Cyclodissipativity and Power Factor Improvement for Full Nonlinear Loads

Dunstano del Puerto-Flores, Romeo Ortega, and Jacquelien M. A. Scherpen

Abstract— **In recent research, a cyclodissipativity characterization of the problem of power factor compensation (PFC) for nonlinear loads with non-sinusoidal source voltage has been presented. Using this characterization the classical capacitor and inductor compensators can be interpreted in terms of energy equalization. This brief note focuses on the extension of this approach. In particular, one result is to show that power factor compensation is equivalent to a new cyclodissipativity condition. Another result is to consider general lossless linear filters as compensators and to show that the power factor is improved if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the nonlinear load is ensured.**

I. INTRODUCTION

In Electrical engineering, a classical problem is optimizing energy transfer from an alternating current (ac) source to a load. The power factor, defined as the ratio between the real or the active power (average of the instantaneous power) and the apparent power (the product of rms values of the voltage and current), captures the energy transmission efficiency for a given load, [1]. The standard approach to improve the power factor is to place a lossless compensator between the source and the load.

The task of designing compensators that aim at improving the power-factor (PF) for nonlinear time-varying loads operating in non-sinusoidal regimes is far from clear. Most of the approaches used to improve PF are based on *ad-hoc* definitions of reactive power, [2], and a lack of consensus on these definitions produces misunderstanding of power phenomena in circuits with nonsinusoidal voltages and currents.

Recently, in [3] a new framework for analysis and design of (possibly nonlinear) PF compensators for electrical systems operating in non-sinusoidal (but periodic) regimes with nonlinear time-varying loads was presented. This framework proceeds from the aforementioned, universally accepted, definition of PF and does not rely on any axiomatic definition of reactive power. It is shown that PF is improved if and only if the compensated system satisfies a certain cyclodissipativity property, [4] [5]. The supply rate in [3] depends explicitly on the load, but unfortunately the load is typically unknown. Hence, the result may not be used for compensator synthesis. One contribution of our work is the proof that PF improvement can also be characterized in terms of a new cyclodissipativity property where the supply rate is independent of the load and is solely determined by the compensator.

In [3] the case of LTI capacitive or inductive compensation was studied, showing that PF improvement is equivalent to energy equalization. In [6] we have studied the concept of weighted real power and showed that the power factor by general lossless LTI filters is improved if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. However, in [6] we have assumed that the resistors in the load are linear. Here we extend the result to the case where the resistor are nonlinear.

J.M.A. Scherpen and D. del Puerto-Flores are with the Deparment of Discrete Technology and Production Automation, Rijksuniversiteit Groningen, Nijenborgh 4, 9747 AG, Groningen, The Netherlands. (e-mail:{j.m.a.scherpen,d.del.puerto.flores}@rug.nl)

R. Ortega is with the Laboratoire des Signaux et Systèmes, SUPELEC, Plateau de Moulon, 91192, Gif-sur-Yvette, cedex, France. (e-mail: ortega@lss.supelec.fr)

II. A CYCLODISSIPATIVITY CHARACTERIZATION OF POWER-FACTOR COMPENSATION

This section introduces the identification of the key role played by cyclodissipativity in PF compensation.

A. Framework

We consider the energy transfer from an n-phase ac generator to a load, see Figure 1. The voltage and current of the source are denoted by the column vectors $v_s(t), i_s(t) \in \mathbb{R}^n$ and the load is described by a (possibly nonlinear and time varying) n-port network $\mathfrak{N}$. We make the following assumptions.

Assumption 1: All signals are assumed to be periodic and have finite power, that is, they belong to

$$\mathcal{L}_2^n = \left\{ x : [0,T) \to \mathbb{R}^n : \|x\|^2 := \frac{1}{T}\int_0^T |x(\tau)|^2 d\tau < \infty \right\}$$

where $|\cdot|$ is the Euclidean norm. We also define the inner product in $\mathcal{L}_2^n$ as

$$\langle x, y\rangle := \frac{1}{T}\int_0^T x^\top(t) y(t) dt.$$

Assumption 2: The source is ideal in the sense that v_s remains unchanged for all loads Y_ℓ.

