($\hat{C}_{xx}$), 1.9 percent. ($\hat{C}_{xy}$), 2.8 percent. ($\hat{C}_{yx}$) and 1.8 percent. ($\hat{C}_{yy}$). Superimposed on each graph are the square roots of the appropriate diagonal elements from the error covariance matrix **P**. These elements provide a first-order approximation to the estimated variance in the states and hence $\pm\sqrt{P_{ii}}$ is an approximate measure of the standard deviation of state estimate $\hat{x}_i$.

Increasing the running speed (and hence the excitation frequency) has little effect upon the accuracy of estimation, as shown by the results in Fig. 2(*b*) for a running speed of 4500 rev/min ($\epsilon_0 = 0.569$, $Q_j = 0.380$). Reducing the speed starts to affect convergence at speeds below 2000 rev/min as shown by the results in Fig. 2(*c*) for a running speed of 1500 rev/min ($\epsilon_0 = 0.737$, $Q_j = 3.426$) and this slower rate of convergence is reflected in the values of the $\pm\sqrt{P_{ii}}$ terms.

A key assumption in producing the above results is that the values of (e/c) and φ are known exactly. Where the unbalance is introduced artificially this is a reasonable assumption, however where natural unbalance is exploited its effective magnitude and angular position would be harder to assess. Consequently a further set of tests was performed involving deliberately-induced errors in the assumed values of (e/c) and φ.

Typical results (for a running speed of 3000 rev/min) are shown in Fig. 3. The error surfaces in Fig. 3 show the amount of bias introduced into each of the coefficient estimates when the assumed values of (e/c) and φ are in error. It appears that the four coefficients are not as sensitive to phase angle errors as in a linear processing scheme which used *explicit* phase angle measurements [8]. In the experiments described in reference [8] it was suggested that a 5 deg error in phase angle could (typically) produce a 30 percent error in the coefficient estimates. Using the nonlinear processing scheme, worst case errors of 30 percent (in estimates C_{xy} *or* C_{yx}) only occur when the phase error exceeds 10 deg *or* the magnitude error exceeds 50 percent.

Discussion

This note has discussed an approach to the identification of linearized journal bearing dynamics under normal operating conditions. Given that an effective technique is already available for estimating the four oil-film stiffness terms, a method must now be developed for extracting reliable estimates of the four damping terms, preferably without the need to conduct further experiments. We have suggested that this can be achieved by reformulating the problem so that an existing algorithm can be applied to estimate the damping terms from noisy measurements of the displacement responses to synchronous excitation. These responses are acquired automatically in any identification experiment and hence the approach could be applied retrospectively to refine estimates of the damping terms.

The feasibility of the approach has been tested under controlled conditions by generating data from a linearized model of a simple rotor-bearing system and demonstrating that the governing equations can be reconstructed. In practice the effectiveness of such an approach will depend upon the robustness of the algorithm when processing actual operating data and the accuracy with which the unbalance parameters can be assessed. The effects of modeling errors (introduced by the linearization process) and other disturbances not considered in this note (for example, surface roughness of the shaft and bearings) will obviously be reflected in the eventual results. It is hoped that the results presented here will encourage experimental work to quantify such effects.

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Control of Systems Subject to Small Measurement Disturbances

D. D. Sworder[1]

When a controlled system is subject to external disturbances of large magnitude, the regulator often uses a mixed strategy, combining a feedforward link to neutralize the primary impact of the disturbance, and a feedback link to reduce the effect of any residual. If there is a possibility of error in measuring the disturbance, the problem becomes more complex. A sophisticated regulator might even try to "learn" the true value of the disturbance. Attempts to design such an intelligent regulator usually lead to intractable synthesis equations. This paper provides a simple alternative for the case in which the measurement of the disturbance is close to the true value. A study of a simple model of a solar-powered boiler shows that the performance of the proposed regulator is near that attainable by much more complicated controllers.

Introduction

Although the study of linear systems receives disproportionate attention in the literature, the fact remains that nonlinear equations are necessary to adequately describe many important control processes. Linear feedback theory may still find an application even in this instance. If the nonlinear system operates near a known nominal condition, a nominal state trajectory and a nominal actuating signal can be determined. Under appropriate conditions, the deviations from the nominal path can be described by a linear model. Linear synthesis algorithms can then be applied to this perturbation model, and a regulator which causes the system to track its nominal trajectory can be deduced thereby.

To be more specific about these ideas, consider the problem of synthesizing a feedwater flow rate regulator for control of a solar receiver. Certain aspects of this problem were addressed in [1] and [2]. On the California desert, there is in operation a 10 MWe generating system which uses a field of movable mirrors (heliostats) to focus the sun's energy on

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panels mounted on a central tower. Water flows through these panels, and is transformed into superheated steam which is then used to drive a conventional turbine/generator.

The equations which relate the response of the thermodynamic variables describing the panels to a feedwater flow rate input are highly nonlinear. Further, as the insolation on the panels changes from early morning conditions to those of a sunny summer day, the operating point of the panels sweeps through a wide range. For this reason the synthesis of a feedwater flow rate regulatory is inherently a nonlinear design problem.

