Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2010, Article ID 405321, 18 pages
doi:10.1155/2010/405321

*Research Article*

# A Class of Impulsive Pulse-Width Sampler Systems and Its Steady-State Control in Infinite Dimensional Spaces

**JinRong Wang,[1] W. Wei,[1] and YanLong Yang[2]**

[1] *College of Science, Guizhou University, Guiyang, Guizhou 550025, China*
[2] *College of Technology of Guizhou University, Guiyang, Guizhou 550004, China*

Correspondence should be addressed to JinRong Wang, wjr9668@126.com

Received 11 November 2009; Accepted 25 January 2010

Academic Editor: Paul Eloe



This paper investigates a class of impulsive pulse-width sampler systems and its steadystate control in the infinite dimensional spaces. Firstly, some definitions of pulse-width sampler systems with impulses are introduced. Then applying impulsive evolution operator and fixed point theorem, some existent results of steady-state of infinite dimensional linear and semilinear pulse-width sampler systems with impulses are obtained. An example to illustrate the theory is presented in the end.

## 1. Introduction

In the design of distributed parameter control systems, one of the important problems is to choose controller and actuator. As the dimension of an industrial controller in actual applications is finite, it restricts us to consider the distributed parameter system with a finite dimensional output. In industrial process control systems, on-off actuators have been in engineer's good graces because of the cheap price and the high reliability.

The interest in the pulse-width sampler control systems was aroused as early as 1960s. It was motivated by applications to engineering problems and neural nets modeling. In modern times, the development of neurocomputers promises a rebirth of interest in this field. The theory of pulse-width sampler control systems is treated as a very important branch of engineering and mathematics. Nevertheless, it can supply a technical-minded mathematician with a number of new and interesting problems of mathematical nature. There are some results such as steady-state control, stability analysis, robust control of pulse-width sampler systems [1–7], integral control by variable sampling based on steady-state data, and adaptive sampled-data integral control [8–11].

On the other hand, in order to describe dynamics of population, subject to abrupt changes as well as other phenomena, such as harvesting, diseases and so forth, some authors have used impulsive differential equations to describe the model since the last century. The reader can refer the basic theory of impulsive differential equations in finite dimensional spaces to Lakshmikantham's book [12]. Meanwhile, the impulsive evolution equations and its optimal control problems on infinite dimensional Banach spaces have been investigated by many authors including Ahmed, Liu, Nieto, and us (see for instance [13–25] and references therein).

However, to our knowledge, the pulse-width sampler systems with impulse on infinite dimensional spaces have not been investigated extensively. In this paper, we first study the following steady-state control of infinite dimensional linear system with impulses

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t) + Cu(t), \quad t \neq \tau_k, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad k = 1, 2, \ldots, \\ z(t) &= K_1 x(t), \end{aligned} \tag{1.1}$$

where the state variable $x(t)$ takes values in a reflexive Banach space $X$, $A$ is the infinitesimal generator of a $C_0$-semigroup $\{T(t), t \geq 0\}$ on the state space X, $f(t) = f \cdot 1(t)$ is $T_0$-periodic step disturbance of the system and $f \in X$. Control variable $u(t) \in \mathbb{R}^q$, the input $C : \mathbb{R}^q \to X$ is a bounded linear operator. There is only one time sequences $\{\tau_k \mid k \in \mathbb{Z}_0^+\}$ satisfing $0 < \tau_1 < \tau_2 < \cdots < \tau_k \cdots$ and $\lim_{k\to\infty}\tau_k = \infty$, $B_k : X \to X$, $0 < \tau_1 < \tau_2 < \cdots < \tau_\delta < T_0$, $\tau_{k+\delta} = \tau_k + T_0$, $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$, $x(\tau_k^+) = \lim_{h\to 0^+} = x(\tau_k + h)$ and $x(\tau_k^-) = x(\tau_k)$represent , respectively the right and left limits of $x(t)$ at $t = \tau_k$. $K_1 : X \to \mathbb{R}^p$ is a given bounded linear operator; $z(t)$ is the $p$ dimensional output of the system (1.1).

We, then, study the following steady-state control of infinite dimensional semilinear system with impulses

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t, x(t)) + Cu(t), \quad t \neq \tau_k, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad k = 1, 2, \ldots, \\ z(t) &= K_1 x(t), \end{aligned} \tag{1.2}$$

where $f : [0, \infty) \times X \to X$ is $T_0$-periodic continuous function.

Suppose that control signal $u(t)$ is the output of the $q$ dimensional pulse-width sampler controller, and $v(t)$ is the input of the $q$ dimensional pulse-width sampler controller, which is the output of some dynamical controller

$$\dot{v}(t) = Jv(t) + K_2 z(t), \tag{1.3}$$

where $J$ is a $q \times q$ matrix, $K_2$ is a $q \times p$ matrix, $J$ is determined by the dynamic characteristics of the controller, and $K_2$ is called the feedback matrix which will be chosen in the latter (see Theorem 3.4 and Theorem 3.8). The output signal $u(t) = (u_1(t), u_2(t), \ldots, u_q(t))^T$ and the input signal $v(t) = (v_1(t), v_2(t), \ldots, v_q(t))^T$ of the pulse-width sampler satisfy the following dynamic relation:

$$u_i(t) = \begin{cases} \operatorname{sign} \alpha_{n_i}, & nT_0 \le t < (n + |\alpha_{n_i}|)T_0, i = 1, 2, \ldots, q; \\ 0, & (n + |\alpha_{n_i}|)T_0 \le t < (n+1)T_0, n = 0, 1, \ldots, \end{cases} \tag{1.4}$$

$$\alpha_{n_i} = \begin{cases} v_i(nT_0), & |v_i(nT_0)| \le 1, i = 1, 2, \ldots, q; \\ \operatorname{sign} v_i(nT_0), & |v_i(nT_0)| \ge 1, n = 0, 1, \ldots, \end{cases} \tag{1.5}$$

where $T_0$ is called the sampling period of the pulse-width sampler which is the same as the period of $f$ and $\tau_k$, $k = 1, 2, \ldots$.

We end this introduction by giving some definitions.

*Definition 1.1.* The closed-loop system (1.1), (1.3)–(1.5) is called *linear pulse-width sampler control system with impulses*. The closed-loop system (1.2), (1.3)–(1.5) is called *semilinear pulse-width sampler control system with impulses*.