The universally accepted definition of PF is given as [1]:

Definition 1: The PF of the source is defined by

$$PF := \frac{P}{S}, \tag{1}$$

where

$$P := \langle v_s, i_s\rangle, \tag{2}$$

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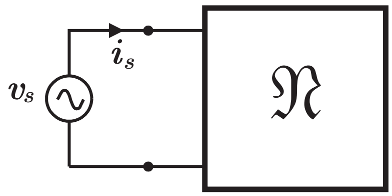

Fig. 1. Illustrating power delivered to a (possibly nonlinear and time varying) load from an n-phase ac ideal generator.

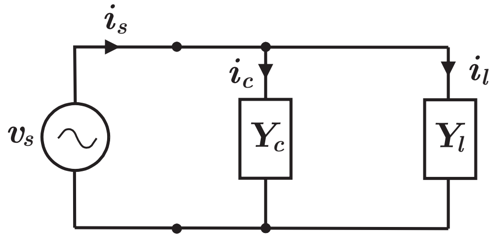

Fig. 2. Schematic diagram of shunt PF compensation configuration.

is the active (real) power,[1] and $S := \|v_s\|\|i_s\|$ is the apparent power.

Under Assumption 2, the apparent power S is the highest average power delivered to the load among all loads that have the same rms current $\|i_s\|$. From (1) and the Cauchy–Schwartz inequality, it follows that $P \leq S$. Hence $PF \in [-1, 1]$ is a dimensionless measure of the energy-transmission efficiency. Cauchy–Schwartz also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that v_s and i_s are collinear. If this is not the case, $P < S$ and compensation schemes are introduced to maximize the PF.

B. The Power-Factor compensation problem

The PF compensation configuration considered in the paper is depicted in Figure 2, where $Y_c, Y_\ell : \mathcal{L}_2^n \to \mathcal{L}_2^n$ are the admittance operators of the compensator and the load, respectively. That is,

$$i_c = Y_c(v_s) \quad i_\ell = Y_\ell(v_s)$$

where i_c, $i_\ell \in \mathcal{L}_2^n$, are the compensator and load currents, respectively. In the simplest LTI case the operators Y_c, Y_ℓ can be described by their admittance transfer matrices, which we denote by $\hat{Y}_c(s)$, $\hat{Y}_\ell(s) \in \mathbb{R}^{n\times n}(s)$, respectively, where s represents the complex frequency variable $s = j\omega$.

The uncompensated PF, that is, the value of PF when $Y_c = 0$, is clearly given by

$$PF_u := \frac{\langle v_s, i_s\rangle}{\|v_s\|\|i_s\|}. \tag{3}$$

Following standard practice, we consider only lossless compensators, that is,

$$\langle Y_c(v_s), v_s\rangle = 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{4}$$

We recall that, if Y_c is LTI, this is equivalent to

$$\mathrm{Re}\{\hat{Y}_c(j\omega)\} = 0. \tag{5}$$

[1] Also called average power [7].

where $\mathrm{Re}\{\hat{Y}_c(j\omega)\}$ is the real part of the admittance transfer matrix $\hat{Y}_c(j\omega)$.

C. Power-Factor compensation and cyclodissipativity

Dissipativity provides us with a useful tool for the analysis of nonlinear systems, which relates nicely to Lyapunov and $\mathcal{L}_2$ stability, [8], [9], [10]. In accordance with physical concepts, a system is called dissipative if it does not produce energy, in some abstract sense. Typical examples of dissipative systems are: passive electrical networks, mechanical systems, viscoelastic materials, etc.

The concept of cyclodissipativity is inspired by the fact that cyclodissipative systems exhibit a dissipative behavior in cyclic motions. As explained in [4], cyclodissipativity is understood here in terms of the available generalized energy. The idea is borrowed from thermodynamics, where the notion is formulated in a conceptually clearer manner than in circuits and systems theory. Thermodynamical systems define cyclodissipative systems as do, for example, less "physical" systems as electrical systems with positive resistors and capacitors and inductors with either sign.