Actually this complexity is more apparent than real. The operating point of a single panel changes in response to changes in panel insolation. There are insolation sensors on the panel which provide the regulator with a means of calculating the nominal values of the system state and actuating signal. Because the diurnal variations are so slow, the nominal operation condition can be thought of as essentially unvarying and a constant coefficient linear perturbation model can be defined. On the basis of this model, a simple PID regulator can be produced. The gains of the regulator can be varied as appropriate.

This same design philosophy can be extended to operations in a more demanding environment. As clouds pass over the heliostat field, abrupt and possibly large amplitude changes occur in the operating point. It is no longer acceptable to consider the operating level as unvarying. Indeed, the transients occasioned by the motion of clouds on a partly cloudy day give the system a special character. Still, the insolation sensors give an indication of the current value of the desired operating point, and the perturbation dynamics can now be descibed by random motion between a set of linear models. References [1] and [2] present simple solution algorithms for this problem under differing hypotheses on the perturbation dynamics. Reference [3] indicates that the linear models provide a good indication of the performance attained by the nonlinear system.

While the applicability of such design procedures is not limited to solar central receivers, consideration of the peculiarities of this specific system does illustrate some of the issues involved. There are, in truth, certain features of the regulator synthesis problem which are not well captured in the models of [1]–[3]. Of principal interest here are errors in measuring the insolation on the panels. The solar flux falls across the panel in a distributed manner. There are six flux sensors placed at various points on the panel which give an accurate, but local estimate of the insolation. The "sensed" value of insolation is found by a "high select" logic, and is actually the maximum value of the sensor readings. The maximum is used because the result of an insufficient feedwater flow rate could be structural damage to the panel and supporting framework. Hence there is a bias toward overestimating the true value of effective insolation.

The preceding paragraph illustrates a generic problem encountered in the use of the linear design algorithms. The regulator is explicitly dependent upon the operating condition of the system. Not only does the operating point of the system determine the dynamics of the perturbation model, but it also indexes the nominal state trajectory and actuating signal. If an error is made in identifying the true operating condition, the synthesis model will not only have incorrect dynamics, but in addition the correction term, which is the output of the feedback link, will be referenced to an incorrect "nominal."

The latter effect is usually neglected in linear synthesis algorithms, and often with good reason. If the operating condition is unchanging, an error in computing the nominal will be reflected as a bias in the perturbation model. The "I" (integration) in the PID controller makes the closed loop model insensitive to such effects.

For the class of systems under consideration here, an integration is not an effective tool. The operating point changes too rapidly to "average" out biases. Rather, this problem is of the form of classical dual control [4] in which the controller must simultaneously regulate the process, and "learn" the true value of the incorrectly measured parameters.

Dual control problems have proven to be so intractable that few synthesis examples exist which have been carried out to completion. There are, fortunately, some restrictions on the design of the feedwater flow rate regulator which yield a natural approximation to the optimal control. First, the measurement of insolation, while not error free, is at least "close" to the true value of effective insolation on the panel. Furthermore, the software purchased for this application does not have sufficient flexibility to incorporate "learning."

Abstracting these properties, it will be assumed that there is only a small error in the sensed value of the operating point, and that the performance improvement obtainable from processing past data is negligible. By dispensing with the "learning" function of the regulator, a simple design algorithm is obtained. The resulting regulator has been tested on a low order model of a solar central receiver with a 10 percent positive mean bias in the sensed insolation level. By comparing the performance of the proposed regulator with that of a (unrealizable) regulator which utilizes perfect knowledge of the operational level, it is shown that there is only slight performance degradation attributable to misclassification of the true value of insolation.

Problem Description

The global model of the system to be controlled will be assumed to be given by an ordinary differential equation of the form

$$\dot{\xi}=f(\xi,w,t,r);t\geq t_0 \tag{1}$$
$$\xi(t_0)=\xi_0$$

where ξ is the vector system state, w the vector actuating signal, and r a scalar indicator variable representing the exogenous influences on the sytsem. In the case of the solar-powered boiler, (1) would represent a lumped mass approximation to the partial differential equations which more accurately describe the panel. The vector ξ would include enthalpies, temperatures, and pressures. The feedwater flow rate would be represented by w, and the insolation would be indexed by r.

As mentioned earlier, linear control system synthesis procedures are often based upon the linear perturbation model associated with (1). Suppose r is a constant, r_0, and suppose that (x_n, v_n) satisfies

$$\dot{x}_n=f(x_n,v_n,t,r_0) \tag{2}$$
$$x_n(t_0)=x_0$$

and represents the desired operating path of the system corresponding to the specific value of r. Then (x_n, v_n) is the nominal trajectory for the system corresponding to r_0.

The actual system response (x_p, v_p) will be subject to a variety of external influences. The difference between these two sets of functions gives the error variables (x, v);

$$x(t)=x_p(t)-x_n(t;r_0) \tag{3}$$
$$v(t)=v_p(t)-v_n(t;r_0)$$

Under appropriate conditions, the error variables satisfy a linear set of differential equations.

$$\dot{x}=F_{r_0}x+G_{r_0}v \tag{4}$$
$$x(t_0)=x_p(t_0)-x_n(t_0)$$

The perturbation model (4) can be used in a synthesis algorithm to find v. The actuating signal is simply the sum $v + v_n(t;r_0)$.