*Definition 1.2.* In the closed-loop system (1.1), (1.3)–(1.5) (or system (1.2), (1.3)–(1.5)), the $q$ dimensional vector $\alpha_n = (\alpha_{n_1}, \alpha_{n_2}, \ldots, \alpha_{n_q})^T$ is called the duration ratio of the pulse-width sampler in the $n$th sampling period, $n = 0, 1, \ldots$.

We defined a closed cube $\Omega$ in $\mathbb{R}^q$ as

$$\Omega = \left\{ \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_q)^T \in \mathbb{R}^q \mid |\alpha_i| \le 1, i = 1, 2, \ldots, q \right\}, \tag{1.6}$$

then we have $\alpha_n \in \Omega$, for $n = 0, 1, \ldots$.

*Definition 1.3.* In the closed-loop system (1.1), (1.3)–(1.5) (or system (1.2), (1.3)–(1.5)), if there exists a $q$ dimensional vector

$$\alpha = \left( \alpha_{n_1}, \alpha_{n_2}, \ldots, \alpha_{n_q} \right)^T \in \Omega, \tag{1.7}$$

and a corresponding periodicity rectangular-wave control signal $u(t) = u(t, \alpha)$ defined by

$$u_i(t) = u_i(t, \alpha) = \begin{cases} \operatorname{sign} \ \alpha_i, & nT_0 \le t < (n + |\alpha_{n_i}|)T_0, i = 1, 2, \ldots, q; \\ 0, & (n + |\alpha_{n_i}|)T_0 \le t < (n+1)T_0, n = 0, 1, \ldots. \end{cases} \tag{1.8}$$

such that the closed-loop system (1.1), (1.3)–(1.5) (or system (1.2), (1.3)–(1.5)), has a corresponding $T_0$-periodic trajectory $x(\cdot) = x(\cdot, \alpha) : x(t + T_0, \alpha) = x(t, \alpha)$, $t \ge 0$, then the control signal (1.8) is called the *steady-state control* with respect to the disturbance $f$. The $T_0$-periodic trajectory $x(\cdot)$ is called steady-state corresponding to steady-state control $u(\cdot)$ and the constant vector $\alpha \in \Omega$ of steady-state control (1.8) is called to be a *steady-state duration ratio*.

*Definition 1.4.* In the closed-loop system (1.1), (1.3)–(1.5) (or system (1.2), (1.3)–(1.5)), if there exists some $\alpha \in \Omega$ such that

$$\lim_{n\to\infty} \alpha_n = \alpha, \quad \text{where} \quad \alpha_n = \left(\alpha_{n_1}, \alpha_{n_2}, \dots, \alpha_{n_q}\right)^T, \alpha = \left(\alpha_1, \alpha_2, \dots, \alpha_q\right)^T, \tag{1.9}$$

then system (1.1), (1.3)–(1.5) (or system (1.2), (1.3)–(1.5)), corresponding to the disturbance $f$ is called to be *stead-state stable*.

Further, system (1.1), (1.3)–(1.5) (or system (1.2), (1.3)–(1.5)), corresponding to the perturbation $f$ is called *stead-state stabilizability* if we can choose a suitable $T_0 > 0$ and $K_2$ such that system (1.1), (1.3)–(1.5) (or system (1.2), (1.3)–(1.5)), is stead-state stable.

## 2. Mathematical Preliminaries

Let $\mathrm{L}(X,X)$ denote the space of linear operators from $X$ to $X$, $\mathrm{L}_b(X,X)$ denote the space of bounded linear operators from $X$ to $X$, $\mathrm{L}_b(\mathbb{R}^q,X)$ denote the space of bounded linear operators from $\mathbb{R}^q$ to $X$, and $\mathrm{L}_b(X,\mathbb{R}^p)$ denote the space of bounded linear operators from $X$ to $\mathbb{R}^p$. It is obvious that $\mathrm{L}_b(X,X)$, $\mathrm{L}_b(\mathbb{R}^q,X)$, and $\mathrm{L}_b(X,\mathbb{R}^p)$ is the Banach space with the usual supremum norm.

Define $\widetilde{D} = \{\tau_1, \dots, \tau_\delta\} \subset [0,T_0]$, where $0 < \tau_1 < \tau_2 < \cdots < \tau_\delta < T_0$. We introduce $PC([0,T_0];X) \equiv \{x : [0,T_0] \to X \mid x \text{ is continuous at } t \in [0,T_0] \setminus \widetilde{D},\ x \text{ is continuous from left and has right hand limits at } t \in \widetilde{D}\}$, and $PC^1([0,T_0];X) \equiv \{x \in PC([0,T_0];X) \mid \dot{x} \in PC([0,T_0];X)\}$. Set

$$\|x\|_{PC} = \max\left\{\sup_{t\in[0,T_0]} \|x(t+0)\|,\ \sup_{t\in[0,T_0]} \|x(t-0)\|\right\}, \qquad \|x\|_{PC^1} = \|x\|_{PC} + \|\dot{x}\|_{PC}. \tag{2.1}$$

It can be seen that endowed with the norm $\|\cdot\|_{PC}(\|\cdot\|_{PC^1})$, $PC([0,T_0];X)(PC^1([0,T_0];X))$ is a Banach space.

We introduce the following assumption [H1].

(i) [H1.1 ] $A$ is the infinitesimal generator of a $C_0$-semigroup $\{T(t), t \geq 0\}$ on X with domain $D(A)$.

(ii) [H1.2 ] There exists $\delta$ such that $\tau_{k+\delta} = \tau_k + T_0$.

(iii) [H1.3 ] For each $k \in \mathbb{Z}_0^+$, $B_k \in \mathrm{L}_b(X,X)$ and $B_{k+\delta} = B_k$.

We first recall the homogeneous linear impulsive periodic system

$$\begin{aligned} \dot{x}(t) &= Ax(t), \quad t \neq \tau_k, \\ \Delta x(t) &= B_k x(t), \quad t = \tau_k, \end{aligned} \tag{2.2}$$

and the associated Cauchy problem

$$\begin{aligned} \dot{x}(t) &= Ax(t), \quad t \in [0, T_0] \setminus \widetilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k), \quad k = 1, 2, \ldots, \delta, \\ x(0) &= \overline{x}. \end{aligned} \tag{2.3}$$

