in figure 1, network representation of the optimization problem is proposed. Some dummy links and nodes are added to the original network to represent production cost, transportation cost, fixed cost of facility as well as revenue. For simplicity, linear inventory holding cost function is designed by incorporating such cost on fixed cost of facility and transportation cost function. Part of inventory holding cost which covers the cost of holding some mandatory constant amount of products could be incorporated in fixed cost of facility, while the cost of handling some variable amount of products could be incorporated on the transportation cost. This simplification is viable due to

the concurrence of concave function of transportation cost and the linear form of the inventory holding cost.

Some dummy links and nodes are also included to represent revenue and points of sales of commercial product which are permitted to deal with in Consolidation Centers (CC) and Distribution Centers (DC).

In order to guarantee the equivalency of the amount of total supply and total demand, we add either of Excess Supply Control Sub Network or Excess Demand Control one into our basic network representation, depends on which condition exists at the beginning of the optimization process. Surely, those sub-networks are not required when supply and demand are in balance. One example of network representations is shown in figure 2. Furthermore, the figure explains the followings:

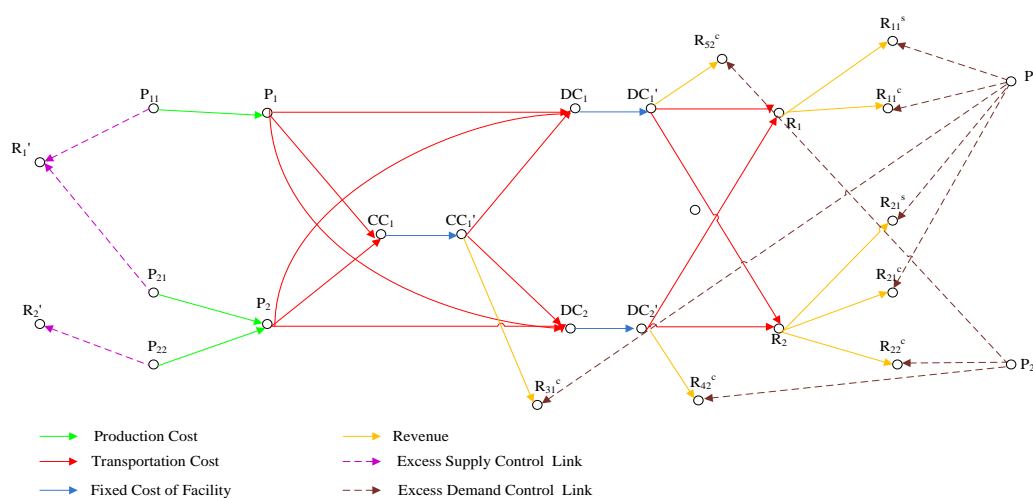


Figure 2 Network Representation of Proposed Model

- Links between node $P_{pm} - P_p$ (p and m indicate the plants and products, respectively) represent production cost function to produce product- m in plant- p .
- Links between $P_p - CC_c, P_p - DC_d, CC'_c - DC_d$, as well as those of between $DC'_d - R_r$ (c, d and r indicates the consolidation centers, distribution centers and retailers, respectively) represent transportation cost function between plant- p and consolidation center- c , plant- p and distribution center- d , consolidation center- c and distribution center- d , as well as distribution center- d and retailer- r , respectively.
- Links between $CC_c - CC'_c$ and $DC_d - DC'_d$ represent fixed cost of facility of associated consolidation center and distribution center, respectively.
- Links between $CC'_c - R_{rm}, DC'_d - R_{rm}$, as well as $R_r - R_{rm}$ represent revenue from product- m which are sold in consolidation center- c and distribution center- d , as well as in retailer- r .
- Links between $P_{pm} - R'_m$ represent cost to eliminate some amount of production of product- m in plant- p due to the excess of supply.
- Links between $P'_m - R^s_{rm}$ and $P'_m - R^c_{rm}$ represent cost of “unsatisfied demand” on product- m of subsidy and of commercial in retailer- r due to the excess of demand.

2) Network Valuation

This step values all nodes and links with their related functions. Each link of network representation is to be valued as its associated cost function as well as revenue designation. All nodes are valued with its associated flow requirement (π_i) and these values should be in

disaggregate form (that is, flow requirement of each node for each type of product). However, we consider the quantity of products flow over network representation as an aggregate flow of all types of products. Due to the cost function on each link associated to production cost and each link associated to revenue are set exclusively to certain product, the aggregate flow over such links can be assumed as disaggregate flow. From this point, it can be understood that the disaggregate value of π in each node associated to both kinds of links is similar to its aggregate value.