The cursory discussion above describes the conventional relation between the linear system model (4) and the actuating signal. Note that the error variables (3) and the linear model (4) are indexed by the realized value of r. If there is an error in identifying the true operating point, the regulator will be uncertain as to the correct nominal trajectory. This ambiguity is reflected in its output. Suppose that the true operating point is r^* while the current measurement of the operating point is r. Then the regulator will add its correction to an incorrect nominal $v_n\ (t;\ r)$ instead of the true nominal $v_n\ (t;\ r^*)$. Denote by v the difference between $v_p\ (t)$ and $v_n\ (t;\ r)$. Then if the two nominal functions v_n are close

$$\begin{aligned}\dot{x} &= F_{r^*}x + G_{r^*}v + G_{r^*}(v_n(t;r) - v_n(t,r^*)) \qquad (5)\\ &= F_{r^*}x + G_{r^*}v + G_{r^*}\Delta_v\end{aligned}$$

The last term in (5) is a bias created by the regulator's inability to determine the true operating level.

There is a similar confusion in the determination of the state error. From (3)

$$\begin{aligned}x(t) &= x_p(t) - x_n(t;r) + \Delta_x(r,r^*) \qquad (6)\\ \Delta_x(t,r,r^*) &= x_n(t;r) - x_n(t;r^*)\end{aligned}$$

Denote by $y(t)$ the "measured" state error; i.e., the error the regulator thinks exists in the system state variables based upon its sensed value of the operating point, r.

$$y(t) = x_p(t) - x_n(t;r) \qquad (7)$$

Then

$$y(t) = x(t) - \Delta_x(t,r,r^*) \qquad (8)$$

The dynamic equation for the measured state follows directly from (5) and (8)

$$\dot{y} = F_{r^*}y + G_{r^*}v + \rho_1(r,r^*) \quad \text{if } r^*(t) = r^*(t^-) \qquad (9)$$

where

$$\rho_1(r,r^*) = G_{r^*}\Delta_v(r,r^*) + F_{r^*}\Delta_x(r,r^*) - \dot{\Delta}_x(r,r^*)$$

As discussed in the introduction, the peculiarity of this system which gives it its character is the discontinuous nature of r. In the case of the solar central receiver, these discontinuities are created by the passage of clouds over the heliostat field. Suppose that the set of permissible values of r is made discrete; i.e.,

$$r(t) \in S = \{1, \ldots, s\}$$

Then, from (8) it is clear that when there is a change in operation point $(r(t) = j \neq r(t^-) = i)$, there will be an associated discontinuity in y; i.e.,

$$\begin{aligned}y(t) &= y(t^-) + \lambda_{ij} \qquad (10)\\ \lambda_{ij} &= x_n(t;i) - x_n(t;j)\end{aligned}$$

where it has been assumed that x_p is continuous across discontinuities in r.

To provide a quantitative description of the variations in r, it will be assumed that r is a finite state Markov process described by a transition matrix $Q = [q_{ij}]$; [5].

$$\text{Prob}\,(r(t+\Delta) = j \,|\, r(t) = i) = \begin{cases} 1 + q_{ii}\Delta + o(\Delta) & i = j \\ q_{ij}\Delta + o(\Delta) & i \neq j \end{cases} \qquad (11)$$

The random process r^* will change in concert with r, and it will be assumed that the probability distribution of $r^*(t)$ depends on $r(t)$ and is conditionally independent of the preceding values of r.

The regulator tends to misclassify its operating condition. Its sensors indicate an operating level of $r(t)$, when in fact the operating level is $r^*(t)$. If the regulator was ignorant of this possibility of error, it would incorrectly identify its perturbation model and nominal trajectory. Equations (9) and (10) give the dynamic equations for the sensed errors.

Although y is the measured state error, the coefficients in the equation of equation (9) are not known by the regulator because of the uncertainty surrounding the value of r^*. The third term in (9) is a bias induced by the sensor error. If it were not for the variability of r, this bias would be of no consequence because its effect could be rendered negligible by placing an integrator in the forward path. In this application, however, the need to respond to sudden changes in r works against the use of a lag compensation.

As a performance index a quadratic weighting of state errors and actuating signal will be used

$$J = E\left\{x(t_f)P_f x(t_f) + \int_{t_0}^{t_f} (x'Mx + v'Nv)\,dt \,|\, y(t_0), r(t_0)\right\} \qquad (12)$$

subject to

$$P_f, M \geq 0;\ N > 0$$

Under appropriate technical assumptions, the feedback regulator minimizing J has desirable stability and sensitivity properties. The class of regulators generated by criteria like (12) is attractive in many instances in which the explicit form for J is difficult to justify.

The design problem described above bears striking similarity to those discussed in [1] and [2], but differs in a very important way. The indicated references were concerned with a Markov decision problem, and were solved using the classical method of dynamic programming [6]. Despite the fact that r is assumed to be a Markov process, y will, in general, not be. A memory of past values of y and r would, if properly interpreted, given an indication of the actual value of r^*. For this reason, the problem as posed is a dual control problem [4] with all of the attendant computational difficulties.

