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REPETITIVE CONTROL BASED DISTURBANCE CANCELLATION USING ITERATIVE BASIS FUNCTION FEEDBACK WITH WAVELET FILTERING

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ABSTRACT

This paper presents repetitive control laws in real time using matched basis functions. These laws adjust the command given a feedback control system in order to eliminate tracking errors, resulting from in general a periodic disturbance and a non-periodic disturbance. The periodic error can be reduced by linear basis functions while the non-periodic error by the projection algorithm along with the wavelet filtering. The control laws do not use a system model, but instead the control action is chosen to be a linear combination of chosen input basis functions, and the corresponding output basis functions are obtained, nominally by experiment. The repetitive control laws use the projection algorithm to compute the output components on the output basis functions, and then the corresponding input components are adjusted accordingly. The output signals are reconstructed via the wavelet filtering before they are feedback to the controller. Numerical experiments show that the repetitive controllers are quite effective. In particular, the output tracking errors are further reduced because of the introduction of the wavelet filtering when compared to the previous work. In general, the repetitive control laws developed here can be used for the purpose of precision machinery control.

INTRODUCTION

There are many practical situations in which a feedback control system is subject to a periodic disturbance, and one would like to have a method of totally eliminating the influence of the disturbance on the output. Here we present methods of canceling tracking errors of feedback controllers to periodic commands. The algorithms are examples of repetitive control.

This research was motivated by the problem of improving the focus of the 4 GeV continuous electron beam accelerator running over a 5 mile path, at the Thomas Jefferson National Accelerator Facility. The electromagnets used in accelerating and bending the electron beam use DC supply, and this comes from rectifying 60 Hz AC. The rectification is not perfect, and residual fluctuations in the DC supply cause fluctuations in the beam. The fundamental at 60 Hz is most important, but one is interested in eliminating 11 harmonics as well. This application is similar to the objective of the very early work in repetitive control, reference [1].

The approach used here is to make use of basis functions in formulating the repetitive control law. Learning control is similar to repetitive control, except that the system is restarted from the same initial condition at the start of every period. Many of the methods used in learning control are appropriate to repetitive control, and vice versa [2, 3]. References [4-8] apply basis functions in learning control. The use of basis functions in repetitive control, updating in a batch form, is discussed in [9], which produces a different kind of bridge between learning and repetitive control than in [3, 2]. References [10, 11] also discuss the use of basis functions in repetitive control. This paper makes use the matched basis function concept discussed in [4, 5, 6, 9] which is advantageous because it eliminates spillover [9]. We develop repetitive control methods for matched basis functions for application in real time. To do so we need a real time method of finding the components of the output on the output basis functions. We investigate the use of the projection algorithm and the wavelet filtering. The wavelet filtering is superimposed on the projection algorithm: in each time step the output signal computed by the projection algorithm is decomposed and then reconstructed via the wavelet filtering. The effect of the wavelet filtering is studied through the comparison of the tracking errors of the output signals computed by the projection algorithm and by the projection algorithm with wavelet filtering. The addition of the wavelet filtering is used to further cancel the non-periodic disturbance for accuracy promotion [12]. For on-line applications, the output tracking error of the numerical example shown in this study appears further reduced at the expenses of slower convergence. For off-line applications, on the other hand, when the machines are shutdown for maintenance or software test, the output tracking error appears the same result with that computed via the pure projection algorithm. In real-life and on-line applications, the superposition of the wavelet filtering on the projection algorithm proves quite effective in precision repetitive control.

THE MATCHED BASIS FUNCTION SYSTEM MODEL

The repetitive control action aims to find a time function which when added to the command to the feedback control system eliminates the effects of the periodic disturbance or periodic tracking error in responding to a periodic command. We agree to limit the adjustments of the command input that to ones lying in a space spanned by a chosen set of periodic basis functions. As in [9] we presume that we can perform experiments applying these chosen basis functions individually to the existing feedback control system. In these experiments we wait until steady state response is reached, record the periodic output, and the result is the set of matching output basis functions. When there is a periodic disturbance present, we must conduct two experiments with different amplitudes multiplying the input basis function, and then take the differences of inputs and the differences of outputs, in order to eliminate the effect of the disturbance in defining the output basis functions. The mathematics is general for any chosen set of basis functions, but an important special case is sinusoidal basis functions, as apply to the problem of eliminating 60 Hz and harmonics in the electron beam focus problem [13].