Flow requirement of node P_{pm} is set as production capacity of plant- p to produce product- m . Flow requirement of node R_{rm}^s and R_{rm}^c is set as demand of retailer- r on product- m subsidy or m -commercial. In addition, flow requirement of node P'_m is set as total amount of demand on product- m that should be reduced in case of excess demand and R'_m is set as total amount of product- m that should be reduced in production in case of excess supply. Since the rest of nodes are functioned as transshipment points, consequently its flow requirement is set as zero for each of products.

As stated in equation (9) and (10), the total link-in flow in each of transshipment point is limited by its node capacity. However, in network representation, we release those requirements in order to catch as much as possible flow entering the node. It is expected that the optimal flows will show the real requirement on the size of the warehouses that should be opened.

Regarding demand satisfaction, particularly of public demand (see equation 4 & 5), which is becoming important in case of Excess Demand, we set extremely high unit cost function to the links of excess demand control sub-network that related to nodes of subsidy (public) demand. It means that such a high “unsatisfied-demand cost” will avoid unfulfillment of public demand.

3) Network Assignment

As the network representation and its associated link and node values are determined, the original problem of equation (1) ~ (17) simply could be expressed as Minimum Concave cost Multi-commodity Flow – Network Representation (MCMF-NR) problem. Consequently, the objective function and its constraints are changed. Nevertheless, the transformation warrants that both problems are correspond. Mathematical programming of the MCMF-NR then becomes:

$$\min Z (x_{ij}) = \sum_{(i,j) \in A} \varphi_{ij} (x_{ij}) \tag{18}$$

subject to:

$$\sum_i x_{il} - \sum_j x_{lj} = \pi_l, \quad \forall l \in N \tag{19}$$

$$x_{ij} \geq 0, \quad \forall (i, j) \in A \tag{20}$$

where:

$$\pi_l = \sum_{m \in M} \pi_{lm}, \quad \forall l \in N \tag{21}$$

$$\pi_l = 0, \quad \forall l \notin D_s, \forall l \notin S_c \tag{22}$$

For Excess Demand Case:

$$\begin{aligned} S_c(i) &= \{(i) \in N : i = P_{pm} \cup P'_m, \forall p, \forall m\} \text{ and} \\ D_s(i) &= \{(i) \in N : i = R_{rm}^s \cup R_{rm}^c, \forall r, \forall m\} \end{aligned} \tag{23}$$

For Excess Supply Case:

$$\begin{aligned} Sc(i) &= \{(i) \in N : i = P_{pm}, \forall p, \forall m\} \text{ and} \\ Ds(i) &= \{(i) \in N : i = R_{rm}^s \cup R_{rm}^c \cup R_m', \forall r, \forall m\} \end{aligned} \quad (24)$$

For supply and demand in balance case :

$$\begin{aligned} Sc(i) &= \{(i) \in N : i = P_{pm}, \forall p, \forall m\} \text{ and} \\ Ds(i) &= \{(i) \in N : i = R_{rm}^s \cup R_{rm}^c, \forall r, \forall m\} \end{aligned} \quad (25)$$

Subscripts:

- p : indicate the Plants
- r : indicate the Retailers
- m : indicate the Products

Superscripts :

- s : indicate the subsidy / public products
- c : indicate the commercial products

Sets :

- N : Set of nodes of network representation
- A : Set of links of network representation
- Sc : Set of source nodes of network representation
- Ds : Set of destination nodes of network representation
- M : Set of products

Decision Variables :

- x_{ij} is aggregate flow on link i to j

Input Parameters :

- π_{lm} : is flow requirement of node- l on product- m

Input Functions:

- $\varphi_{ij}(\cdot), \forall (i, j) \in A$: is the link cost function of link $i-j$

Regarding Gallo and Zangwill theorems, our MCMF-NR problem could be solved by considering the extreme points of feasible set of MCMF-NR, in which they correspondence to spanning tree. Optimality of total cost function (equation (18)) is evaluated through searching of the extreme flows which are corresponding to Destination Spanning Tree (DST) which gives the minimum value to the objective function.

The assignment process is based on the characteristics of concave cost network flow problems, that is if flows are assigned to arcs when adjacent nodes have a greater supply or demand than other nodes, then the cost may be reduced. In addition, the total cost tends to be reduced if the arc flows tend to be zero or the allowed for maximum amount. In other words, an all-or-nothing assignment rule should help to reduce transshipment costs. Combinatorial problem of MCMF-NR will be solved by branch and bound procedure, in which the partitioning is based on the inclusion of subsets of arcs.

Due to the transformation from MCMF to MCMF-NR, the location decision variables X_c and Y_d of the original problem have to be eliminated. Moreover, the decision to open or close one facility (that is transshipment warehouse) depends on the link flow of the associated link

representing its fixed cost of facility. Small amount of such link flow could be an indication for closing such warehouse, and conversely.