In the current implementation of the operational feedwater flow rate regulator, these complications were avoided by the simple artifice of treating r and r^* as being identical. Indeed, no attempt to estimate r^* could be made since the regulator software did not have the flexibility to retain or manipulate the past values of measured system variables. An explicit use of the state derivatives in such a calculation would have been deemed inappropriate because of unmodelled state and measurement noise.

The objective of this design study is to use the current measurements of the operational state of the system $(y(t), r(t))$ in such a way as to achieve satisfactory performance. The optimal regulator of this class will not be sought, but as will become clear, if r and r^* are close, the deviation from optimal will not be large.

The Synthesis Algorithm

A brief look at the classical dynamic programming algorithm of stochastic control shows immediately the difficulties which prevent the designer from finding the explicit form of the optimal regulator. Let $\mathbf{F}_t$ be the information pattern (the σ-field) generated by the observations at the regulator; $\{y(s), r(s);\ t_0 \leq s \leq t\}$. Let $V(t;\ \mathbf{F}_t)$ be the value function

$$V(t;\mathbf{F}_t) = \min_U\ E\left\{\int_t^{t_f} (x'Mx + v'Nv)\,dt + x'(t_f)P_f x(t_f) \,|\, \mathbf{F}_t\right\}$$

where U is a suitably selected class of control policies. Then, formally, [6]

$$V(t;\mathbf{F}_t) = \min_U\ E\Big\{((y+\Delta_x)'M(y+\Delta_x) + v'Nv)\,dt$$

$$+ V(t+dt;\mathbf{F}_{t+dt}) \mid \mathbf{F}_t \Big\} \qquad (13)$$

The convoluted form of $V(t;\mathbf{F}_t)$ precludes an explicit solution to (13). Since $v(t)$ is adapted to $\mathbf{F}_t$, the requirement that the regulator "learn" the value of r^* is implicit in the minimization. This fact so complicates this quadratic optimization that actually solving (13) appears to be beyond hope for realistic system models.

Many investigators have studied (13), or more commonly, its discrete time equivalent. Simplification of (13) has been achieved by identifying as two separate tasks the operation of identification and regulation. Such approximations differ in their degree of complexity and the activeness with which they seek to identify the unmeasured system parameters [4].

For the class of systems under study here, even the simplest of these algorithms would be far too elaborate. For reasons discussed in more detail earlier, no memory is included in the regulator, and $v(t)$ must of necessity be a function of $(x(t), r(t), t)$ alone. While the regulator structure is thus constrained, the problem is simplified by the fact that r and r^* are "close," i.e., while the deviation terms in (9) are not negligible, they are small. It will, therefore, be assumed that $V(t;\mathbf{F}_t)$ is only weakly dependent on the past values of $(y(s), r(s))$; i.e., the value function admits to the approximation

$$V(t;\mathbf{F}_t) \cong V(t,y(t),r(t)) \qquad (14)$$

Note that in contrast with the current implementation, r is not identified with r^*. Instead, the error terms in (9) are retained. Equation (14) simply implies that r and r^* are close enough that the potential performance improvement attributable to learning is inconsequential.

Under this simplifying assumption, an approximate solution to (13) is easily produced using classical methods;

$$v = -N^{-1}\bar{G}'_i P_i (y+\alpha_i) \quad \text{if } y(t)=y, r(t)=i \qquad (15)$$

$$V(t,y,i) = (y+\alpha_i)' P_i (y+\alpha_i) + \beta_i \qquad (16)$$

where

$$\dot{P}_i = -P_i\bar{F}_i - \bar{F}'_i P_i + P_i \bar{G}_i N^{-1} \bar{G}'_i P_i - M - \sum_j q_{ij} P_j \qquad (17)$$

$$P_i(t_f) = P_f$$

$$\dot{\alpha}_i = (\bar{F}_i + P_i^{-1}M)\alpha_i - (\bar{\rho}_1(i) + P_i^{-1}M\bar{\Delta}_x(i))$$

$$+ \sum_j q_{ij} P_i^{-1} P_j (\alpha_i - \alpha_j - \lambda_{ij}) \qquad (18)$$

$$\alpha(t_f) = \bar{\Delta}_x(i)$$

$$\dot{\beta}_i = -\sum_j q_{ij}\beta_j - \gamma_i$$

$$\beta_i(t_f) = 0 \qquad (19)$$

where

$$\gamma_i = \alpha'_i M \alpha_i + \overline{\Delta'_x M \Delta_x}(i) - \alpha'_i M \bar{\Delta}_x(i) - \bar{\Delta}'_x(i) M \alpha_i$$

$$+ \sum_j q_{ij} (\alpha_i - \alpha_j - \lambda_{ij})' P_j (\alpha_i - \alpha_j - \lambda_{ij}) \qquad (20)$$

and where for a random variable $g(r, r^*)$

$$\bar{g}_r \text{ or } \bar{g}(r) = E\{g(r,r^*) \mid r\}. \qquad (21)$$

Equations (15) gives the near optimal regulator and (17)–(20) give the design equations for the functions which characterize the closed loop system. The regulator has the simple form of linear feedback controller with a bias. The gain term $-N^{-1}\ \bar{G}'_i P_i$ is precisely what would be appropriate if the random matrices (F_{r^*}, G_{r^*}) were replaced by their mean values $(\bar{F}_r, \bar{G}_r)$. This might be thought of as a "certainty equivalence" gain.