The Basis Functions and the System Model

Wen and Longman [13] define T_u as a matrix of *p* rows, each column representing one period of the corresponding chosen discrete-time basis function, and its value is $\sin(\frac{2\pi f * i}{p})$ or $\cos(\frac{2\pi f * i}{p})$, where $i = 0, 1, \cdots, p-1$. The T_y is the corresponding matrix of output basis functions, giving the output that would be obtained if there were no periodic disturbance. The matrices α and β are defined as column matrices of the coefficients of the output and the input basis functions, respectively. If the input is the linear combination defined by $\underline{u} = T_u \beta$, then the steady state response is $\underline{y} = T_y \alpha = T_y \beta$. This makes the steady state system response model in basis function space into an identity matrix *I*,

$$\alpha = I\beta \tag{1}$$

LINEAR REPETITIVE CONTROL LAWS IN TERMS OF COMPONENTS ON MATCHED BASIS FUNCTIONS

A general linear repetitive control law for real time implementation is [13]

$$\beta(k+1) = \beta(k) + \Gamma(\alpha * - \hat{\alpha}(k)) \tag{2}$$

where Γ is a square matrix of learning gains, $\hat{\alpha}(k)$ is the column vector of current estimates of the output components on the output basis functions, and $\alpha *$ is the desired trajectory written in terms of components on the output basis functions, which can be written in terms of the *p* time step history of one period of the desired trajectory, $\underline{y}*$, according to $\alpha * = T_y^+ \underline{y}*$, where T_y^+ is the Moore-Penrose pseudoinverse.

CHOICES FOR REAL TIME COMPUTATION OF COMPONENTS ON BASIS FUNCTIONS

Here we seek to do real time updates, which recursively determine some estimate of the components on the output basis functions every time step, $\hat{\alpha}(k)$. There are various ways to do this, including the projection algorithm common in adaptive control, and various forms of recursive least squares [14-16]. In this study, we utilize the projection algorithm, and the $\hat{\alpha}(k)$ can be represented as [13]

$$\hat{\alpha}(k) = \hat{\alpha}(k-1) + \Phi(k)[y(k) - T_y^T(k)\hat{\alpha}(k-1)] \tag{3}$$

where $\Phi(k)$ can be determined a prior, and is not a function of data. The $T_y^T(k)$ is determined from T_y whose columns contain one period of the periodic basis functions. In the following, we discuss the form of $\Phi(k)$ in the projection algorithm. We pay particular attention to, because in this case the $\Phi(k)$ becomes a periodic function of time step, a property that facilitates the stability analysis.

Projection Algorithm

The projection algorithm at time step $k - 1$ has an estimate $\hat{\partial}(k-1)$, and chooses to improve this estimate by finding that $\hat{\partial}(k)$ lying on the hypersurface $y(k) - T_y^T(k)\hat{\partial}(k)$, that is closest to $\hat{\partial}(k-1)$ in a Euclidean sense. One can use a Lagrange multiplier to minimize $J = \left\|\hat{\partial}(k) - \hat{\partial}(k-1)\right\|^2$ subject to the constraint $y(k) - T_y^T(k)\hat{\partial}(k) = 0$. The result is [13]

$$\Phi(k) = \frac{aT_y(k)}{c + T_y^T(k)T_y(k)} \tag{4}$$

where $a = 1$ and $c = 0$. These values can be altered. A $c > 0$ is used to eliminate the possibility of dividing by zero when this is a possibility, and modifying a can be used to make the trade-off between the rate of convergence and the amount of smoothing needed. The rate of convergence is important in time varying situations, which is the case in the repetitive control problems here. The algorithm is attractive because of its simplicity. The fact that it does not fully use all of the data, means that its convergence is likely to be slower than some algorithm that uses all of the information available.

There is an alternative formulation which we also consider, that writes an individual optimization as above for each pair of basis functions associated with a given frequency, i.e. for a sine and a cosine input basis function for each frequency, together with their matched output basis functions. So the equations (3) are decoupled frequency by frequency. And then a separate decoupled learning equation (2) is also used for each frequency. To increase the accuracy of the repetitive control based projection algorithm, the wavelet filtering is introduced.