Definition 2: Given a mapping $w : \mathcal{L}_2^n \times \mathcal{L}_2^n \to \mathbb{R}$. The n-port system of Figure 1 is cyclo–dissipative with respect to the supply rate $w(v_s, i_s)$ if and only if

$$\int_0^T w(v_s(t), i_s(t))dt > 0. \tag{6}$$

for all $(v_s, i_s) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$.

Remark 1: In words, a system is cyclodissipative when it can not create (abstract) energy over closed paths in the state-space. It might, however, produce energy along some initial portion of such a trajectory; if so, it would not be dissipative.

To place or results in context, and make the paper self–contained, we recall the following results from [3].

Proposition 3: Consider the system of Figure 2 with fixed Y_ℓ. The compensator Y_c improves the PF if and only if the system is cyclo–dissipative with respect to the supply rate

$$w(v_s, i_s) := (Y_\ell(v_s) + i_s)^\top (Y_\ell(v_s) - i_s). \tag{7}$$

Proof: From Kirchhoff's current law $i_s = i_c + i_\ell$, the relation $i_c = Y_c(v_s)$, and the lossless condition (4), it follows that $\langle v_s, i_s\rangle = \langle v_s, i_\ell\rangle$. Consequently, (1) becomes

$$PF = \frac{\langle v_s, i_\ell\rangle}{\|v_s\|\|i_s\|}. \tag{8}$$

Comparing the equation above with (3) we conclude that $PF > PF_u$ if and only if

$$\|i_s\|^2 < ||i_\ell||^2 = \|Y_\ell(v_s)\|^2, \tag{9}$$

where we used $i_\ell = Y_\ell(v_s)$ for the right hand side identity. Finally, note that (6) with (7) is equivalent to (9), which yields the desired result. ■

Corollary 4: Consider the system of Figure 2 Then Y_c improves the PF for a given Y_ℓ if and only if Y_c satisfies

$$2\langle Y_\ell(v_s), Y_c(v_s)\rangle + \|Y_c(v_s)\|^2 < 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{10}$$

Dually, given Y_c, the PF is improved for all Y_ℓ that satisfy (10).

Proof: Substituting $i_s = (Y_\ell + Y_c)(v_s)$ in (9) yields (10). ■

Remark 2: The key advantage of cyclodissipativity is that it restricts the set of inputs of interest to those generate periodic solutions (a feature that is intrinsic in PF compensation problems) it furthermore deals with "abstract" energies.

III. A New Cyclo–Dissipativity Condition for Power-Factor Compensation

Unfortunately, the supply rate in [3] depends explicitly on the load, which is typically unknown. Hence, the result may not be used for compensator synthesis. One contribution of our work is the proof that PF improvement can also be characterized in terms of a new cyclodissipativity property where the supply rate is independent of the load and is solely determined by the compensator.

Proposition 5: Consider the system of Figure 2 with fixed Y_c. The PF is improved for all Y_ℓ such that the system is cyclo–dissipative with respect to the supply rate

$$w(v_s, i_s) := (Y_c(v_s))^2 - 2i_s^\top Y_c(v_s). \tag{11}$$

Proof: We have shown above that $PF > PF_u$ if and only if $||i_s||^2 < ||i_\ell||^2$. Using the fact that $i_s = i_c + i_\ell$, the latter inequality can be written as

$$||i_c + i_\ell||^2 < ||i_\ell||^2, \tag{12}$$

which is equivalent to

$$||i_c||^2 + 2\langle i_c, i_\ell\rangle < 0. \tag{13}$$

Substituting $i_\ell = i_s - i_c$ in (13) yields

$$||i_c||^2 - 2\langle i_c, i_s\rangle > 0. \tag{14}$$

The proof is completed replacing $i_c = Y_c v_s$. ■

The supply rate (7) depends on Y_ℓ that is usually unknown. Hence, the result of Proposition 3 can only be used for analysis of a given known load—as done in [11] for a TRIAC controlled rectifier. On the other hand, the supply rate (11) depends on Y_c, that is to be designed. Current research is under way to exploit this new cyclo–dissipativity property to synthesize PF compensators.