It is interesting to note that if the conditional distribution of r^* given r is symmetrical about r, and if the matrices (F_{r^*}, G_{r^*}) are locally linear functions of $(r-r^*)$ about (F_r, G_r), then the equation for P_i becomes that associated with $r \equiv r^*$. In such a circumstance, the analyst is justified in neglecting the misclassification errors in computing the gain.

The bias term in the regulator $\{\alpha_i, i \in S\}$ depends upon the mean value of the modal matrices $(\bar{F}_i)$ as well as the weighted mean of the bias terms $(\bar{\rho}_1(i) + P_i^{-1}M\bar{\Delta}_x(i))$. If r^* is symmetrically distributed about r and if Δ_x and Δ_v are locally linear, $\bar{\Delta}_x$ will be zero. Even in the case, however, $\bar{\rho}_1(i)$ may be nonzero because of the correlation which exists between say F_{r^*} and $\Delta_x(r, r^*)$. Thus, α_i is not predictable from a certainty equivalence argument even in the simplest situation. A nonzero mean bias in r $(E(r^* \mid r) \neq r)$ compounds this effect.

The effect of misclassification is most evident in the equation of β_i. Both first and second order moments of Δ_x serve as driving terms for β_i. In addition there is an implicit dependence of γ_i on $\bar{\rho}_1$ and $\bar{\Delta}_x$ through $\{\alpha_i; i \in S\}$.

Equations (15)–(21) give the design equation for a suboptimal regulator of a system subject to errors in measurement of its operating level. The equations may be solved by direct integration, and the resulting regulator is easy

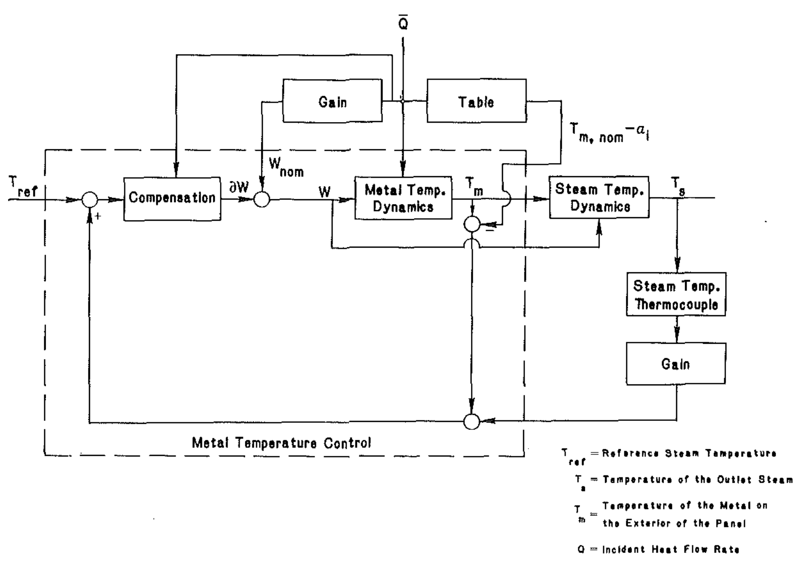

Fig. 1 A simplified steam temperature control loop

to implement. Because the form of the regulator precludes "learning," the performance of the system as given by (16) may be inferior to that obtainable by other causal regulators. Still, if r and r^* are close, the performance degradation will be small.

The precise penalty attributable to the failure to learn r^* is difficult to quantify because a method for finding the performance of the optimal controller is not available. Some idea of this loss can be obtained by comparing the performance of (15) with the performance of an optimal control when $r \equiv r^*$. This latter is easily produced from (15)-(21) by setting $\Delta_\nu \equiv 0$, $\Delta_x \equiv 0$.

If $\{F_i, G_i; i \in S\}$ is constant and if $(T_f - t_0)$ is large, there is an even simpler way to estimate impact of the failure to better estimate r^*. Denote the steady state rate of growth of V as

$$\hat{V}(y,i) = \lim_{(t-t_0)\to\infty} \frac{1}{t-t_0} V(t,y,i) \tag{22}$$

Under weak conditions on Q it is known that $\hat{V}(t, y, i)$ is independent of (y, i) and that [2]

$$\hat{V} = \mathbf{p}(\infty)'\gamma \tag{23}$$

where γ is a vector with components γ_i (see (20)) and $\mathbf{p}(\infty)$ is the vector of equilibrium probabilities of r. If γ_{OPT} is the value of (20) when $\Delta_x \equiv 0$, and $\Delta_v \equiv 0$, a simple figure of merit for (15) would then be

$$H = \frac{\mathbf{p}(\infty)'(\gamma_{(20)} - \gamma_{\mathrm{OPT}})}{\mathbf{p}(\infty)'\gamma_{\mathrm{OPT}}} \tag{24}$$

Since perfect identification is not possible, H represents a conservative measure of the loss attributable to the simple form of the regulator. If $H < .25$ perhaps, it would be difficult to justify the development of a more sophisticated regulator to replace (15).

