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Enhanced Axial Dispersion in Oscillating Pipe Flow With Different Solute Concentrations at Its Ends

The problem of the enhanced axial contaminant dispersion by sinusoidal oscillation of fluid in a cylindrical pipe connected to reservoirs filled with contaminated groundwater involving different concentrations of species has been examined. It is found that the axial contaminant transfer along the pipe can be greatly enhanced by the fluid oscillation, which leads to a rapid removal of contaminants from groundwater and develops a relative low concentration region in the neighborhood of the subsurface collector. The enhanced dispersion coefficients, D_{eff} , of several heavy metal ions and organic compounds with water as carrying fluid were computed, and the results show they are about four to six orders of magnitude larger than those in the absence of fluid oscillation. The operational energy cost is low. For example, by operating the presented U-type enhanced mass pump, whose total cross-section area in one leg is 0.0154 m^2 , it takes 15 days to extract and remove acetone from a contaminated groundwater body that is 12 m below grade and 10 m^3 in volume, such that the acetone concentration can be decreased from 650 mg/l to 325 mg/l (i.e., 50 percent lower than its original level) and the averaged daily (24 hours) operational energy cost is only 12.6 kJ (1.56 Btu).

Introduction

This paper presents a study into the problem of enhanced axial dispersion by sinusoidal oscillation of fluid in pipes connected to a clean water tank at one end and to a subsurface reservoir (collector) filled with contaminated groundwater at the other end, in which the concentrations of various hazardous contaminants are uncontrolled and, hence, time-dependent. The described pipe system in which the fluids oscillate is called the Enhanced Mass Pump (EMP), which is a key component of our currently studied hydrodynamic enhanced barrier system (HEBS), designed to improve the effectiveness of the existing groundwater containment units. The HEBS generally includes a slurry wall, a subsurface groundwater collector installed parallel to the slurry wall at the contaminated groundwater site, an at-grade tank that supplies unlimited clean water or treatment liquid, and a U-type EMP which can be operated in either oscillating-flow mode or circulating-flow mode.

Figure 1 shows the schematic of a U-type EMP geometry, which essentially consists of two bundles of small diameter, open-ended thin pipes attached to the at-grade tank at one end and to the collector at the other. Assuming the groundwater flowing into the subsurface collector is small and negligible, the extraction of contaminants from groundwater by oscillation of fluid in pipes can be several orders of magnitudes larger than those in the absence of oscillations. This rapid removal of contaminants from groundwater creates a very low contaminant concentration zone at or near the subsurface collector, which acts like a hydrodynamic barrier superimposed on the existing groundwater containment units and effectively prevents the hazardous substances from further dispersing outward into the surrounding groundwater. When it is operated in the circulating flow mode, the contaminated groundwater is continuously pumped into the at-grade tank or treatment units. After proper treatment, the relative "clean" groundwater then will be routed back to the collector and mixed there. This continuing groundwater "replacement" depresses the concentration of contaminant in the groundwater and eventually results in a low concentration zone or a hydrodynamics barrier. Moreover, the HEBS also can be used as an injector to constantly deliver the treatment agents to places where human approach is restricted, such as the radioactive-related areas in power plants and/or groundwater systems.

Theoretical studies of dispersion for unidirectional laminar flows in straight pipes have been made by Taylor (1953) and showed that when a small quantity of diffusing substance is introduced into a fluid flowing along a circular pipe, the spreading of the cloud of the species is enhanced by the flow of the fluid. A similar effect occurs for oscillating flows, as pointed out by Bowden (1965) and later by Harris (Harris and Goren, 1967). The enhanced axial dispersion of contaminants in the presence of laminar oscillatory flow within a thin pipe was studied by Chatwin (1975), who suggested that the assumption of constant time-averaged axial contaminant gradient can be made. Watson (1983) showed that the effective axial dispersion of contaminants is proportional to the square of the tidal displacement, and the theoretical prediction was successfully tested experimentally by Joshi et al. (1983) and by Jaeger (Jaeger and Kurzweg, 1983; Jaeger et al., 1991). In the 1980s, studies were extended to the biomechanics area by Jaeger and Kurzweg (1983) and to the heat transfer field by Kurzweg and Zhao (1984), Kaviany (1986), and Zhang and Kurzweg (1991; Zhang, 1988). This later problem led to the interesting observation that the effective axial heat conduction rates increase some five orders of magnitude over those in the absence of oscillations, and the process does not require a convective mass exchange. In these studies, a constant concentration (or temperature in heat transfer) existing in the end-reservoir fluids was assumed. For many applications, including this one, however, the solutes concentration in the end-reservoirs are time-depen-