IV. Weighted Power Equalization and Power Factor Compensation for RLC Loads

In this section we extend Proposition 5 in [3], where the PF compensators are assumed to be capacitors or inductors, to general lossless LTI filters. Similarly to [3], we assume that the load is a nonlinear RLC circuit consisting of lumped dynamic elements (n_L inductors, n_C capacitors) and static elements (n_R resistors). Capacitors and inductors are defined by the physical laws and constitutive relations [7]:

$$i_C = \dot{q}_C, \quad v_C = \nabla H_C(q_C), \tag{15}$$

$$v_L = \dot{\phi}_L, \quad i_L = \nabla H_L(\phi_L), \tag{16}$$

respectively, where i_C, v_C, $q_C \in \mathbb{R}^{n_C}$ are the capacitors currents, voltages and charges, and $i_L, v_L, \phi_L \in \mathbb{R}^{n_L}$ are the inductors currents, voltages and flux–linkages, $H_L : \mathbb{R}^{n_L} \to \mathbb{R}$ is the magnetic energy stored in the inductors, $H_C : \mathbb{R}^{n_C} \to \mathbb{R}$ is the electric energy stored in the capacitors, and ∇ is the gradient operator. We assume that the energy functions are twice differentiable and for linear capacitors and inductors,

$$H_C(q_C) = \frac{1}{2} q_C^\top C^{-1} q_C,\ H_L(\phi_L) = \frac{1}{2}\phi_L^\top L^{-1}\phi_L,$$

respectively, with $L \in \mathbb{R}^{n_L \times n_L}$, $C \in \mathbb{R}^{n_C \times n_C}$. To avoid cluttering the notation we assume L, C are diagonal matrices. Finally, we distinguish between two sets of nonlinear static resistors: n_{R_i} current–controlled resistors and n_{R_v} voltage–controlled resistors, for which the characteristics are given by the following one-to-one real-valued functions:

$$v_{R_i} = \hat{v}_{R_i}(i_{R_i}), \tag{17}$$

and

$$i_{R_v} = \hat{i}_{R_v}(v_{R_v}), \tag{18}$$

respectively, where i_{R_i}, $v_{R_i} \in \mathbb{R}^{n_{R_i}}$ are the currents, voltages of the current-controlled resistors, and $i_{R_v}, v_{R_v} \in \mathbb{R}^{n_{R_v}}$ are the currents, voltages of the voltage-controlled resistors, with $n_R = n_{R_i} + n_{R_v}$.

Recalling the definition of real power (2) we introduce the following.

Definition 6: Given a compensator admittance Y_c the weighted (real) power of a single–phase circuit with port variables $(v, i) \in \mathcal{L}_2 \times \mathcal{L}_2$ is given by

$$P^w := \langle Y_c(v), i\rangle. \tag{19}$$

If Y_c is LTI

$$P^w = \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}[k]\hat{I}^*[k] \tag{20}$$

where $\hat{V}[k], \hat{I}[k]$ are the k-th spectral lines of v and i, respectively, and $\hat{Y}_c[k] := \hat{Y}_c(k\omega_0)$, with $\omega_0 := \frac{2\pi}{T}$. That is, P^w is the sum of the power components of the circuit modulated by the frequency response of Y_c—hence the use of the "weighted" qualifier.[2]

The aforementioned definition motivates the next lemma.

Lemma 1: Consider a nonlinear time invariant (TI) current-controlled {voltage-controlled} one-port resistor characterized by (17) {(18)} and a fixed LTI lossless compensator Y_c with $n = 1$. Let $\hat{Y}_c(j\omega)$ denote the associated admittance transfer function. If $\hat{Y}_c(j\omega)$ has a zero at the origin, then the weighted averaged power along periodic trajectories satisfies

$$P^w_{R_i} := \langle Y_c v_{R_i}, i_{R_i}\rangle = 0, \tag{21}$$

$\{P^w_{R_v} := \langle Y_c v_{R_v}, i_{R_v}\rangle = 0\}$ for all admissible pair $(v_{R_i}, i_{R_i}) \in \mathcal{L}_2 \times \mathcal{L}_2$ $\{(v_{R_v}, i_{R_v}) \in \mathcal{L}_2 \times \mathcal{L}_2\}$, and for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of $\hat{Y}_c(j\omega)$.