An Example

To illustrate the foregoing analysis consider a simple model of a solar-powered boiler. Figure one shows a simplified block diagram of the steam temperature regulator used to control a single boiler panel on the central receiver. The isolation Φ on the panel acts to increase the exterior metal temperature of panel (T_m), and this in turn affects the temperature of the superheated steam (T_s). By comparing the reference temperature (T_{ref}) with the measured steam temperature, an error signal is generated. The nominal feedwater flow rate is proportional to insolation ($W_{\mathrm{nom}} = K\Phi$). The compensation acts on the sensed error to provide a correction (δW) to this nominal flow rate.

The steam temperature thermocouple sits in a protective well in the primary steam exit pipe, and is very slow to react to changes in outlet steam temperature. For this reason, the metal temperature inner loop must be responsive to sudden changes in loop conditions. This inner loop will be of primary importance in this section.

Reference [7] provides both linear and nonlinear models for a solar powered once-through boiler. These models have been used for deriving and evaluating various receiver control strategies. A simple model for the superheat section of the boiler can be written as

$$M_m C_m \dot{T}_m = \Phi - A_b u_b \left(\frac{W}{W_{\mathrm{ref}}}\right)^{.8} (T_m - T_f) \tag{25a}$$

$$h_{\mathrm{out}} = h_{\mathrm{in}} + \frac{A_b u_b}{W}\left(\frac{W}{W_{\mathrm{ref}}}\right)^{.8} (T_m - T_f) \tag{25b}$$

$$T_f = \frac{T_{\mathrm{out}} + T_{\mathrm{sat}}}{2} \tag{25c}$$

$$T_{\mathrm{out}} = f(P, h_{\mathrm{out}}) \tag{25d}$$

where in this specific instance

M_m = metal mass, $\mathrm{lb_m}$
C_m = specific heat/metal/btu/lb-°F
A_b = surface area, $\mathrm{ft^2}$
U_b = heat transfer coef., Btu/$\mathrm{ft^2}$-°F-s
h_{in} = input enthalpy = 496.68 Btu/$\mathrm{lb_m}$
P = outlet pressure = 1535 psia
W_{ref} = reference flow rate = 3.25 $\mathrm{lb_m}$/s

Equation (25a) relates changes in the panel metal temperature to changes in insolation (Φ) and an average fluid temperature (T_f). Equation (25b) relates the outlet enthalpy (h_{out}) to the inlet flow rate (W) and the associated temperatures. The fluid temperature is an average in the superheat region (25c). The outlet temperature is related to the outlet enthalpy by a nonlinear relation given symbolically in (25d).

Equation (25) is a simple thermodynamic model, but it illustrates nonlinear behavior of the panel. For example, metal temperature and output enthalpy are related to a fractional power of the input flow rate. For the specific panel to be studied, $M_m C_m = 46$, $A_b u_b = 9.2$. Suppose that $T_{\mathrm{sat}} = 600$°F and that the nominal output steam temperature is 960°F. The nominal values of the actuating signal and the panel temperature follow directly from (25). Let $(h_{\mathrm{out}} - h_{\mathrm{in}})_{\mathrm{nom}} = \Delta h_{\mathrm{nom}}$. Then

$$W_{\mathrm{nom}} = \frac{\Phi}{\Delta h_{\mathrm{nom}}} \tag{26a}$$

$$T_{m,\mathrm{nom}} = T_{f,\mathrm{nom}} + \frac{(W_{\mathrm{ref}}\Delta h_{\mathrm{nom}})^{.8}}{A_b u_b}\Phi^{.8} \tag{26b}$$

The nominal flow rate is directly proportional to the insolation while the panel temperature is proportional to Φ to a fractional power.

Suppose the receiver is operating on a partly cloudy day. For simplicity it will be assumed that the measured insolation takes on one of two widely separated values.

$$\Phi = \begin{cases} \Phi_1;\ 4.92\times10^2 \text{ Btu/s, dense cloud; } r=1 \\ \Phi_2;\ 3.49\times10^3 \text{ Btu/s, unobscured sun; } r=2 \end{cases} \tag{27}$$

The nominal operating point of the panel is contingent on the value of Φ. Equation (25) can be linearized about the nominal pair given in (26);

$$\dot{x} = F_i x + G_i v \tag{28}$$

where $x = \delta T_m$, $v = \delta W$ and

$$F_i = \frac{1}{M_m C_m}\left[\frac{(W_{\mathrm{ref}}\Delta h_{\mathrm{nom}})^{.8}}{A_b u_b}\frac{1}{\Phi_i^{.8}} + \frac{\Delta h_{\mathrm{nom}}}{2C_p}\frac{1}{\Phi_i}\right]^{-1} \tag{29a}$$

$$G_i = \frac{\Delta h_{\mathrm{nom}}}{M_m C_m}\frac{\left(\frac{1}{\Phi_i} + \frac{1.6\,C_p\,W_{\mathrm{ref}}^{.8}}{A_b u_b \Delta h_{\mathrm{nom}}^2}\frac{1}{\Phi_i^{.8}}\right)}{\left(\frac{1}{\Phi_i} + \frac{2\,C_p W_{\mathrm{ref}}^{.8}}{A_b u_b \Delta h_{\mathrm{nom}}^2}\frac{1}{\Phi^{.8}}\right)} \tag{29b}$$

where C_p is the specific heat under outlet conditions.