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dent. Studies of the enhanced axial dispersion by oscillation of fluid in pipes that are connected to end reservoirs filled with a variable solutes concentration is of particular interest in designing the hydrodynamics enhanced barrier unit for groundwater remediation at hazardous waste landfill sites. The EMP works like a smart mass pump, it extracts and removes the contaminants only from the source, not the whole water body. Apparently, this will greatly reduce the "clean-up" costs in the remediation activities at a hazardous wastes landfill site.

Formulation of the Problem

1. End-Reservoir Concentration. Because of the U-type EMP design (Fig. 1), the fluid static pressure is self-balanced during the oscillations, and the analysis of EMP can be simplified by examining only its one leg. Since gravity force is selfbalanced, it will not appear in the analysis. Figure 2 shows the schematic of a one-leg EMP rotated 90 deg from its vertical orientation. This figure presents one leg of the system in Fig. 1. The system in Fig. 2 has been previously analyzed (Zhang, 1988), although not for this application, and provides a mathematical approach to the problem. The mechanisms associated with Tank 1 in Fig. 2 do not exist in the collector of Fig. 1. In operation, the system shown in Fig. 1 will be removing contaminated water in one leg while injecting cleaner water from the other leg. The precise response of a contaminated aquifer to these imposed flows depends upon the aquifer's hydraulic characteristics. More often than not, contamination occurs in a surficial aquifer. It is expected, but not known as yet, that the water in the screened sub-surface collector will be refreshed through the screening by the surrounding contaminated aquifer water. The major focus of planned bench and pilot-scale experiments is to gain an understanding of this part of the system and what limitations it imposes.

With this understanding of the differences between the proposed system and previously analyzed mechanisms, we assume that the time-dependent groundwater solute concentration in or near the subsurface collector is $C_1(t)$ and that it is C_{01} at t =0. The solute concentration in the at-grade tank fluid is C_2 , which is held constant by properly adjusting the secondary flow rate, Q_2 . The contaminated groundwater of concentration C_1^i flows freely into the subsurface collector at a constant flow rate, Q_1 , and exits from it at the same flow rate; the solute concentration is the same as that in the collector (i.e., $C_1(t)$). The oscillator near the at-grade tank can set the working fluid in pipes into sinusoidal axial motion at an angular frequency, ω . The tidal displacement Δx , which is proportional to the amplitude of the applied periodical pressure, is always kept relatively small so that the condition of zero net convection

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between the pipe ends is maintained. In practice, the volume of the subsurface collector, V_1 , is often quite large, so that the variation of solute concentration in the collector within a single oscillatory period is very small and negligible, which assures a nearly constant time-averaged axial solute concentration gradient between the pipe ends. With these assumptions, the timeaveraged enhanced axial contaminant flux in oscillatory flow can be estimated by

$$q_{eff}(t) = D_{eff}\gamma A \tag{1}$$

where $\gamma = (C_1(t) - C_2)/L$ is the time-averaged axial concentration gradient between end-reservoirs, A is the total cross section area of pipes, L is the pipe length, and D_{eff} is the effective dispersion coefficient obtained from Equation 4. The concentration in the contaminated groundwater at or near the subsurface collector is described by the Volterra integral equation

$$C_1(t) = C_{01} + \frac{1}{V_1} \int \left\{ Q_1 [C_1^i - C_1(t)] - q_{eff} \right\} dt \quad (2)$$

By solving Eq. (2) with the initial condition of $C_1(t) = C_{01}$, the time-dependent concentration, $C_1(t)$, in the subsurface collector has the form

$$C_1(t) = (C_{01} - C_{tr})e^{-\kappa t} + C_{tr}$$
(3)

where $C_{tr} = (C_1^i Q_1 + C_2 Q_p)/(Q_1 + Q_p)$, $\kappa = (Q_1 + Q_p)/V_1$, and $Q_p = D_{eff}A/L$. The time required to achieve such a solute concentration can also be easily estimated from Eq. (3).