If $\overline{x} \in D(A)$ and $D(A)$ is an invariant subspace of $B_k$, using [18, Theorem 5.2.2, page 144], step by step, one can verify that the Cauchy problem (2.3) has a unique classical solution $x \in PC^1([0, T_0]; X)$ represented by $x(t) = S(t, 0)\overline{x}$, where

$$S(\cdot, \cdot) : \Delta = \{(t, \theta) \in [0, T_0] \times [0, T_0] \mid 0 \leq \theta \leq t \leq T_0\} \longrightarrow \mathrm{L}_b(\mathrm{X}, \mathrm{X}) \tag{2.4}$$

given by

$$S(t, \theta) = \begin{cases} T(t - \theta), & \tau_{k-1} \leq \theta \leq t \leq \tau_k, \\ T\left(t - \tau_k^+\right)(I + B_k)T(\tau_k - \theta), & \tau_{k-1} \leq \theta < \tau_k < t \leq \tau_{k+1}, \\ T\left(t - \tau_k^+\right)\left[\coprod_{\theta < \tau_j < t}(I + B_j)T\left(\tau_j - \tau_{j-1}^+\right)\right](I + B_i)T(\tau_i - \theta), & \\ & \tau_{i-1} \leq \theta < \tau_i \leq \cdots < \tau_k < t \leq \tau_{k+1}. \end{cases} \tag{2.5}$$

The operator $\{S(t, \theta), (t, \theta) \in \Delta\}$ is called impulsive evolution operator associated with $\{B_k; \tau_k\}_{k=1}^{\infty}$.

The properties of the impulsive evolution operator, $\{S(t, \theta), (t, \theta) \in \Delta\}$ associated with $\{T(t), t \geq 0\}$ and $\{B_k; \tau_k\}_{k=1}^{\infty}$, are collected here.

**Lemma 2.1** (see [26, Lemma 2.1] [27]). *Let assumption [H1] hold. The impulsive evolution operator $\{S(t, \theta), (t, \theta) \in \Delta\}$ has the following properties.*

(1) *For $0 \leq \theta \leq t \leq T_0$, $S(t, \theta) \in L_b(X, X)$, there exists a $M_{T_0} > 0$ such that* $\sup_{0 \leq \theta \leq t \leq T_0} \|S(t, \theta)\| \leq M_{T_0}$.

(2) *For $0 \leq \theta < r < t \leq T_0$, $r \neq \tau_k$, $S(t, \theta) = S(t, r)S(r, \theta)$.*

(3) *For $0 \leq \theta \leq t \leq T_0$, $n \in Z^+$, $S(t + nT_0, \theta + nT_0) = S(t, \theta)$.*

(4) *For $0 \leq \theta \leq t \leq T_0$, $n \in Z^+$, $S(t + nT_0, 0) = S(t, 0)[S(T_0, 0)]^n$.*

(5) *For $0 \leq \theta < t$, there exists an $M \geq 1$, $\omega \in \mathbb{R}$ such that*

$$\|S(t, \theta)\| \leq M \exp\left\{\omega(t - \theta) + \sum_{\theta \leq \tau_n < t} \ln(M\|I + B_n\|)\right\}. \tag{2.6}$$

The exponential stability of the impulsive evolution operator $\{S(t, \theta), t \geq \theta \geq 0\}$ will be used throughout the paper; we recall them as the following definitions and lemmas.

*Definition 2.2.* $\{S(t,\theta),\ t \geq \theta \geq 0\}$ is called exponentially stable if there exist $K \geq 0$ and $\nu > 0$ such that

$$\|S(t,\theta)\| \leq K e^{-\nu(t-\theta)}, \quad t > \theta \geq 0. \tag{2.7}$$

Assumption [H2]: $\{T(t), t \geq 0\}$ is exponentially stable, that is, there exist $K_0 > 0$ and $\nu_0 > 0$ such that

$$\|T(t)\| \leq K_0 e^{-\nu_0 t}, \quad t > 0. \tag{2.8}$$

Two important criteria for exponential stability of a $C_0$-semigroup are collected here.

**Lemma 2.3** (see [26, Lemma 2.4]). *Assumptions [H1] and [H2] hold. There exists* $0 < \lambda < \nu_0$ *such that*

$$\prod_{k=1}^{\delta} (K_0 \|I + B_k\|) e^{-\lambda T_0} < 1. \tag{2.9}$$

*Then* $\{S(t,\theta), t \geq \theta \geq 0\}$ *is exponentially stable.*

**Lemma 2.4** (see [26, Lemma 2.5]). *Assume that assumption [H1] holds. Suppose*

$$0 < \mu_1 = \inf_{k=1,2,\ldots,\delta} (\tau_k - \tau_{k-1}) \leq \sup_{k=1,2,\ldots,\delta} (\tau_k - \tau_{k-1}) = \mu_2 < \infty. \tag{2.10}$$

*If there exists* $\alpha > 0$ *such that*

$$\omega + \frac{1}{\mu} \ln(M\|I + B_k\|) \leq -\gamma < 0, \quad k = 1, 2, \ldots, \delta, \tag{2.11}$$

*where*

$$\mu = \begin{cases} \mu_1, & \gamma + \omega < 0, \\ \mu_2, & \gamma + \omega \geq 0, \end{cases} \tag{2.12}$$

*then* $\{S(t,\theta), t \geq \theta \geq 0\}$ *is exponentially stable.*

*Remark 2.5* (see [26, Theorem 3.2]). If $\{S(t,\theta), t \geq \theta \geq 0\}$ is exponentially stable, then $[I - S(T_0, 0)]$ is inverse and $[I - S(T_0, 0)]^{-1} \in \mathrm{L}_b(X, X)$.

## 3. Steady-State Control

In this section, we study the steady-state control of pulse-width sampler control system with impulses. First we introduce the following assumptions.

[H3]: $f(t)$, $t \geq 0$, is $T_0$-periodic step perturbation.

[H4]: Control signal $u(t)$ is $T_0$-periodic, which is defined by the rectangular wave signal $u(t,\alpha)$, $\alpha \in \Omega$ given by (1.8).

Similar to the proof of Theorem 3.2 [26], one can obtain the following results immediately.

**Lemma 3.1.** *Assumptions [H1], [H3], and [H4] hold. Suppose $\{S(t,\theta), t \geq \theta \geq 0\}$ is exponentially stable; for every $u(t,\alpha)$, system* (1.1) *has a unique $T_0$-periodic PC-mild solution*

$$x(t,\alpha) = S(t,0)x_0 + \int_0^t S(t,\theta)\big(f(\theta) + Cu(\theta,\alpha)\big)d\theta + \sum_{0\leq\tau_k<t} S\big(t,\tau_k^+\big)c_k, \tag{3.1}$$

*where*

$$x_0 = [I - S(T_0,0)]^{-1}\int_0^{T_0} S(T_0,\theta)\big(f(\theta) + Cu(\theta,\alpha)\big)d\theta, \qquad [I - S(T_0,0)]^{-1} \in L_b(X,X), \tag{3.2}$$

*which is globally asymptotically stable.*

By Lemma 3.1, we have the following results.