The model given by (25) is, of course, rather primitive. The dynamics are essentially first order, and the temperature gradient across the panel is not explicitly identified. The nonlinearity in (25) contains a power law in W with value 0.8 in (25a) and -0.2 in (25b) with a product nonlinearity in both (25a) and (25b). Equation (29) gives the coefficients of the linearized model. It is evident that F_i is strongly dependent upon Φ_i. At high insolation levels, the metal temperature error responds (relatively) quickly to changes in the flow rate, while if Φ_i is small, the system is more dilatory. The dependence of G on Φ is more complex in appearance, but in fact G is only weakly dependent upon Φ.

The indicated description of the solar-powered boiler is typical of a large class of systems which have a well defined, smoothly varying, linear model whose parameters are determined by the operating point of the system. A fixed parameter, linear model would predict closed loop performance quite well if it were not for the large and abrupt variations which occur in operating point. The size of these changes and the unexpectedness of their occurrence limits the utility of a time invariant linear model.

A feedforward control strategy can be quite effective in reducing the size of the transients associated with the variation in Φ. Ideally, this would operate as shown in Fig. 1 with the measured value of Φ used to determine the nominal flow rate (W_{nom}), the nominal output ($T_{m,\text{nom}}$) and the appropriate compensation. The feedforward link would avoid the delay associated with waiting for the effect of insolation changes to be reflected in metal or steam temperature changes.

For reasons discussed earlier, this feedforward path may introduce errors of its own because the true value of Φ cannot be measured without error. To illustrate the effect of this type of error, consider a specific example in which there exists the possibility of a positive bias in the measurement of Φ; i.e.,

$$\begin{aligned}&\text{Prob}\,(\Phi_r=\Phi_{r^*})=0.5\\&\text{Prob}\,(\Phi_r=1.2\Phi_{r^*})=0.5\end{aligned}\qquad(30)$$

The measured value of insolation, Φ_r, is equal to the true value, Φ_{r^*}, half of the time. The rest of the time, the measurement has a 20 percent positive bias. While there are only two possible values of Φ_r (see (27)), the actual insolation level will take on one of four different levels.

To model the behavior of r, cloud data from the Barstow, CA site of the receiver is available. Near noon on a day in May of 1979 it became partly cloudy. The mean interval that clouds obscured the heliostats was approximately 2.3 minutes. The mean duration of a clear interval was 4.3 minutes. A simple Markov model matching the sample means is

$$Q=\begin{bmatrix}-0.43 & 0.43\\ 0.23 & -0.23\end{bmatrix}$$

The performance weights in (12) are to some extent arbitrary. For the purposes of this example consider the infinite horizon problem with $M = 3.55$ and $N = 100$. This choice of performance index leads to satisfactory closed loop response in the deterministic problem, and it is a natural first choice in the stochastic problem.

To better understand the effect of the classification errors, it is convenient to build up a hierarchy of control policies, and to study their performance. A lower bound on the performance attainable in this system is given by the case in which there are no errors in measuring Φ; i.e.,

$$\text{Prob}\,(\Phi_r=\Phi_{r^*})=1\qquad(31)$$

The solution to this idealized problem is given by the algorithm of [2]

$$v_{\text{OPT}}=\begin{cases}0.1577(y-5.914)\ \text{lb}_\text{m}/\text{min if } \Phi_r=\Phi_1\\ 0.0809(y+2.491)\ \text{lb}_\text{m}/\text{min if } \Phi_r=\Phi_2\end{cases}\qquad(32)$$

The long term average rate of cost growth is

$$\hat{V}_{\text{OPT}}=2021\qquad(33)$$

The performance given by (33) represents a goal to be sought, but measurement errors will prevent its attainment. Actually, it is something of a misnomer to refer to the regulator of (32) as optimal. It uses "perfect" learning which is precluded by hypothesis in this example. It also has to contend with only two operating levels, compared with the four levels of Φ_{r^*} in (30). Still $\hat{V}_{\text{OPT}}$ does given an optimistic indication of the performance attainable from a regulator which has the ability to remember and manipulate past observations.

The coefficients of the feedback regulator described in (15)–(21) can be evaluated to yield

$$v_1=\begin{cases}.1597\,(y-2.049)\ \text{lb}_\text{m}/\text{min} & \text{if } \Phi_r=\Phi_1\\ .08551\,(y+7.984)\ \text{lb}_\text{m}/\text{min} & \text{if } \Phi_r-\Phi_2\end{cases}\qquad(34)$$

and

$$\hat{V}=2278\qquad(35)$$

It is evident from (33) and (35) that the performance deterioration attributable with the errors in measuring Φ_{r^*} is slight when the control policy given in (15)–(21) is used. In fact H (see 24)) is only 13 percent. Hence the use of a more sophisticated regulator than (34) would be hard to justify. The gains of v_1 and v_{OPT} are quite close, but the biases differ significantly.