2 Effective Axial Dispersion Coefficient. The values of the effective axial dispersion coefficient, D_{eff} , can be determined from existing theory of contaminant dispersion by oscillating flow in pipes, as first done by Harris and Goren (1967) and later in more detail by Chatwin (1975) and Watson (1983), provided the restriction is made that one is dealing with dilute mixtures where the flow hydrodynamics are essentially controlled by the properties of the carrier fluid. By neglecting the direct axial molecular diffusion and end effects, and assuming laminar conditions, the effective axial dispersion coefficient for incompressible flow in a cylindrical pipe of radius *a* is

$$D_{eff} = \Phi(\alpha, \sigma) \omega \Delta x^2 \tag{4}$$

where ω is the oscillating frequency and Δx is the tidal displacement. The dimensionless enhanced axial dispersion coefficient, $\Phi(\alpha, \sigma)$, can be estimated from (Jaeger and Kurzweg, 1983):

$$\Phi(\alpha, \sigma) = \frac{1}{4\alpha\sigma(1 - \sigma^{-2})} \left[F_{I}(\alpha) - \frac{F_{I}(\alpha\sqrt{\sigma})|F(\alpha)|^{2}}{\sqrt{\sigma}|F(\alpha\sqrt{\sigma})|^{2}} \right]$$
$$\div \left(\left| 1 + \frac{2}{\alpha}F(\alpha) \right|^{2} \right) \quad (5)$$

where $\alpha = a\sqrt{\omega/v}$ is the Womersley number, $\sigma = v/D_m$ is the Schmidt number, v is the kinematic viscosity of the carrier fluid,



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Fig. 3 Effective axial dispersion coefficient versus Womersley number

a is the pipe radius, D_m is the molecular diffusivity, and $F_l(\alpha)$ represents the imaginary part of the complex function

$$F(\alpha) = -\frac{J'_0(\sqrt{-i\alpha})}{J_0(\sqrt{-i\alpha})} = i \left[\frac{ber'\alpha + ibei'\alpha}{ber\alpha + ibei\alpha} \right]$$
(6)

The first order Bessel function, J_0 , is related to the Kelvin functions by $J_0(\sqrt{-i\alpha}) = ber\alpha + ibei\alpha$. The enhanced axial dispersion coefficient $\Phi(\alpha, \sigma)$ described in Eq. (5) can be derived by reformulation of Watson's original result, first derived by Jaeger and Kurzweg (1983). It has the advantage that derivatives no higher than the first order of the Kelvin functions are needed.

3 Power Input Calculation. The rate of work done by the piston (or oscillator) to support the oscillation of the carrier fluid per unit length can be found by integrating the product of pressure and velocity over the pipe cross-section; the mean rate of power input for a single pipe is (Jaeger et al., 1991):

$$W_{pipe}(\alpha) = \frac{\pi \alpha^4 F_I(\alpha)}{8 \left| 1 + \frac{2}{\alpha} F(\alpha) \right|^2} \frac{\rho v^3 L \Delta x^2}{a^4}$$
(7)

The power required to extract a unit mass of contaminants from the end-reservoir and deliver it to the other end-reservoir by the carrier fluid through a parallel bundle of n pipes (radius r = a) within the EMP conduit is

$$W_{\rm EMP} = \frac{nW_{\rm pipe}}{q} = \pounds(\alpha, \sigma) \frac{\mu\omega L}{\gamma a^2}$$
(8)

where q = mass of contaminants delivered by n pipes and

$$\mathcal{L}(\alpha, \sigma) = \frac{2}{\sigma \alpha^2 (1 - \sigma^{-2})} \left[1 - \frac{F_I(\alpha \sqrt{\sigma}) |F(\alpha)|^2}{\sqrt{\sigma} F_I(\alpha) |F(\alpha \sqrt{\sigma})|^2} \right]$$
(9)

Equation (8) shows that the power required to abstract a unit of contaminant from groundwater with EMPs is directly proportional to the viscosity of the carrier fluid, the pipe length, and the oscillation frequency, while inversely proportional to time-averaged axial concentration gradient and the pipe's crosssection. For a given pipe length and axial concentration gradient, which are often constrained by design conditions, the use of lighter carrier fluid or a lower oscillation frequency will result in a more cost-effective manner of operation. However, to optimize the performance of the EMP device, one has to choose a compatible pipe diameter for which a reasonable oscillation frequency produces maximum mass transfer per energy cost. This is called tuning the apparatus. Rapid removals can be accomplished by increasing the oscillation frequency and the tidal displacement. However, if this is done in an untuned condition, the rapid removal may occur but an energy penalty must be paid and the possibility of turbulent flow must be considered. Tuning reduces the energy penalty by making maximum use of axial dispersion, and in reasonable systems does not lead to turbulent flow.