WAVELET TRANSFORM

Many applications use the wavelet decomposition taken as a whole. The common goals concern the signal or image clearance and simplification, which are parts of de-noising or compression [17].

Fourier Transform and Short-Time Fourier Transform

When analyzing a signal, first understanding its property is necessary. The signal measured in the lab is generally a time domain signal, whose property cannot be apprehend until the time history of the signal is transformed to the frequency domain using the Fourier transform. However, there is a problem in the Fourier spectrum because it shows no relationship between frequency and time, i.e. there is no information of the corresponding time for a specific frequency in the spectrum.

In order to solve the disadvantage of the Fourier transform, Dennis Gabor in 1946 adapted the Fourier transform to analyze only a small section of the signal at a time—a technique called *windowing* the signal. Gabor's adaptation, called the Short-Time Fourier Transform (STFT), mapped a signal into a two-dimensional function of time and frequency [17, 18]. Although the STFT obtained the mapping of the signal in the time domain to the frequency domain, its accuracy was limited to the size of the window for the signal. This limitation was not good for the signal containing both the high and low frequency. The wavelet analysis therefore appeared.

Wavelet Analysis

The wavelet was proposed by Morlet *et al* in early 1980's [19, 20]. It improved the STFT by using variable sized-regions of windows according to their frequency locations. Yet the representation of the wavelet was not in the form of time-frequency but time-scale.

The wavelet transform can be classified to two categories: one is the continuous wavelet transform (CWT) and the other is the discrete wavelet transform (DWT). The CWT is used only for property analysis, and the DWT is practically used in the digital implementation [21].

To introduce the wavelet function, let us first understand the definition of the mother wavelet. The average value of the mother wavelet $\psi(t)$ should be zero, i.e. $\psi(t)$ oscillates and satisfies [22]:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \tag{5}$$

where t denotes time. The mother wavelet $\psi(t)$ in equation (5) can be amplified by a scale a and be displaced by a distance b to obtain the following wavelet function [22]:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a}) \quad a, b \in \Re,\ a \neq 0 \tag{6}$$

where a represents the scale factor and b the translate factor.

For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content, on the other hand, imparts flavor or nuance [17]. For the signal of the repetitive control, the disturbance is embedded in the low-frequency components. Hence, in order to eliminate the non-periodic disturbance, we select the high-frequency components of the signal and remove the low-frequency parts.

Wavelet transform uses the concept of the wavelet function and decomposes the original signal, via its filtering, into two parts: the approximations and the details, a process which is called the wavelet decomposition. The approximations are the high-scale, low-frequency components of the signal while the details are the low-scale, high-frequency components [17]. The parts of the approximations can be further decomposed as shown in Fig. 1, in which S represents the original signal, and

cA_i denotes the coefficient of the approximations at the i-th decomposed level while cD_i denotes the coefficient of the details at the i-th decomposed level, respectively, and $S=cD_1+cD_2+\cdots+cD_i+cA_i$. The wavelet reconstruction (or wavelet inverse transform) refers to the combination of the decomposed components, using either the approximations or details or their mixed version with selected level components, to reconstruct the original signal via a filter. In this study, we choose all components of the details (cD_i), having the property of low-scale and high-frequency, and remove all components of the approximations to reconstruct the output signal in order to cancel the non-periodic low-frequency disturbance.

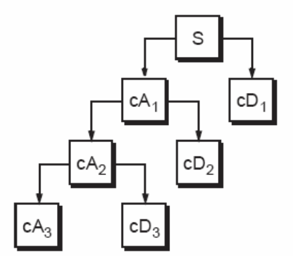

Fig. 1 Illustration of the wavelet decomposition tree (From Matlab: Wavelet Toolbox User's Guide)

Wavelet Function

Wavelet filters includes several families, each with its own property. Each family consists of several wavelet functions with different numbers of vanishing moments [12]. From the literature record, the Haar function [23] was first wavelet function proposed by Alfréd Haar in 1909. Other wavelet functions include the Morlet wavelet [24], Meyer wavelet [25], Biorthogonal wavelet, Mallat wavelet [26], Daubechies wavelet [27], Coiflets wavelet, Symlets wavelet, and Mexican Hat wavelet, etc.