**Theorem 3.2.** *Under the assumptions of Lemma 3.1, if the sampler periodic $T_0$ has the following properties:*

$$i\omega_n \in \rho(J), \quad \omega_n = \frac{2n\pi}{T_0}, \quad n = 0, \pm 1, \pm 2, \ldots, \tag{3.3}$$

*where $\rho(J)$ is the resolvent set of the matrix $J$, $i$ satisfies $i^2 = -1$, then the following open-loop control system*

$$\begin{aligned}\dot{x}(t,\alpha) &= Ax(t,\alpha) + f(t) + Cu(t,\alpha), \quad t \neq \tau_k,\\ \Delta x(t,\alpha) &= B_k x(t,\alpha) + c_k, \quad t = \tau_k,\\ z(t) &= K_1 x(t),\\ \dot{v}(t,\alpha) &= Jv(t,\alpha) + K_2 z(t,\alpha)\end{aligned} \tag{3.4}$$

*has a unique $T_0$-periodic PC-mild solution $v(t,\alpha)$ given by*

$$v(t,\alpha) = e^{Jt}\left[\left(I - e^{JT_0}\right)^{-1}\int_0^{T_0} e^{J(T_0-s)}K_2 z(s,\alpha)ds\right] + \int_0^t e^{J(t-s)}K_2 z(s,\alpha)ds, \tag{3.5}$$

*Proof.* By (3.3), we know that $e^{i\omega_n T_0} = e^{i2n\pi} = 1$, that is $1 \in \rho(e^{JT_0})$. Thus $(I - e^{JT_0})^{-1}$ exists and is bounded. It is not difficult to see that

$$v(t,\alpha) = e^{Jt}v_0 + \int_0^t e^{J(t-s)}K_2 z(s,\alpha)ds, \tag{3.6}$$

where $v_0 = v(0,\alpha)$.

Consider

$$y = (I - e^{JT_0})^{-1}\int_0^{T_0} e^{J(T_0-s)}K_2 z(s,\alpha)ds, \tag{3.7}$$

which is the unique solution of the following equation:

$$y = e^{Jt}y + \int_0^t e^{J(t-s)}K_2 z(s,\alpha)ds. \tag{3.8}$$

Let

$$v_0 = y = \left(I - e^{JT_0}\right)^{-1}\int_0^{T_0} e^{J(T_0-s)}K_2 z(s,\alpha)ds; \tag{3.9}$$

it comes from Lemma 3.1 that

$$z(t+T_0,\alpha) = z(t,\alpha), \quad t \geq 0. \tag{3.10}$$

It is easy to verify that

$$v(t,\alpha) = e^{Jt}\left[(I - e^{JT_0})^{-1}\int_0^{T_0} e^{J(T_0-s)}K_2 z(s,\alpha)ds\right] + \int_0^t e^{J(t-s)}K_2 z(s,\alpha)ds \tag{3.11}$$

is just the $T_0$-periodic $PC$-mild solution $v(t,\alpha)$ of open-loop control system (3.4).

In order to discuss the existence of steady-state control of system (1.1), we define a map $G : \Omega \in \mathbb{R}^q \to \mathbb{R}^q$ given by

$$G(\alpha) = \left(I - e^{JT_0}\right)^{-1}\int_0^{T_0} e^{J(T_0-s)}K_2 K_1 x(s,\alpha)ds, \quad \alpha \in \Omega, \tag{3.12}$$

where $x(\cdot,\alpha)$ is the $T_0$-periodic $PC$-mild solution of system (1.1) corresponding to $\alpha \in \Omega$. Then we have the following result. □

**Lemma 3.3.** *Under the assumptions of Theorem 3.2, there exists a constant* $\overline{M} > 0$ *such that*

$$\|G(\alpha) - G(\overline{\alpha})\| \leq \overline{M}\|K_2\|\|\alpha - \overline{\alpha}\|, \quad \alpha, \overline{\alpha} \in \Omega. \tag{3.13}$$

*Proof.* Suppose $x_1(t,\alpha)$ and $x_2(t,\overline{\alpha})$ are the $T_0$-periodic $PC$-mild solution of system (1.1) corresponding to $\alpha$ and $\overline{\alpha} \in \Omega$, respectively, then

$$\begin{aligned} x_1(0) - x_2(0) &= x_1(T_0) - x_2(T_0) \\ &= S(T_0,0)(x_1(0) - x_2(0)) + \int_0^{T_0} S(T_0,\theta)C(u(\theta,\alpha) - u(\theta,\overline{\alpha}))d\theta. \end{aligned} \tag{3.14}$$

Thus,

$$\|x_1(0) - x_2(0)\| \leq \left\|[I - S(T_0,0)]^{-1}\right\|\|S(T_0,\theta)\|\|C\|_{L_b(\mathbb{R}^q,X)}\int_0^{T_0}\|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q}d\theta. \tag{3.15}$$

For $0 \leq \theta \leq t \leq T_0$, we obtain

$$\begin{aligned} \|x_1(t,\alpha)-x_2(t,\overline{\alpha})\| &\leq \|S(t,0)\|\|x_1(0)-x_2(0)\| + \|S(T_0,\theta)\|\|C\|_{L_b(\mathbb{R}^q,X)}\int_0^{T_0}\|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q}d\theta \\ &\leq K\|C\|_{L_b(\mathbb{R}^q,X)}(\|Q\|K + 1)\int_0^{T_0}\|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q}d\theta \\ &\leq \overline{M}_1\int_0^{T_0}\|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q}d\theta, \end{aligned} \tag{3.16}$$

where

$$\overline{M}_1 = K\|C\|_{L_b(\mathbb{R}^q,X)}(\|Q\|K + 1), \quad Q = [I - S(T_0,0)]^{-1}. \tag{3.17}$$

By elementaly computation,

$$\begin{aligned} \|G(\alpha) - G(\overline{\alpha})\| &\leq \left\|(I - e^{JT_0})^{-1}\right\|\left\|e^{JT_0}\right\|\|K_2\|\|K_1\|_{L_b(X,\mathbb{R}^p)}\int_0^{T_0}\|x_1(s,\alpha) - x_2(s,\overline{\alpha})\|ds \\ &\leq \left\|(I - e^{JT_0})^{-1}\right\|\left\|e^{JT_0}\right\|\|K_2\|\|K_1\|_{L_b(X,\mathbb{R}^p)}\overline{M}_1 T_0\int_0^{T_0}\|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q}d\theta \\ &\leq \overline{M}_2\|K_2\|\int_0^{T_0}\|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q}d\theta, \end{aligned} \tag{3.18}$$

where

$$\overline{M}_2 = \left\| \left(I - e^{JT_0}\right)^{-1} \right\| \left\| e^{JT_0} \right\| \|K_1\|_{L_b(X,\mathbb{R}^p)} \overline{M}_1 T_0. \tag{3.19}$$