4) Scenario Testing

At this level, a set of scenarios on distribution system can be tested to elaborate properties of the proposed model. Some models of distribution system, that is distribution network, are exercised as part of optimality search and learning phase of various properties of the model.

5. THE ALGORITHM

According to the step-wise approach described in section 4, the following general algorithm is suggested :

Step 0: (Initialization) Given a network $G = (N, A)$ of MCMF-NR, then set the unit cost at the current flow of each link. If a link has no flow, then its flow is set to be one to approximate the link cost.

Step 1: (Set Tree) Choose one destination node- j of set D_s . Start from j find possible path that connects destination node- j to one of source node- i and it forms a tree of G .

Step 2: (Flow Assignment) Assign the maximum amount of flow that can be sent from source node- i to destination node- j , that is, $f_{max} = \min(rs_i, rd_j)$, where rs_i is the remaining supply for node- i and rd_j is the remaining demand for node- j . Deduct f_{max} from source node- i and destination node- j , and update the link flow along the path between i and j .

Step 3: (Z-Value) Compute the value of Z . Let $Z^* = Z$ be the value of local optimum which corresponds to tree - T^* .

Step 4: (Finalization) If all destination node has been chosen, then STOP; otherwise go to step 1.

6. ILLUSTRATIVE EXAMPLE

We demonstrate our proposed approach on network representation to solve the problem of location decision on a small example with two plants, one consolidation center, two distribution centers and two retailers. The physical distribution network is as shown in figure 1. Figure 3 indicates network representation of our problem. Due to this example is designated as excess demand problem, we add dummy plant P_A' to accommodate "unsatisfied demand" on product A and dummy plant P_B' to accommodate those of product B. Definitely, we must not eliminate demand of subsidy product, hence we set extremely high unit cost on link $R_{1A}^s - P_A$ and $R_{2A}^s - P_A$. Furthermore, we set unit cost of links of commercial products associated to P_A' and P_B' similar to its selling price. This notion means that the higher the selling price is, the lesser the possibility of the demand unsatisfied.

Plant 1 produces product A only, while plant B produces product A and B. Demands of retailer 1 consist of product A of subsidy and product A of commercial one. Demands of retailer 2 consist of product A of subsidy, product A of commercial and product B of commercial. The number in the bracket denotes plant capacity, included the capacity of dummy plant P_A and P_B , as well as demand on retailer node. Those number also represent the flow requirement of each node. In our NR model, capacity of the warehouse is not limited.

Cost functions of links between $P_{pm} - P_p$ is set as production cost function of each product- m in plant- p . Those links form concave function. Cost functions of links between $P_p - CC_c, P_p - DC_d, CC'_c - DC_d$ are set as transportation cost function and those are in concave