The bias in v_{OPT} is the result of the regulator's attempt to prepare the system for the inevitable transition from one operating level to another. Thus, v_{OPT} attempts to maintain a higher operating temperature when $\Phi = \Phi_1$ ($T_m = 1022.4$°F) than would be the case if no change in Φ_r were expected ($T_m = 1016.5$°F). In this way v_{OPT} minimizes the disturbance induced by a change in Φ_r. Similarly, if $\Phi_r = \Phi_2$, a lower operating temperature is called for ($\alpha_2 = 2.5$°F).

The situation facing the regulator given by (34) is more complex. The bias effects mentioned in the preceding paragraph are still relevant. Further, there are the biases due to misclassification in the feedback link. Because $\Phi_r \geq \Phi_{r^*}$, the regulator knows the nominal temperature based upon r is greater than or equal to the true nominal. A reasonable, but myopic, policy might be to modify the bias in the regulator of (34) by adding the mean error in measuring Φ; i.e.,

$$v_{\text{MYP}}=\begin{cases}0.1577\,(y-1.679)\ \text{lb}_\text{m}/\text{min if } \Phi_r=\Phi_1\\ 0.0809\,(y+8.766)\ \text{lb}_\text{m}/\text{min if } \Phi_r=\Phi_2\end{cases}\qquad(36)$$

Equation (36) is a reasonably good approximation to (34), but since (36) is no easier to calculate or to implement than (34) is, there seems to be no advantage to using it.

The comparison of $\hat{V}_1$ with $\hat{V}_{\text{OPT}}$ indicates the ability to learn the exact value of r conveys little performance improvement in this example. To gain a understanding of the sensitivity of performance to the correct biases in v, consider a regulator which is designed without regard to either the random variations in Φ or the errors in measuring Φ. If $r \equiv r^*$ is assumed to be constant, classical LQ-regulator theory provides a direct synthesis algorithm. When applied to the model given by (25), a linear regulator results

$$v_{\text{LQ}}=\begin{cases}0.1623\,y\ \text{lb}_\text{m}/\text{min} & \text{if } \Phi_2=\Phi_1\\ 0.07915\,y\ \text{lb}_\text{m}/\text{min} & \text{if } \Phi_r=\Phi_2\end{cases}\qquad(37)$$

The regulator identifies r with r^* and assumes that the extant conditions are permanent.

The LQ-regulator has gains which differ little from those derived for (31). To see how it performs, suppose (pessimistically) that the sensor measuring r^* has the 20 percent bias discussed earlier and that r is a random process described by Q. The long term growth in cost is easily computed

$$\hat{V}_{\text{LQ}}=3530$$

The performance deterioration attributable to the neglect of biases is 74.7 percent. On the other hand, the biases in (31) make it rather insensitive to the actual sample path of r.

Conclusions

This paper provides the analytical basis for an easily designed and simply implemented regulator which is useful in systems in which there are possible errors in the measurement of the nominal operating point. Under the assumption that these errors are relatively small, it is shown that a complicated regulator which has the ability to "learn" the unknown parameters of the system may be unnecessary. For the feedwater flow rate regulator discussed earlier, perfect information provides only slight performance improvement over the proposed regulator.

As the example indicates, neglectng the measurement errors and the random variation in operating point may give rise to unacceptable performance. A simplified approach which simply replaces the random biases by their mean values is of some use, but it does not lead to a simpler implementation, and for that reason its use does not seem warranted.

The utility of the design procedure proposed here depends upon the measurement error being small. If the error in the measurement of r^* could be large or if the regulator has the capacity for sophisticated data manipulation, an "active adaptive" regulator may be justified. Many situations have constraints which preclude a solution of such versatility. In this event, (15) provides a useful approximation.

As indicated in the example, the primary effect of the measurement errors is in the calculation of the regulator bias. Equation (18) would be simplified if $r \equiv r^*$ because then $(\bar{\rho}_1(i) + P_i^{-1} M \bar{\Delta}_x(i))$ would be zero. The size of this term gives an immediate indication of the relevance of the measurement errors in the regulator synthesis. From the definition of ρ_1, if F_r and G_r are slowly varying functions of r, and if $\bar{\Delta}_x$ and $\bar{\Delta}_v$ are nearly zero, a design algorithm neglecting the error in measuring r^* will suffice. When there is a significant bias in r, the utility of the regulator proposed here will be heightened.

The development of the regulator and inferences derived from the example depend upon the dynamic hypotheses implicit in (2). This is a problem shared by all model dependent analytical procedures. Because the model selected for this paper contains a family of possible realizations (sample functions of r), one would expect that the regulator given by (15)–(21) will tend to be less sensitive to errors in the equations of system evolution than would a regulator "tuned" to a specific, time invariant system model. Once the principal sources of modeling ambiguity are identified, a sensitivity study will provide a quantitative indication of the performance changes associated with parameter variations.

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