(i) For $\alpha_l \overline{\alpha}_l > 0$. Without loss of generality, we suppose that $0 < \alpha_l < \overline{\alpha}_l$, then we have

$$\int_0^{T_0} \|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q} d\theta \le \int_{\alpha_l T_0}^{\overline{\alpha}_l T_0} \|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q} d\theta \le T_0 \|\alpha - \overline{\alpha}\|. \tag{3.20}$$

(ii) For $\alpha_l \overline{\alpha}_l < 0$. For example, $\alpha_l < 0 < \overline{\alpha}_l$, $|\overline{\alpha}_l| > \alpha_l$, we have

$$\int_0^{T_0} \|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q} d\theta \le \int_{\alpha_l T_0}^{|\overline{\alpha}_l| T_0} \|u(\theta,\alpha) - u(\theta,\overline{\alpha})\|_{\mathbb{R}^q} d\theta \le 2T_0 \|\alpha - \overline{\alpha}\|. \tag{3.21}$$

By (3.18), (3.20) and (3.21), there exists a constant $\overline{M} > 0$ such that

$$\|G(\alpha) - G(\overline{\alpha})\| \le \overline{M} \|K_2\| \|\alpha - \overline{\alpha}\|, \quad \alpha, \overline{\alpha} \in \Omega. \tag{3.22}$$

□

By Lemma 3.3, we have the following result immediately.

**Theorem 3.4.** *Under the assumptions Theorem 3.2, one can choose a suitable* $\|K_2\|$ *such that the systems* (1.1), (1.3)–(1.5) *have a unique steady-state and the fixed point of* $G$ *is just the conducting vector.*

*Proof.* Let $x(t,\alpha)$ be the $T_0$-periodic $PC$-mild solution of system (1.1) corresponding to $\alpha \in \Omega$, then

$$x(0) = x(T_0) = S(T_0,0)x(0) + \int_0^{T_0} S(T_0,\theta)\big(f(\theta) + Cu(\theta,\alpha)\big) d\theta, \tag{3.23}$$

that is,

$$x(0) = [I - S(T_0,0)]^{-1} \int_0^{T_0} S(T_0,\theta)\big(f(\theta) + Cu(\theta,\alpha)\big) d\theta. \tag{3.24}$$

By virtue of [H3], we can suppose that $\|f(t)\| \le f_0$, $t \ge 0$, then

$$\|x(0)\| \le \left\| [I - S(T_0,0)]^{-1} \right\| \|S(T_0,\theta)\| \int_0^{T_0} \Big( \|C\|_{L_b(\mathbb{R}^q,X)} q + f_0 \Big) d\theta \le K\|Q\| \Big( \|C\|_{L_b(\mathbb{R}^q,X)} q + f_0 \Big) T_0 \equiv \overline{M}_3. \tag{3.25}$$

It comes from

$$G(\alpha) = \left(I - e^{JT_0}\right)^{-1}\int_0^{T_0} e^{J(T_0-s)}K_2K_1S(t,0)x(0)ds$$
$$+ \left(I - e^{JT_0}\right)^{-1}\int_0^{T_0} e^{J(T_0-s)}K_2K_1\left(\int_0^{t} S(t,s)\left(f(s) + Cu(s,\alpha)\right)ds\right)ds \tag{3.26}$$

that

$$\|G(\alpha)\| \leq \overline{M}_4\|K_2\|, \tag{3.27}$$

where

$$\overline{M}_4 = \left\|\left(I - e^{JT_0}\right)^{-1}\right\|\left\|e^{JT_0}\right\|\|K_1\|_{L_b(X,\mathbb{R}^p)}T_0\overline{M}_3\left(K + \frac{1}{\|Q\|}\right). \tag{3.28}$$

Using Lemma 3.3 and (3.27), it is not difficult to verify that $G : \Omega \to \Omega$ is a contraction map when

$$0 < \|K_2\| < \frac{1}{\max\left(\overline{M}, \overline{M}_4\right)}. \tag{3.29}$$

By the application of contraction mapping principle, $G$ has a unique fixed point $\alpha^* \in \Omega$. Obviously, the $T_0$-periodic $PC$-mild solution of system (1.1) corresponding to $\alpha^*$ is just the unique steady-state. □

Next, we investigate the steady-state control of system (1.2), (1.3)–(1.5). We need to introduce the following assumption [H5].

(i) [H5.1 ] $f : [0,\infty) \times X \to X$ is measurable for $t \geq 0$, and for any $x, y \in X$, there exists a positive constant $L_u > 0$ such that

$$\|f(t,x) - f(t,y)\| \leq L_u\|x - y\|. \tag{3.30}$$

(ii) [H5.2 ] $f(t,x)$ is $T_0$-periodic in $t$. That is, $f(t + T_0, x) = f(t,x)$, $t \geq 0$.

**Lemma 3.5.** *Under the assumptions [H1], [H4] and [H5], the impulsive evolution operator* $\{S(t,\theta), t \geq \theta \geq 0\}$ *is exponentially stable, that is, there exists a constant* $K > 0$ *and* $\nu > 0$ *such that*

$$\|S(t,\theta)\| \leq Ke^{-\nu(t-\theta)}, \quad t > \theta \geq 0, \tag{3.31}$$

*where* $\nu > (L_u K T_0 + \ln K)/T_0$*, then system* (1.2) *has a unique* $T_0$*-periodic PC-mild solution* $x(\cdot, \alpha)$ *corresponding to control* $u(\cdot, \alpha)$ *given by*

$$x(t, \alpha) = S(t, 0)x_0 + \int_0^t S(t, \theta)\big(f(\theta, x(\theta)) + Cu(\theta, \alpha)\big)d\theta + \sum_{0 \le \tau_k < t} S(t, \tau_k^+)c_k \tag{3.32}$$