4 Limits of Laminar Flow. The foregoing analysis is based upon the assumption of laminar oscillating flow over the full length of the pipe. Transition to turbulent flow occurs when the Reynolds number exceeds a critical value that is generally accepted to be approximately 2,000, although some studies indicate larger values are possible (Hino et al., 1976; Ohmi and Iguchi, 1982). Thus, as a minimum condition for laminar flow:

$$\text{Re} = vD/\nu \le 2,000$$

Assuming water as the working fluid, $\nu = 10^{-6}$ sq m/s yields:

$$vD \leq 0.002 \text{ sq m/s}$$

for laminar flow to occur.

The general range of reasonable small pipe diameters is 0.003 to 0.0125 m ($\frac{1}{8}$ in to $\frac{1}{2}$ in). Thus, fluid velocity within these pipes cannot exceed 0.67 to 0.16 m/s, respectively, if laminar flow is to be assured. If we assume a Δx of 4 m, oscillation frequency would be limited to approximately 0.17 to 0.04 rad/s to assure laminar flow.

Using a Womersley number of 0.14, the oscillation frequency is limited to 0.0022 to 0.00013 rad/s (about 8 to 0.5 rad/hr), respectively. In the untuned case where the Womersley number is 1.0, the oscillation frequency is limited to 0.11 to 0.0064 rad/ s (approximately 400 to 23 rad/hr), respectively. Based on this, the maintenance of laminar flow must be considered but appears not to limit the present analysis.

Results and Discussion

The function $\Phi(\alpha, \sigma)$, described in Eq. (5), has been programmed, and the results of computer evaluations of the effective axial dispersion coefficients as a function of Womersley number, α , for several different values of Schmidt numbers, σ , are given in Fig. 3. The curves show a single maximum that shifts to a lower value of the Womerslev number as σ is increased. These curves are called tuning curves, since for a fixed oscillating frequency their peak indicates the pipe diameter at which contaminants extracted from the end reservoirs is optimized. Physically, the tuning condition implies that the optimum enhanced axial contaminants transfer can be achieved only when the radial molecular diffusion time is equal to one half of the oscillation period, or $\sigma \alpha^2 = \pi$. For Schmidt numbers greater than 1, these tuned values of $\Phi = D_{eff}/\omega\Delta x^2$ are approximately 0.02. The observed drop-off in the peak values of $\Phi(\alpha, \alpha)$ σ) for very small σ is attributed to an inability to store contami-

 Table 1
 Tuning values of several heavy metals and organic compounds

	Cs	Ag	Cd	Pyridine	Acetone	HCl
$\sigma (10^{-4} \text{ m}^2/\text{s}) \Phi_{\text{max}} \alpha (\text{for tuned } \Phi_{\text{max}})$	489.3	610.4	635.1	1734.5	785.9	381.06
	0.0191	0.0196	0.0195	0.0195	0.0192	0.0196
	0.158	0.158	0.158	0.1	0.14	0.2

¹ With water as working fluid ($\nu H_2 O = 10^{-6} \text{ m}^2/\text{s}$)

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Fig. 4 The relative acetone concentrations versus oscillating frequency under tuned and untuned conditions (t = 15 days, $V_1 = 10$ m³, $\Delta x = 4$ m)

nants in the thin boundary layer that occurs in a fluid of low kinematic viscosity.

Table 1 presents the enhanced axial dispersion coefficient at the tuning point for several toxic heavy metals and organic compounds, which often are observed in contaminated groundwater. As a comparison, the tuning curve for HCl also is tabulated. The tuned values of the enhanced axial dispersion coefficients are seen to lie between 0.0191 and 0.0196, which agrees with the observations illustrated in Fig. 3. Note that, for the three metals, the tuning occurs at a Womersley number of 0.158, while for pyridine the Womersley number is 0.1, and for acetone, 0.14. Since optimum contaminants extraction from contaminated groundwater, for fixed oscillation frequency, can occur only at or near the tuning point, Fig. 3 suggests a pipe size for which a given oscillation frequency produces a tuned condition. In engineering practices, however, the tuning condition may not be achievable and the system may have to be operated at a larger α . Although this will decrease D_{eff} , the enhancement can be still very large.