In this study, we utilize the Haar function as the wavelet filter. It is the pioneered wavelet function and the simplest one. Haar wavelet is discontinuous, and resembles a step function. Based on the on-line test as shown in Fig. 5-3 and 5-4, satisfactory results can be obtained using the Haar wavelet with the signal decomposition level up to level 5. In terms of wavelet families, the results of the Haar wavelet are similar to the Daubechies wavelet but the Haar wavelet has faster computation speed. Thus, the following numerical example uses the Haar wavelet with signal decomposition level 5.

The Haar wavelet's mother wavelet can be represented as:

$$\psi(t)=\begin{cases}1 & 0\le t<\frac{1}{2}\\ -1 & \frac{1}{2}\le t<1\\ 0 & \text{otherwise}\end{cases} \tag{7}$$

with corresponding scaling function:

$$\phi(t)=\begin{cases}1 & 0\le t<1\\ 0 & \text{otherwise}\end{cases} \tag{8}$$

and the mother wavelet's graph:

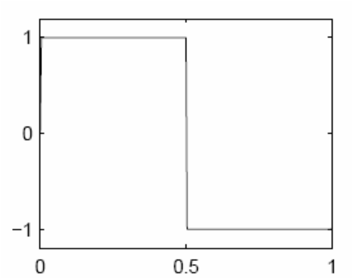

Fig. 2 Haar wavelet's mother wavelet (From Matlab: Wavelet Toolbox User's Guide)

EXAMPLES

To study and illustrate the behavior of the repetitive control laws developed here, we consider a third order transfer function model that was found to represent rather well the input to output behavior of feedback controllers for each axis of a Robotics Research Corporation robot [13]

$$T(s)=\frac{a\omega^2}{(s+a)(s^2+2\zeta\omega s+\omega^2)} \tag{9}$$

where $a=8.8$ corresponding to a break frequency of 1.4 Hz, and $\omega=37$ rad/sec, giving the first robot undamped natural vibration frequency at 5.9 Hz. This system is simple, yet it is sufficiently complex that it is found to be a good test case for studying learning and repetitive control laws. For the robot the damping ratio is $\zeta=0.5$ which is sufficiently fast that there is more than one settling time in a one second period. In order to accentuate the potential difficulties associated with transients crossing from one period to the next, we artificially decrease the damping to $\zeta=0.05$. This makes a time constant of 1.85 sec, and the setting time is often taken as four time constants or 7.4 sec. Thus, transients take nearly seven and a half periods to decay, and it is far from being quasi steady state using reasonable learning gains. The sample rate is taken as 64 Hz, and the closed loop governing equation for the feedback controller is given by the difference equation associated with (9) for this sample rate, assuming a zero order hold on the input. As is common in repetitive control problems, the desired trajectory is zero, and there is a periodic disturbance added to the output, i.e. it is a disturbance rejection problem.

Procedure

First, we convert the transfer function in equation (9) to the state-space form:

$$T(s)=\frac{B(s)}{A(s)}=\frac{b_1 s^{n-1}+\cdots+b_{n-1}s+b_n}{a_1 s^{n-1}+\cdots+a_{n-1}s+a_n} \\ =C(sI-A)^{-1}B+D \tag{10}$$

where $n=1+$ order of transfer function, and then in the controller canonical form:

$$\dot{x}=Ax+Bu \\ y=Cx+Du \tag{11}$$

so as to obtain the parameters A, B, C and D for the transfer function. The flow chart can be depicted in Fig. 3.

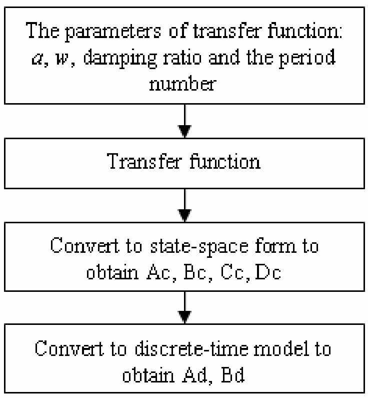

Fig. 3 Flow chart of converting transfer function to state-space

The following process of the repetitive control is to incorporate other parameters into the algorithm, whose complete flow chart is illustrated in Fig. 4.