*and is also exponentially stable.*

*Proof.* Suppose that $x_1(t, \alpha)$ $(x_2(t, \overline{\alpha}))$ is the $PC$-mild solution of system (1.2) corresponding to initial value $x_1 = x_1(0)$ $(x_2 = x_2(0))$, respectively, then

$$\begin{aligned} \|x_1(t) - x_2(t)\| &\le Ke^{-vt}\|x_1 - x_2\| + L_u K \int_0^t e^{-v(t-\theta)}\|x_1(\theta) - x_2(\theta)\|d\theta \\ &\le Ke^{-vt}\|x_1 - x_2\| + L_u K \int_0^t \|x_1(\theta) - x_2(\theta)\|d\theta. \end{aligned} \tag{3.33}$$

By Gronwall inequality, we can deduce

$$\|x_1(t) - x_2(t)\| \le Ke^{(L_u K - \nu)t}\|x_1 - x_2\|, \quad t \in [0, T_0]. \tag{3.34}$$

Define a map $H : X \to X$ given by

$$H(t)\overline{x} = x(t) = S(t, 0)\overline{x} + \int_0^t S(t, \theta)\big(f(\theta, x(\theta)) + Cu(\theta, \alpha)\big)d\theta + \sum_{0 \le \tau_k < t} S(t, \tau_k^+)c_k. \tag{3.35}$$

Then we can verify that

$$\|H(T_0)x_1 - H(T_0)x_2\| \le Ke^{(L_u K - \nu)T_0}\|x_1 - x_2\|. \tag{3.36}$$

It comes from

$$\nu > \frac{L_u K T_0 + \ln K}{T_0} \tag{3.37}$$

that $H(T_0)$ is a contraction map on $X$. Thus, by the application of contraction mapping principle again, $H(T_0)$ has a unique fixed point $x^* \in X$ satisfying

$$H(T_0)x^* = x^*. \tag{3.38}$$

Using (2), (3), (4) of Lemma 2.1, one can verify that

$$H(nT_0) = [H(T_0)]^n, \quad n \in \mathbb{N}. \tag{3.39}$$

By virtue of (3.34), (3.38), and (3.39), we know that $x(\cdot) = x(\cdot, x^*)$ is just the $T_0$-periodic $PC$-mild solution of (1.2) which is exponentially stable. □

Similar to the proof of Lemma 3.1, using Lemma 3.5, we can obtain the following result immediately.

**Lemma 3.6.** *Under the assumptions of Lemma 3.5, if $T_0$ also satisfies* (3.3), *then the open-loop control system*

$$\begin{aligned}\dot{x}(t,\alpha) &= Ax(t,\alpha) + f(t,x(t,\alpha)) + Cu(t,\alpha), \quad t \neq \tau_k,\\ \Delta x(t,\alpha) &= B_k x(t,\alpha) + c_k, \quad t = \tau_k,\\ z(t,\alpha) &= K_1 x(t,\alpha),\\ \dot{v}(t,\alpha) &= Jv(t,\alpha) + K_2 z(t,\alpha).\end{aligned} \tag{3.40}$$

*has a unique $T_0$-periodic $PC$-mild solution $v(\cdot,\alpha)$.*

In order to discuss the existence of steady-state control of system (1.2), we define a map $G : \Omega \in \mathbb{R}^q \to \mathbb{R}^q$ given by

$$\widetilde{G}(\alpha) = \left(I - e^{JT_0}\right)^{-1} \int_0^{T_0} e^{J(T_0-s)} K_2 K_1 x(s,\alpha)ds, \quad \alpha \in \Omega, \tag{3.41}$$

where $x(\cdot,\alpha)$ is the periodic solution of system (1.2) corresponding to $\alpha \in \Omega$. Then we have the following results.

**Lemma 3.7.** *Under the assumptions of Lemma 3.6, there exists a constant $\widehat{M} > 0$ such that*

$$\left\|\widetilde{G}(\alpha) - \widetilde{G}(\overline{\alpha})\right\| \leq \widehat{M}\|K_2\|\|\alpha - \overline{\alpha}\|, \alpha, \overline{\alpha} \in \Omega. \tag{3.42}$$

*Proof.* Suppose that $x_1(t,\alpha)$ and $x_2(t,\overline{\alpha})$ are the $T_0$-periodic $PC$-mild solutions corresponding to $\alpha$ and $\overline{\alpha} \in \Omega$ with the initial value $x_1(0)$ and $x_2(0)$, respectively, then

$$\begin{aligned}x_1(0) - x_2(0) &= x_1(T_0) - x_2(T_0)\\ &= S(T_0,0)(x_1(0) - x_2(0)) + \int_0^{T_0} S(T_0,\theta)\left(f(\theta,x_1(\theta)) - f(\theta,x_2(\theta))\right)d\theta\\ &\quad + \int_0^{T_0} S(T_0,\theta)C(u(\theta,\alpha) - u(\theta,\overline{\alpha}))d\theta.\end{aligned} \tag{3.43}$$

Thus,

$$x_1(0) - x_2(0) = [I - S(T_0, 0)]^{-1}\left[\int_0^{T_0} S(T_0, \theta)\big(f(\theta, x_1(\theta)) - f(\theta, x_2(\theta))\big)d\theta + \int_0^{T_0} S(T_0, \theta)C(u(\theta, \alpha) - u(\theta, \overline{\alpha}))d\theta\right]. \tag{3.44}$$

Furthermore,

$$\|x_1(0) - x_2(0)\| \le \|Q\|M_{T_0}\left[L_u\int_0^{T_0}\|x_1(\theta) - x_2(\theta)\|d\theta + \|C\|_{L_b(R^q,X)}\int_0^{T_0}\|u(\theta, \alpha) - u(\theta, \overline{\alpha})\|_{\mathbb{R}^q}d\theta\right]. \tag{3.45}$$

For $0 \le \theta \le t \le T_0$, we have

$$\begin{aligned}\|x_1(t, \alpha) - x_2(t, \overline{\alpha})\| \le{}& \|Q\|M_{T_0}^2 L_u\int_0^{T_0}\|x_1(\theta, \alpha) - x_2(\theta, \overline{\alpha})\|d\theta \\ &+ M_{T_0}\|C\|_{\mathrm{L}_b(\mathbb{R}^q,X)}(\|Q\|M_{T_0} + 1)\int_0^{T_0}\|u(\theta, \alpha) - u(\theta, \overline{\alpha})\|_{\mathbb{R}^q}d\theta \\ &+ M_{T_0}L_u\int_0^{t}\|x_1(\theta, \alpha) - x_2(\theta, \overline{\alpha})\|d\theta.\end{aligned} \tag{3.46}$$