[2] Since the spectral lines of real signals satisfy $\hat{F}[-k] = \hat{F}^*[k]$, the weighted power is a real number.

Proof: From the Foster's reactance theorem, see [12] and [13], the impedance function of LTI lossless can be written in the form

$$\hat{Z}(s) = \frac{g(s^2+\omega_{z_1}^2)(s^2+\omega_{z_2}^2)\cdots}{s(s^2+\omega_{p_1}^2)(s^2+\omega_{p_2}^2)\cdots},$$

where $g > 0$ and $0 \leq \omega_{z_1} < \omega_{p_1} < \omega_{z_2} < \omega_{p_2} \cdots$. Furthermore, ω_{z_1} can be zero or not depending upon whether $\hat{Z}(s)$ has a zero or a pole at the origin. We have that $\hat{Y}_c(s) = \frac{1}{\hat{Z}_c(s)}$. Since Y_c admits a factorization $Y_c = Y_{c_1}(Y_{c_2})$, then

$$\langle i_{R_i}, Y_c v_{R_i}\rangle = \langle i_{R_i}, Y_{c_1}(Y_{c_2} v_{R_i})\rangle,$$
$$\langle i_{R_v}, Y_c v_{R_v}\rangle = \langle i_{R_v}, Y_{c_1}(Y_{c_2} v_{R_v})\rangle = \langle i_{R_v}, Y_{c_2}(Y_{c_1} v_{R_v})\rangle,$$

where we used the fact that Y_{c_1} and Y_{c_2} commute[3]. For a lossless n-ports we have that Y_c is skew Hermitian, i.e., $\hat{Y}_c(s)+\hat{Y}_c^*(s) = 0$ for all $s = j\omega$, where Y_c^* is the adjoint (or the conjugate transpose) of Y_c, see [13] and [14]. Consider the case of the nonlinear TI current-controlled resistor. By the assumption that $\hat{Y}_c(s)$ has a zero at the origin, we define $\hat{Y}_{c_1}(s) = s$ and thus we have

$$\langle i_{R_i}, Y_c v_{R_i}\rangle = \langle Y_{c1}^* i_{R_i}, Y_{c2} v_{R_i}\rangle,$$

Since Y_{c_1} is skew-Hermitian, and $Y_{c_1} = \frac{d}{dt}$, then the last expression become

$$\langle i_{R_i}, Y_c v_{R_i}\rangle = -\left\langle \frac{di_{R_i}}{dt}, Y_{c2}\hat{v}_{R_i}(i_{R_i})\right\rangle, \tag{22}$$

and the right-hand side of (22) can be written as

$$\left\langle \frac{di_{R_i}}{dt}, Y_{c2}\hat{v}_{R_i}(i_{R_i})\right\rangle = \frac{1}{T}\int_0^T (Y_{c2}\hat{v}_{R_i}(i_{R_i}))\frac{di_{R_i}}{dt}dt.$$

By substitution, we obtain

$$\left\langle \frac{di_{R_i}}{dt}, Y_{c2}\hat{v}_{R_i}(i_{R_i})\right\rangle = \frac{1}{T}\int_{i_{R_i}(0)}^{i_{R_i}(T)} Y_{c2}\hat{v}_{R_i}(i_{R_i})di_{R_i}.$$

Since the input is periodic with period T, i.e., $i_{R_i}(0) = i_{R_i}(T)$, then the inner product (22) is zero. The convolution $Y_{c2}\hat{v}_{R_i}(i_{R_i})$ is also periodic with period T in steady state, see Theorem 4.1.2 in [15], and the existence and uniqueness of the composition can be proved by Volterra serie, see Theorem 3.2.1 in [15]. An analogous result holds for the case of the nonlinear TI voltage-controlled resistor, i.e., $\left\langle Y_{c_2}\hat{i}_{R_v}(v_{R_v}), \frac{dv_{R_v}}{dt}\right\rangle = 0$. ■

In the previous lemma, the assumption that $\hat{Y}(j\omega)_c$ had a zero at the origin was necessary condition for the result to hold for the case of nonlinear resistor. In particular, where the resistor is linear[4] it can be removed.