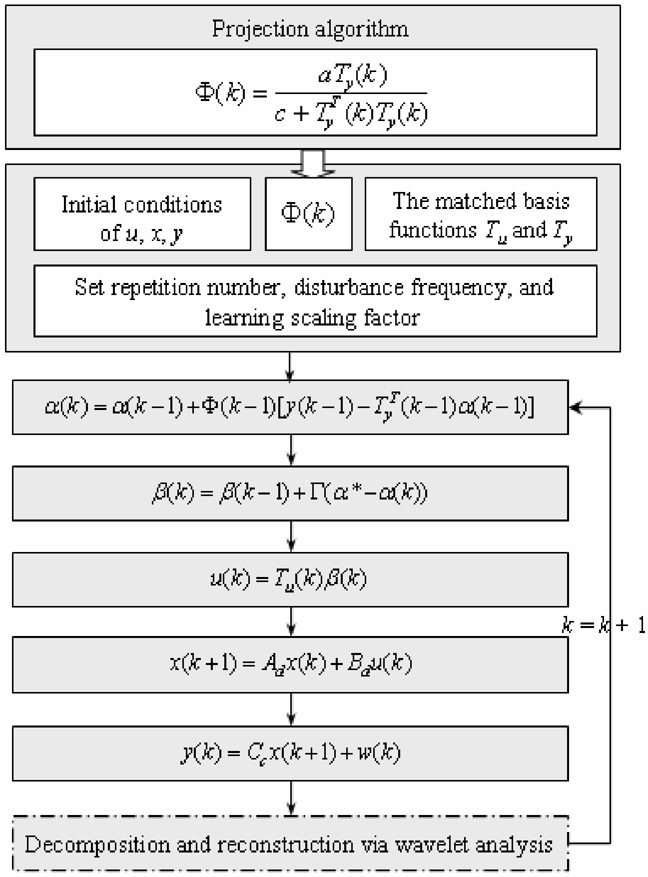

Fig. 4 Complete operational flow chart

For the on-line operation, every loop obtains a predicted trajectory y and, with the desired trajectory y_d , thus the tracking error for each time step k. For the off-line operation, all time steps of the parameter value α are first computed followed by computing all time steps' value for β using available α , and then u and x so as to obtain the predicted output y . It is noteworthy that the Haar wavelet function with signal decomposition level 5 is chosen and the signal reconstruction uses all the components of the details. The wavelet analysis portion in Fig. 4 refers to "Decomposition and reconstruction via wavelet analysis."

Disturbance Rejection using the Projection Algorithm and the Wavelet Filtering

Suppose that there is only one disturbance frequency, a sine wave at 2 Hz with amplitude of 45 (all disturbances treated in the examples are sine waves with this same amplitude). We use input basis functions that are sine and cosine of 2 Hz, and the output basis functions are the associated steady state responses. The results using the projection algorithm are given in Fig. 5-1 to Fig. 5-4. Fig. 5-1 and 5-2 are the results of the off-line projection algorithm and the off-line wavelet analysis. Fig. 5-3 and 5-4 are the results of the on-line projection algorithm and the on-line wavelet analysis. The learning gain Γ is 0.01 time the identity matrix. The other parameters are $c=0$, $a=1$, and the initial ∂ and β are set to zero at the time the repetitive control is turned on after one period. These values apply to all of the projection algorithm results cited here. A gain of 0.01 appears to give the fastest results. Using only the off-line projection algorithm, it takes about 50 repetitions until zero error is reached (a numerical zero). It appears no difference when introducing the off-line wavelet analysis through the comparison of Fig. 5-1 and 5-2. However, when introducing the wavelet analysis to the projection algorithm in on-line operation, the RMS tracking error and the output coefficients error both are more reduced at the expenses of slower convergence (about 80 repetitions until zero error is reached) as shown in Fig. 5-3 and 5-4. Fig. 5-2 and 5-4 give the difference between the current estimate of components on the output basis functions, and the desired output basis function components. The plot takes the absolute value, and then the logarithm, and changing signs of the errors can produce some of the irregularity seen in these curves.

The off-line repetitive control is used purely for software test and in practical machine operation it must be on-line. Thus the following example gives the on-line real time operation and compares the results of the projection algorithm with and without the wavelet analysis.