By Gronwall inequality again, we obtain

$$\begin{aligned}\|x_1(t, \alpha) - x_2(t, \overline{\alpha})\| \le{}& e^{M_{T_0}L_uT_0}\|Q\|M_{T_0}^2 L_u\int_0^{T_0}\|x_1(\theta, \alpha) - x_2(\theta, \overline{\alpha})\|d\theta \\ &+ e^{M_{T_0}L_uT_0}M_{T_0}\|C\|_{\mathrm{L}_b(\mathbb{R}^q,X)}(\|Q\|M_{T_0} + 1)\int_0^{T_0}\|u(\theta, \alpha) - u(\theta, \overline{\alpha})\|_{\mathbb{R}^q}d\theta.\end{aligned} \tag{3.47}$$

Integrating from 0 to $T_0$, we obtain

$$\int_0^{T_0}\|x_1(t, \alpha) - x_2(t, \overline{\alpha})\|dt \le \frac{M_6}{M_5}\int_0^{T_0}\|u(\theta, \alpha) - u(\theta, \overline{\alpha})\|_{\mathbb{R}^q}d\theta, \tag{3.48}$$

where

$$M_5 = 1 - e^{M_{T_0}L_uT_0}\|Q\|M_{T_0}^2 L_u > 0, \qquad M_6 = e^{M_{T_0}L_uT_0}T_0M_{T_0}\|C\|_{\mathrm{L}_b(\mathbb{R}^q,X)}(\|Q\|M_{T_0} + 1). \tag{3.49}$$

Thus,

$$\begin{aligned}\left\|\widetilde{G}(\alpha)-\widetilde{G}(\overline{\alpha})\right\| &\leq \left\|\left(I-e^{JT_0}\right)^{-1}\right\|\left\|e^{JT_0}\right\|\|K_2\|\|K_1\|_{L_b(X,\mathbb{R}^p)}\int_0^{T_0}\|x_1(s,\alpha)-x_2(s,\overline{\alpha})\|ds \\ &\leq \left\|\left(I-e^{JT_0}\right)^{-1}\right\|\left\|e^{JT_0}\right\|\|K_2\|\|K_1\|_{L_b(X,\mathbb{R}^p)}\frac{M_6}{M_5}\int_0^{T_0}\|u(\theta,\alpha)-u(\theta,\overline{\alpha})\|_{\mathbb{R}^q}d\theta \\ &\leq 2\left\|\left(I-e^{JT_0}\right)^{-1}\right\|\left\|e^{JT_0}\right\|\|K_2\|\|K_1\|_{L_b(X,\mathbb{R}^p)}\frac{M_6}{M_5}T_0\|\alpha-\overline{\alpha}\|.\end{aligned} \tag{3.50}$$

Choosing a constant

$$\widehat{M}=2\left\|\left(I-e^{JT_0}\right)^{-1}\right\|\left\|e^{JT_0}\right\|\|K_1\|_{L_b(X,\mathbb{R}^p)}\frac{M_6}{M_5}T_0>0, \tag{3.51}$$

then,

$$\left\|\widetilde{G}(\alpha)-\widetilde{G}(\overline{\alpha})\right\|\leq \widehat{M}\|K_2\|\|\alpha-\overline{\alpha}\|,\quad \alpha,\overline{\alpha}\in\Omega. \tag{3.52}$$

□

Using Lemma 3.7, we have the following result.

**Theorem 3.8.** *Under the assumptions of Lemma 3.7, there exists a constant $N_f>0$ such that $\|f(t,x)\|\leq N_f$, if $\|K_2\|$ is sufficiently small, then system* (1.2), (1.3)–(1.5) *has a unique steady-state and the fixed point of $\widetilde{G}$ is just the conducting vector.*

*Proof.* Let $x(t,\alpha)$ be the $T_0$-periodic $PC$-mild solution of system (1.2) corresponding to $\alpha\in\Omega$, then

$$x(0)=[I-S(T_0,0)]^{-1}\int_0^{T_0}S(T_0,\theta)\left(f(\theta,x(\theta))+Cu(\theta,\alpha)\right)d\theta. \tag{3.53}$$

Further,

$$\|x(0)\|\leq K\|Q\|\left(\|C\|_{L_b(\mathbb{R}^q,X)}q+N_f\right)T_0=M_7. \tag{3.54}$$

Let

$$M_8=\left\|\left(I-e^{JT_0}\right)^{-1}\right\|\left\|e^{JT_0}\right\|\|K_1\|_{L_b(X,\mathbb{R}^p)}KT_0\left[M_7+\left(N_f+\|C\|_{L_b(\mathbb{R}^q,X)}q\right)T_0\right]. \tag{3.55}$$

It comes from

$$G(\alpha) = \left(I - e^{JT_0}\right)^{-1} \int_0^{T_0} e^{J(T_0-s)} K_2 K_1 S(t,0) x(0) ds$$
$$+ \left(I - e^{JT_0}\right)^{-1} \int_0^{T_0} e^{J(T_0-s)} K_2 K_1 \left( \int_0^t S(t,s) \left(f(s, x(s,\alpha)) + Cu(s,\alpha)\right) ds \right) ds \tag{3.56}$$

that

$$\|G(\alpha)\| \leq M_8 \|K_2\|. \tag{3.57}$$

It is not difficult to see that $G : \Omega \to \Omega$ is a contraction map when

$$0 < \|K_2\| < \frac{1}{\max\left(\widehat{M}, M_8\right)}. \tag{3.58}$$

By application of contraction mapping principle again, $\widetilde{G}$ has a unique fixed point $\widetilde{\alpha}^* \in \Omega$. Obviously, the $T_0$-periodic $PC$-mild solution of system (1.2) corresponding to $\widetilde{\alpha}^*$ is just the unique steady-state. □

Finally, an example is given for demonstration. Consider the following system

$$\begin{gathered}
\frac{\partial}{\partial t} x(t,y) = \frac{\partial^2 x(t,y)}{\partial^2 y} + bu(t) + f(y) \cdot 1(t), \quad y \in (0,l), 2\pi > t > 0, t \neq \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \\
\Delta x(\tau_i, y) = x(\tau_i^+, y) - x(\tau_i^-, y) = b_k x(\tau_i, y), \quad y \in (0,l), t = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \\
x(t,0) = x(t,l) = 0, \quad t \geq 0, \\
z(t) = \int_0^l k_1 x(t,y) dy,
\end{gathered} \tag{3.59}$$

and the output $v(t)$ satisfies

$$\frac{dv(t)}{dt} = v(t) + k_2 z(t), \tag{3.60}$$

where $b$, $b_k$, $k_1$ and $k_2$ are constants.