Corollary 7: Consider a linear TI one-port resistor and a fixed LTI lossless compensator Y_c with $n = 1$. Let $\hat{Y}_c(j\omega)$ denote the associated admittance transfer function. Then the weighted averaged power along periodic trajectories satisfies

$$P_{R_i}^w := \langle Y_c v_{R_i}, i_{R_i}\rangle = 0, \tag{23}$$

$\{P_{R_v}^w := \langle Y_c v_{R_v}, i_{R_v}\rangle = 0\}$ for all admissible pair $(v_{R_i}, i_{R_i}) \in \mathcal{L}_2 \times \mathcal{L}_2$ $\{(v_{R_v}, i_{R_v}) \in \mathcal{L}_2 \times \mathcal{L}_2\}$, and for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of $\hat{Y}_c(j\omega)$.

Proof: From Parseval's theorem, we have

$$\begin{aligned}\langle Y_c v_{R_i}, i_{R_i}\rangle &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{I}_{R_i}(-j\omega)\hat{Y}_c(j\omega)\hat{V}_{R_i}(j\omega)d\omega \\ &= \frac{R}{2\pi}\int_{-\infty}^{\infty}\mathrm{Im}\{\hat{Y}_c(j\omega)\}|\hat{I}_{R_i}(j\omega)|^2 d\omega\end{aligned}$$

where the second identity is equal to zero, given the fact that $\mathrm{Im}\{\hat{Y}_c(j\omega)\}$ is an odd function of ω and $\mathrm{Re}\{\hat{Y}_c(j\omega)\} = 0$ for almost all ω. An analogous result can be obtained for the weighted averaged power $P_{R_v}^w$ by using the relationship $\hat{I}_{R_v} = G\hat{V}_{R_v}$. ■

Proposition 8: Consider the system of Figure 2 with $n = 1$,[5] a full nonlinear RLC load and a fixed LTI lossless compensator Y_c with admittance transfer function $\hat{Y}_c(j\omega)$ which has a zero at the origin.

i) PF is improved if and only if

$$\frac{1}{2}V_s^w + \sum_{q=1}^{n_L} P_{L_q}^w + \sum_{q=1}^{n_C} P_{C_q}^w < 0 \tag{24}$$

where V_s^w is the rms value of the filtered voltage source, that is,

$$V_s^w := ||Y_c v_s||^2 = \sum_{k=1}^{\infty}|\hat{Y}_c(k)\hat{V}_s(k)|^2$$

and

$$\begin{aligned}P_{C_q}^w &:= \sum_{k=-\infty}^{\infty}\hat{Y}_c[k]\hat{V}_{C_q}[k]\hat{I}_{C_q}^*[k] \\ P_{L_q}^w &:= \sum_{k=-\infty}^{\infty}\hat{Y}_c[k]\hat{V}_{L_q}[k]\hat{I}_{L_q}^*[k],\end{aligned}$$

are the weighted powers of the q–th capacitor and inductor, respectively.

ii) Condition (24) may be equivalently expressed as

$$\left\langle (\frac{1}{p}Y_c)v_L, \nabla^2 H_L v_L\right\rangle - \left\langle i_C, (\frac{1}{p}Y_c)\nabla^2 H_C i_C\right\rangle > \frac{1}{2}V_s^w \tag{25}$$

where $p := \frac{d}{dt}$.