It is pointed out that the effective dispersion coefficient, D_{eff} , at the tuning point is directly proportional to the product of the oscillation frequency and the square of the tidal displacement. Accordingly, the increase in the tidal displacement can always lead to a large augmentation in the solutes extraction rate from the contaminated groundwater, but at a significant energy penalty compared to the tuned condition. To illustrate the theoretical predictions, we have computed, as a sample, the variation of acetone concentration in groundwater at or near the subsurface collectors (see Fig. 2). We assume that the volume of the subsurface collector, V_1 , is 10 m³ and that it is located 12 m below grade. The tidal displacement used in the study is 4 m, the acetone concentration at t = 0 is 650 mg/l, the inflow concentration remains 650 mg/l at all times during the operation, and the inflow rate is $Q_1 = 1.157 \times 10^{-6} \text{ m}^3/\text{s}$. This flow represents the rate at which the water in the underground collector is refreshed. It has little effect unless it becomes too small to maintain contaminant concentration in the collector at levels sufficient to drive the desired diffusion. The selected value for this flow $(0.1 \text{ m}^3/\text{day})$ is somewhat arbitrary, but is felt to be within a range of reasonable values for field conditions. The value is controlled by aquifer conditions and the surface area of the underground collector. Under realistic operating conditions, we only have limited control over the latter of these.

Figure 4 shows the relative acetone concentration, $C_1(t)/C_{01}$, curves as a function of oscillation frequency under tuned ($\alpha = 0.14$) and untuned ($\alpha = 1$) conditions. These curves represent the relative concentrations of acetone in groundwater after 15 days of EMP operation. The acetone concentration decreases drastically as the oscillating frequency increases; for example, if $\omega = 1$ rad/sec, the relative acetone concentration, $C_1(t)/C_{01}$, observed at the end of the 15 days of EMP operation is



Fig. 5 Relative acetone concentration as a function of operation time

approximately 0.001 under untuned conditions, while it is 0.027 under tuned conditions. Actual operation levels would likely be one to four orders of magnitude smaller than this frequency. The relatively lower residual solute concentration of the EMP operating under untuned conditions is due to the use of a larger oscillating frequency for a given small-diameter pipe. However, running at a still higher oscillating frequency would mean the use of even smaller-diameter pipes in the U-type EMP conduit, higher operational energy cost, and hence inefficiency.

The effect of tidal displacement is the larger the displacement, the lower the remaining contaminant concentration remaining in groundwater. In the acetone example, simulation shows acetone concentration in the groundwater after the EMP has been operating for 15 days. Under the untuned condition (Womersley number, $\alpha = 1$, $\omega = 0.001$ rad/s), the acetone concentration decreases from 650 mg/l to 380 mg/l, which is quite low in comparison with the surrounding groundwater.

The depth from the earth's surface to the collector, which is also the pipe length in the EMP, will affect the rate of contaminant removal from groundwater. For a given tidal displacement, the deeper the collector is located, the less efficient the EMP technique becomes. However, if a variable tidal displacement is used, the depth of the water table will not be a problem in the EMP process. For example, the device might be designed in which the tidal displacement is always equal to one-third of the depth. In an acetone example, the acetone concentration in groundwater decreases as the depth of collector increases. When the depth is equal to 30 m, the acetone concentration is reduced to 400 mg/l under tuned condition and to 175 mg/l under untuned conditions, which is about 28 percent of its original level with 15 days of EMP operation.

The residual acetone concentration in groundwater is a function of the length of time that the EMP is operated. As shown in Fig. 5, under untuned conditions, the acetone concentration can be reduced to 50 percent of its original level after 20 days



Fig. 6 Dimensionless operational costs as a function of Womersley number

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 Table 2 Operating energy costs for removal of acetone in a single pipe under tuned and untuned conditions (with EMP leg's conduit diameter of 14 cm [5.5 in])

	Pipe diameter (cm)	Oscillating frequency (rad/s)	Power input for two legs (watts)	Total daily energy costs (J)	Average daily acetone removed (g/day)
Tuned	$0.63 \left(\frac{1''}{4}\right)$	0.002	9.2×10^{-6}	0.79	75.3
$(\alpha = 0.14)$	$0.32 \left(\frac{1''}{8}\right)$	0.008	1.5×10^{-3}	12.67	170.1
Untuned	$0.63 \left(\frac{1''}{4}\right)$	0.099	0.043	3688	82.3
$(\alpha = 1)$	0.32 (1)	0.397	0.683	59013	245.6

of operation and to about 30 percent after 40 days of operation. Under tuned conditions, the corresponding relative acetone concentrations are about 77 percent and 64 percent, respectively. As shown by Eq. (3), the contaminant concentration is an inverse exponential function of time; it will not be a cost-effective way to achieve an even lower concentration level by solely increasing the EMP operation time. A lower concentration of certain solute or hazardous substances in the groundwater can be achieved by adjusting some other factors, such as the total crosssection area of the EMP, the tidal displacement, and the oscillatory frequency.