Let $X = L^2(0,l)$; define

$$\begin{gathered}
(Ax)(y) = x''(y), \quad \text{for arbiartry } x \in D(A), \\
D(A) = \left\{ x \in L^2(0,l) \mid x, x'' \in L^2(0,l), x(0) = x(l) = 0 \right\}.
\end{gathered} \tag{3.61}$$

Then $A$ can generate an exponentially stable $C_0$-semigroup $\{T(t), t \geq 0\}$ in $L^2(0,l)$ and $\|T(t)\| \leq e^{-(\pi/l)t}$, $t \geq 0$. We only choose a suitable positive number $k_2$, then all the assumptions are met in Theorem 3.4, our results can be used to system (3.59).

## Acknowledgment

The authors acknowledge the support from the National Natural Science Foundation of China (no.10961009), Introducing Talents Foundation for the Doctor of Guizhou University (2009, no.031). Youth Teachers Natural Science Foundation of Guizhou University (2009, no.083).

## References

[1] R. Teman, *Infinite Dimensional Dynamical System in Mechanics and Physics*, Springer, New York, NY, USA, 1988.
[2] D. S. Bernstein and C. V. Hollot, "Robust stability for sampled-data control systems," *Systems & Control Letters*, vol. 13, no. 3, pp. 217–226, 1989.
[3] H. X. Zhou and Q. L. Xu, "A PMW temperature control system with a distributed parameter," *Modeling, Simulation and Control*, vol. 32, pp. 53–64, 1998.
[4] Q. G. Li and Y. Zhao, "Steady-state control of a class of parabolic pulse-width sampler systems," *Control Theory & Applications*, vol. 19, no. 4, pp. 516–520, 2002.
[5] L. Hou and A. N. Michel, "Stability and analysis of pulse-width-modulated feedback systems," *Automatica*, vol. 37, no. 9, pp. 1335–1349, 2001.
[6] F. M. Al-Sunni and A. H. Al-Amer, "Robust control of sampled data systems," *IEE Proceedings: Control Theory and Applications*, vol. 145, no. 2, pp. 236–240, 1998.
[7] L. S. Hu, P. Shi, and B. Huang, "Stochastic stability and robust control for sampled-data systems with Markovian jump parameters," *Journal of Mathematical Analysis and Applications*, vol. 313, no. 2, pp. 504–517, 2006.
[8] A. Ilchmann and S. Townley, "Adaptive sampling control of high-gainstabilizable systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 10, pp. 1961–1966, 1999.
[9] N. Özdemir, *Robust and adaptive sampled data I-control*, Ph.D. thesis, University of Exeter, Exeter, UK, 2000.
[10] N. Özdemir and S. Townley, "Integral control by variable sampling based on steady-state data," *Automatica*, vol. 39, no. 1, pp. 135–140, 2003.
[11] Z. Ke, H. Logemann, and S. Townley, "Adaptive sampled-data integral control of stable infinite-dimensional linear systems," *Systems & Control Letters*, vol. 58, no. 4, pp. 233–240, 2009.
[12] V. Lakshmikantham, D. D. Baĭnov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*, vol. 6 of *Series in Modern Applied Mathematics*, World Scientific Publishing, Teaneck, NJ, USA, 1989.
[13] N. U. Ahmed, "Optimal impulse control for impulsive systems in Banach spaces," *International Journal of Differential Equations and Applications*, vol. 1, no. 1, pp. 37–52, 2000.
[14] N. U. Ahmed, "Some remarks on the dynamics of impulsive systems in Banach spaces," *Dynamics of Continuous, Discrete & Impulsive Systems*, vol. 8, no. 2, pp. 261–274, 2001.
[15] N. U. Ahmed, K. L. Teo, and S. H. Hou, "Nonlinear impulsive systems on infinite dimensional spaces," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 54, no. 5, pp. 907–925, 2003.
[16] N. U. Ahmed, "Existence of optimal controls for a general class of impulsive systems on Banach spaces," *SIAM Journal on Control and Optimization*, vol. 42, no. 2, pp. 669–685, 2003.
[17] N. U. Ahmed, "Optimal feedback control for impulsive systems on the space of finitely additive measures," *Publicationes Mathematicae Debrecen*, vol. 70, no. 3-4, pp. 371–393, 2007.
[18] J. H. Liu, "Nonlinear impulsive evolution equations," *Dynamics of Continuous, Discrete and Impulsive Systems*, vol. 6, no. 1, pp. 77–85, 1999.
[19] J. J. Nieto and R. Rodriguez-Lopez, "Boundary value problems for a class of impulsive functional equations," *Computers & Mathematics with Applications*, vol. 55, no. 12, pp. 2715–2731, 2008.
[20] J. J. Nieto and D. O'Regan, "Variational approach to impulsive differential equations," *Nonlinear Analysis*, vol. 10, no. 2, pp. 680–690, 2009.

[21] W. Wei, X. Xiang, and Y. Peng, “Nonlinear impulsive integro-differential equations of mixed type and optimal controls,” *Optimization*, vol. 55, no. 1-2, pp. 141–156, 2006.

[22] X. Xiang and W. Wei, “Mild solution for a class of nonlinear impulsive evolution inclusions on Banach space,” *Southeast Asian Bulletin of Mathematics*, vol. 30, no. 2, pp. 367–376, 2006.

[23] X. Xiang, W. Wei, and Y. Jiang, “Strongly nonlinear impulsive system and necessary conditions of optimality,” *Dynamics of Continuous, Discrete & Impulsive Systems*, vol. 12, no. 6, pp. 811–824, 2005.

[24] X. Yu, X. Xiang, and W. Wei, “Solution bundle for a class of impulsive differential inclusions on Banach spaces,” *Journal of Mathematical Analysis and Applications*, vol. 327, no. 1, pp. 220–232, 2007.

[25] J. R. Wang, X. Xiang, and W. Wei, “Optimal periodic controls for linear impulsive periodic system,” in *Proceedings of International Conference on Computer Science and Software Engineering (CSSE '08)*, vol. 4, pp. 138–141, Wuhan, China, December 2008.

[26] J. R. Wang, X. Xiang, and W. Wei, “Global behaviors and optimal harvesting of a class of impulsive periodic logistic single-species system with continuous periodic control strategy,” *Boundary Value Problems*, vol. 2008, Article ID 192353, 21 pages, 2008.

[27] J. R. Wang, X. Xiang, and Y. Peng, “Periodic solutions of semilinear impulsive periodic system on Banach space,” *Nonlinear Analysis: Theory, Methods & Applications*, vol. 71, no. 12, pp. e1344–e1353, 2009.