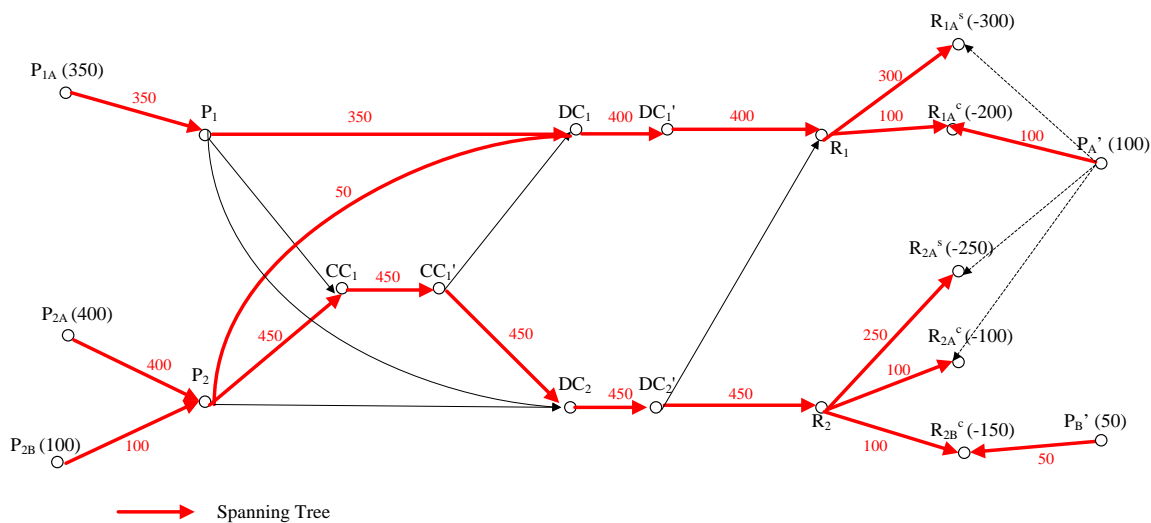


Figure 3 Network Representation of Illustrative Example

function, while transportation cost function between $DC'_d - R_r$ is set as linear function. In this example, it is assumed that the concave cost function of transportation cost is not limited by the vehicle capacity. Furthermore, fixed cost of facility of CC_c and DC_d are set as a fixed number and those number are assumed to be similar for all the warehouses. Links represent revenue ($R_r - R_{rm}^s$ and $R_r - R_{rm}^c$) form linear function. Those are a function of quantity of flow and selling price. We set similar selling price for all subsidy products, no matter the location of the retailer is, whereas the selling prices of commercial ones are set in variety.

Using the algorithm of section 5, manually we enumerate all link flows of some possible destination spanning trees (DST) and its associated objective function. The red lines of figure 3 show one of destination spanning tree and its associated flow. From the assignment, we can see that the demand on product A of commercial in retailer R_1 should be reduced 100 unit, while the demand on product B of commercial in retailer R_2 should be diminished by 50 unit. These reductions is coming from the problem of excess demand.

The decision to open or close certain warehouse depends on the amount of flow on link representing its fixed cost of facility. In our example, it is likely that CC_1, DC_1 and DC_2 are opened due to its significant link flows.

7. CONCLUSION

Regarding mathematical programming of Location Model that we have proposed on our previous research work, we examine the properties of the model and find that it could be classify as Minimum Concave-cost Multicommodity Flow (MCMF) problem. To solve the problem, we make use network representation to simplify the problem. Consequently, our

original problem is converted into MCMF-Network Representation problem. Despite the solution of the proposed model is approached by an aggregate flow, our network representation could derive disaggregate flows, especially for those which concern with production cost and revenue. Flow associated to each of both parameters could not be aggregated due to the diverse production cost (or selling price) among each commodity. In addition, our network representation could deal with the situation where the total supply is not in balance with total demand. The other notable conclusion of this research work is that location decision of MCMF-NR is not represented by the binary number, as employed on most of location models. Location decision is based on link flows associated to fixed-cost of facility. Based on proposed network representation, the MCMF-NR problem is solved through heuristic procedure, in which the core idea is finding the destination of spanning tree. General algorithm is suggested and it then needs to be explored thoroughly, particularly based on the algorithm proposed by Gallo (1979). This research work should be followed by the numerical case in order to validate the model. Eventually, it is realized that this research work is not intended to focus on the sophistication of MCMF solution. It is focused on the implication of the usage of network representation as an alternative approach of location model of PSO-SOC.

REFERENCES

- Dupont L. (2008) Branch and bound algorithm for a facility location problem with concave site dependent costs, **International Journal Production Economics** **112**, 245-254.
- Fontes, D.B.M.M. and Gonzalves, J.F. (2007) Heuristic solutions for general concave minimum cost network flow problem, **Networks 2007-DOI 10.1002/net**, www.interscience.wiley.com.
- Gallo, G., Sandi, C. and Sodini, C. (1980) An algorithm for the min concave cost flow problem, **European Journal of Operational Research** **4**, 248-255.
- Kuhn, H.W. and Baumol, W.J. (1962) An approximate algorithm for the fixed-charge transportation problem, **Naval Res. Logistics Quarterly** **9**, 1-16.
- Larsson, T., Migdalas, A. and Ronnqvist, M. (1994) A Lagrangian heuristic for the capacitated concave minimum cost network flow problem, **European Journal of Operational Research** **78**, 116-129.
- Lin, J.R. (2002) **Strategic Design of Distribution Systems with Economies of Scale in Transportation**, Ph.D. Dissertation, Graduate School of Cornell University.
- Nash, S.G. and Sofer A. (1996) **Linear and Nonlinear Programming**. The McGraw-Hill Companies, Inc.
- Osman, I.H. and Kelly, J.P. (1996) **Meta-Heuristics : An overview, Meta-Heuristics : Theory & Applications**. Kluwer Academic Publishers, Boston, London, Dordrecht.
- Yan, S., Juang, D.S., Chen, C.R. and Lai, W.S. (2005) Global and local search algorithms for concave cost transshipment problems, **Journal of Global Optimization** **33**, 123-156.
- Yan, S. and Luo, S.C. (1999) Theory and methodology. Probabilistic local search algorithms for concave cost transportation network problems, **European Journal of Operational Research** **117**, 511-521.
- Zangwill, W.I. (1968) Minimum concave cost flows in certain networks, **Management Science** **Vol.14, No.7**.