It is important to note that, for a fixed pipe diameter, the energy requirement increases as $\omega^{5/2}$, and the rate of contaminants removed from groundwater under tuned conditions increases in direct proportion to $\omega \Delta x^2$. Therefore, an increasing contaminant removal rate will be accompanied by a more rapidly increasing power import at a high oscillation frequency, thus making the EMP process inefficient there. Fortunately, in most of the discussed cases, the Schmidt numbers of various solutes in groundwater are from several hundreds to more than one thousand, and the tuning points for α are approximately or less than one (Fig. 3). Figure 6 shows the dimensionless energy curve as a function of Womersley number in a single pipe, which can be used to estimate the operational energy requirement of EMP. For the discussed case, the estimated operational energy costs of EMP are presented in Table 2.

Table 2 shows that the energy cost for a single pipe is lower under tuned conditions than under untuned conditions and that the use of larger pipes generally leads to lower energy costs, but they will also decrease the contaminant transport rate. For example, if $\frac{1}{4}$ in. (0.00625 m) diameter pipes are used, the averaged daily solute extraction is 75.3 grams under tuned conditions and 82.3 grams under untuned conditions, their ratio is approximately 1.09. However, the daily energy cost for the untuned operation is about 4,688 times higher than those of tuned operation. This implies that with $\frac{1}{4}$ in. (0.00625 m) pipes, to remove the same amount of hazardous substances from the contaminated groundwater with EMP being operated under untuned conditions will be 4300 times more expensive than under tuned conditions. For $\frac{1}{8}$ in. (0.003175 m) diameter pipes, this ratio is 3,234. However, to achieve the same solute concentration level, the operational energy cost with the U-type EMP being operated under tuned conditions is less than one-third of the pumping cost, which agrees with the findings by Jaeger (Jaeger et al., 1991).

Conclusions

Earlier studies have shown that oscillation of fluid columns in appropriately sized pipes greatly increases axial diffusion. This paper studied the application of this result to the removal of contaminants from groundwater by simulation using a mathematical model.

Application of oscillating pipe flow to the removal of water soluble contaminants from groundwater offers a means of essentially pumping contaminants out of the ground. Using the contaminated water as a working fluid, the contaminants can be removed as they are brought up without the problem of disposal of large amounts of water. The mathematical simulation indicates that this can be a viable method for removal of contamination.

The optimum harnessing of the enhancement of the axial transfer of contaminants in the system can be realized when the ratio of the pipe radius, a, to the boundary layer thickness, $\delta = \sqrt{2\nu/\omega}$, satisfies the tuning condition, $\delta/a = \sqrt{\sigma/\pi}$. This means that the optimum effect for low-energy removal of contaminants from the contaminated reservoir liquid by oscillation of fluids in pipes can be achieved only if the solute radial diffusion time, $t_d = a^2/D_m$, is equal to the half period of the oscillation, $t_h = \pi/\omega$.

The operational energy cost is low; for the discussed acetone case, to reduce the acetone concentration in groundwater to 50 percent of its original level (650 mg/l), the enhanced axial acetone flux over the entire EMP cross-section area (0.031 m^2) is 4.45 g/hour, which is about 2.986×10^5 larger than that in the absence of oscillation, and the power required is only 0.145 watts under tuned conditions and 677 watts under untuned conditions.

The energy analysis shows that operation in a tuned mode will result in slower removal of contamination, but dramatically reduces the energy cost of operation. This opens the possibility of remote, solar-powered units slowly pumping contaminants out of contaminated aquifers for storage in activated carbon or other appropriate absorbent or adsorbent. In some cases, solar energy or an other active agent may be used to directly destroy the contaminants.

The next step is to conduct bench and pilot-scale experiments to verify the simulation and test assumptions made regarding the influence of aquifer characteristics upon the process.

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