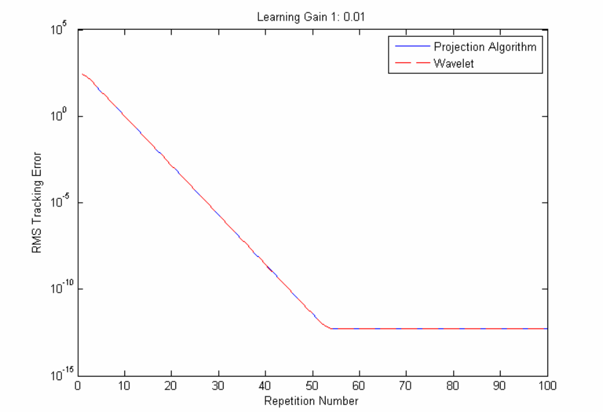

Fig. 5-1 RMS tracking error. A single sine disturbance at 2 Hz. Repetitive control using only 2 Hz basis functions. (with off-line wavelet analysis)

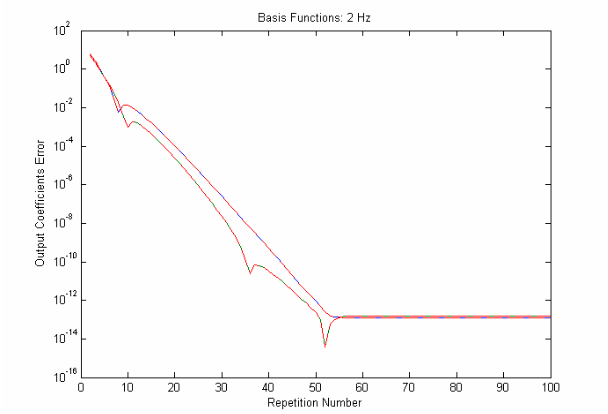

Fig. 5-2 Output coefficients error. A single sine disturbance at 2 Hz. Repetitive control using only 2 Hz basis functions. (with off-line wavelet analysis)

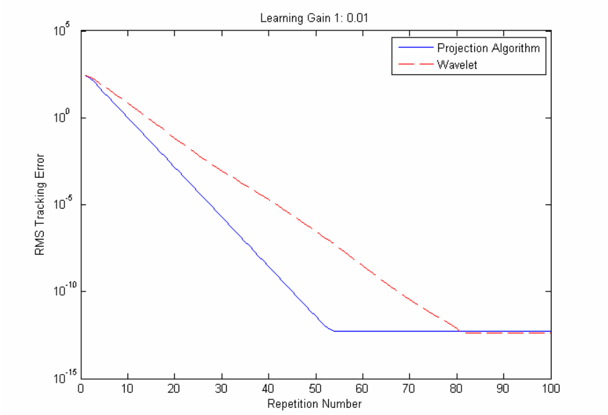

Fig. 5-3 RMS tracking error. A single sine disturbance at 2 Hz. Repetitive control using only 2 Hz basis functions. (with on-line wavelet analysis)

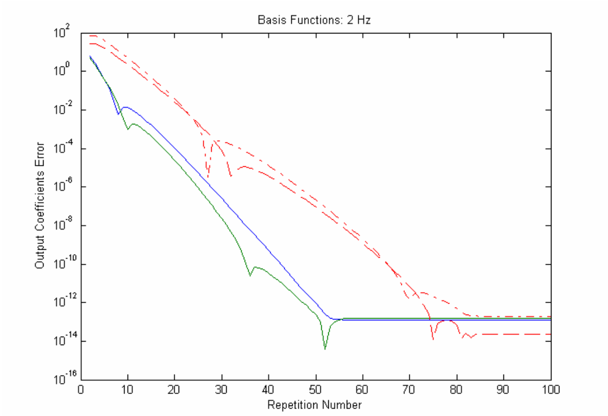

Fig. 5-4 Output coefficients error. A single sine disturbance at 2 Hz. Repetitive control using only 2 Hz basis functions. (with on-line wavelet analysis)