iii) If the capacitors and inductors are linear their weighted powers become

$$\begin{aligned}P_{C_q}^w &:= 2\omega_0\sum_{k=1}^{\infty}\left\{k\ \mathrm{Im}\{\hat{Y}_c[k]\}\sum_{q=1}^{n_C}C_q|\hat{V}_{C_q}[k]|^2\right\} \\ P_{L_q}^w &:= -2\omega_0\sum_{k=1}^{\infty}\left\{k\ \mathrm{Im}\{\hat{Y}_c[k]\}\sum_{q=1}^{n_L}L_q|\hat{I}_{L_q}[k]|^2\right\}.\end{aligned} \tag{26}$$

[3]Since Y_{c_1} and Y_{c_2} are two continuous, linear, time-invariant operators, then there is an invertible operator S such that $SY_{c_1}S^{-1} = H_1$ and $SY_{c_2}S^{-1} = H_2$, where H_1 and H_2 denote multiplication operators. Since H_1 and H_2 commute, then Y_{c_1} and Y_{c_2} commute

[4]A linear resistor is both current- and voltage controlled and is represented by $u_{R_i} = Ri_{R_i}$ (Ohm's law), where R is the resistance, or, similarly, $i_{R_v} = Gv_{R_v}$, where $G(= R^{-1})$ is the conductance.

[5]This condition is imposed, without loss of generality, to simplify the presentation of the result.

where $\mathrm{Im}\{\hat{Y}_c[k]\}$ is the imaginary part of the admittance transfer function $\hat{Y}_c[k]$.

iv) Furthermore, the results i-iii can be extended for a general LTI lossless compensator, if the resistors of the load are linear time-invariants.

Proof: Corollary 4 shows that the PF is improved if and only if (10) holds, which may be equivalently expressed as

$$||Y_c v_s||^2 + 2\langle Y_c v_s, i_\ell\rangle < 0.$$

Applying the generalized form of Tellegen's theorem to the RLC load one gets

$$i_\ell^\top Y_c v_s = i_{R_v}^\top Y_c v_{R_v} + i_{R_i}^\top Y_c v_{R_i} + i_L^\top Y_c v_L + i_C^\top Y_c v_C,$$

see [16], which upon integration yields

$$\langle i_\ell, Y_c v_s\rangle = \langle i_L, Y_c v_L\rangle + \langle i_C, Y_c v_C\rangle \tag{27}$$

where we have used the fact that, because of Lemma (1), $\langle i_{R_v}, Y_c v_{R_v}\rangle = 0$ and $\langle i_{R_i}, Y_c v_{R_i}\rangle = 0$ for nonlinear LTI resistors.

Then, Condition (24) is obtained directly from Definition 6.

Now,

$$\begin{aligned}\langle i_L, Y_c v_L\rangle &= \left\langle \nabla H_L, Y_c \dot{\phi}_L\right\rangle \\ &= -\left\langle \nabla^2 H_L v_L, (\frac{1}{p} Y_c) v_L\right\rangle,\end{aligned}$$

where the first identity follows from the relations (16)and the second uses the well–known property of periodic functions $< f, \dot{g} >= - < \dot{f}, g >$. Similar derivations with the term $\langle i_C, Y_c v_C\rangle$ yield (25).

To prove iii) we use (20), the basic relations for LTI inductors and capacitors

$$\hat{I}_{C_q}[k] = jk\omega_0 C_q \hat{V}_{C_q}[k], \quad \hat{V}_{L_q}[k] = jk\omega_0 L_q \hat{I}_{L_q}[k],$$

and the fact that Y_c satisfies (5).

Finally, the proof of iv) follows directly from Corollary 7. ■

Remark 3: Condition (24) indicates that the PF will be improved if and only if the overall weighted power (supplied plus stored) is negative.

Remark 4: From (25) (or replacing (26) in (24)) we see that PF improvement is equivalent to average power equalization between inductors and capacitor—notice the minus signs—with the gap being determined by the weighted supplied power.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, extensions to the analysis of power factor compensation of nonliear loads based on cyclodissipativity were presented. First, power factor compensation and a new cyclodissipativity condition were shown to be equivalent. Secondly, we have studied the concept of weighted (real) power and showed that the power factor by general LTI compensators is improved if and only if a certain equalization condition between the weighted powers of compensator and load is ensured. Furthermore, we extended the result to the case where the resistors are nonlinear.

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