Multiple Learning Gains for Multiple Basis Functions

Now suppose that there are two sinusoidal disturbances, one at 2 Hz and one at 4 Hz. Fig. 6-1 and 6-2 learns both the 2 Hz and the 4 Hz signal simultaneously, but it uses separate projection algorithm equations for each, and it uses separate learning control equations for each. The learning gains are optimized for best performance. The results show that it is quite beneficial to use different learning gains for different basis functions. It appears that if you find the right gains, that there is no penalty for learning more than one frequency at a time. The convergence is again reached around 60 repetitions. After the addition of the wavelet as shown in Fig. 6-1, the converged steady state of the RMS tracking error spears more stable than the pure projection algorithm though with slower convergence rate (about 80 repetitions). In addition, the output coefficients error is more reduced as shown in Fig. 6-2.

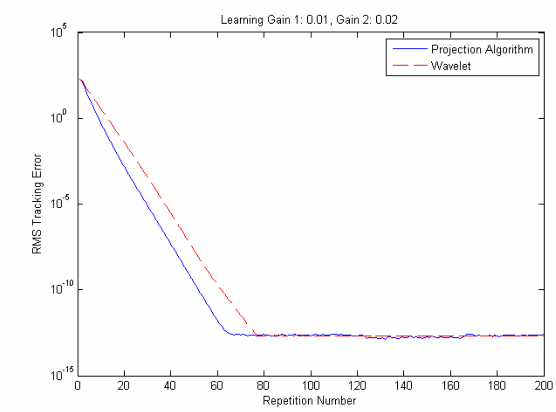

Fig. 6-1 RMS tracking error. Disturbances at both 2 and 4 Hz. Repetitive control using decoupled projection algorithms for both frequencies, with learning gain 0.01 for 2 Hz and 0.02 for 4 Hz.

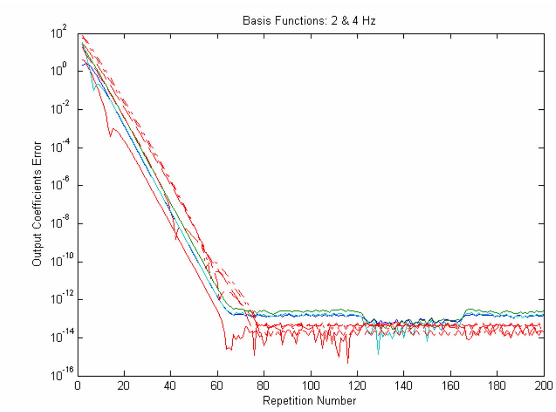

Fig. 6-2 Output coefficients error. Disturbances at both 2 and 4 Hz. Repetitive control using decoupled projection algorithms for both frequencies, with learning gain 0.01 for 2 Hz and 0.02 for 4 Hz.

CONCLUSIONS

This paper has presented a set of repetitive control methods to eliminate the effects of repetitive disturbances in feedback control systems, or to eliminate tracking errors in feedback controllers executing periodic commands. Examples show that the methods are effective in eliminating periodic disturbances. And after introducing the wavelet analysis on-line, the overall performance improves in terms of accuracy at the expenses of slower convergence. Methods are developed to allow one to predict stability of the repetitive control process before turning on the system, and to set the learning gains and other repetitive control parameters a *priori*. Numerical experience indicates that for fast convergence when there are multiple basis functions, it is best to adjust the learning gain associated with each basis function differently rather than use an overall scalar learning gain. In the numerical experiments these were adjusted separately for disturbances at each frequency, and then applied to the system with multiple disturbance frequencies. This adjustment could be done experimentally as well, differencing the results from runs to eliminate the true disturbances, and trying to eliminate a mathematically added disturbance on the output of the frequency of interest.

The use of basis functions in learning control was seen to have advantages in preventing problems of long term instability. Here we apply it in repetitive control, and do so in real time instead of using batch processing. The resulting repetitive control laws make use of matched basis functions, that are experimentally determined, and the control law does not need any additional information such as a difference equation model. One expects these control laws to work when the learning is done slowly so that all signals are quasi steady state.

Concerning the choices of how to find the components of the output on the output basis functions in real time, the projection algorithm worked well. The wavelet analysis superimposed on the projection algorithm is more effective. Possible future research direction is to formulate the signal reconstruction in the wavelet filtering when selecting the components of the decomposed signal. The entailed stability analysis could be addressed.

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