

Nonlinear Effects on the Stresses and Deformations of Bolted Joints

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Axisymmetric finite element modeling of bolted joints was performed to show the effects of the magnitude as well as the radial location of the externally applied load on the member separation radius and the stress on the surface between the two members. The separation radius was found to be nonlinearly related to changes in the magnitude as well as position of the external load. A 27-percent decrease for 24-mm bolts to 39-percent decrease for 8-mm bolts in the separation radius resulted with changes in the load magnitude. The external load varied from zero to the maximum that could be sustained before joint separation for steel members. The change in separation radius for the aluminum members, cast iron members, and a combination of the two materials was on the order of 2–10 percent. For the minimum and maximum external load, the separation radius decreased by 5 and 12 percent, respectively, with an increase in radial position of one to five bolt diameters for the 24-mm bolt models. Changes in the stress on the surface between the members also occurred with changes in magnitude as well as radial position of the external load. The stress was found to be higher near the bolt for larger external loads and also when the radial location of the external load was increased.

Introduction

Historically, the value of the bolt as a fastener would be difficult to estimate. Furthermore, it is probable that bolted connections will continue to increase in usefulness in the future. Thus, greater understanding of their mechanical behavior in joints such as shown in Fig. 1 continues to be of value to design engineers. This paper reports results of ongoing finite element analysis (FEA) research which is directed toward revealing phenomena related to axisymmetric bolted joints. It is probable that these results can be extrapolated to joints with various patterns of the fasteners. The main focus of this paper is on the effect of external loads on the stress distributions and deformations of bolted joints.

Contributions to the literature on bolted joints are numerous. Discussions of some of the early work can be found in Grosse and Mitchell (1990), Wileman et al. (1991), and Lehnhoff et al. (1992, 1994). Kulak et al. (1987), Bickford (1990), and Zahavi (1992) provide many references and other good background material in their texts.

Grosse and Mitchell (1990) have given an excellent discussion of both the geometric complexities and the nonlinearities of bolted joints. They, along with Shigley and Mischke (1989) and various other authors, have noted that the usual design objective is to reduce the portion of the external load carried by the bolt, P_b . Since the external load, P , usually cycles during service, the purpose in reducing P_b is to reduce the cyclic load component the bolt is subjected to and thus extend the fatigue life. By making the right design choices for the preload, F_i , and the stiffnesses k_b and k_m , the bolt force, F_b , can be caused to experience less of the cyclic external load, P . Shigley and Mischke (1989) give the basic joint force equations as

$$C = \frac{k_b}{k_b + k_m} \quad (1)$$

$$P = P_b + P_m \quad (2)$$

$$F_b = CP + F_i = P_b + F_i \quad (3)$$

$$F_m = (1 - C)P - F_i = P_m - F_i \quad (4)$$

These equations are usually taken to be linear because it is assumed that the stiffness values are constants similar to material properties. The stiffnesses are usually obtained by applying a preload, F_i , in the absence of an external load ($P = 0$). The deformations are determined and the stiffnesses are calculated from

$$k_j = \frac{F_j}{\delta_j} \quad (5)$$

where k_j is the stiffness of either the bolt, k_b , or the members, k_m ; F_j is the force in either the bolt, F_b , or the members, F_m ; and δ_j is a measure of either the bolt, δ_b , or the member, δ_m , displacements.

The presence of nonzero variable external loads means that the forces F_b and F_m depend on the external load, P , as well as the preload, F_i . The stiffnesses depend on the loads and displacements and the loads and displacements depend on the stiffnesses with a resulting nonlinear relationship (Zill, 1993). Then when the stresses are calculated the relation

$$\sigma_j = \frac{F_j}{A_j} \quad (6)$$

(where σ_j is either the stress in the bolt, σ_b , or the stress in the member, σ_m ; and A_j is either the effective area of the bolt, A_b , or the effective area, A_m , of the members) can symbolically be used to explain the nonlinearities related to the stresses. Thus, the stress depends on the displacements which are nonlinearly related to the external loads. In FEA, the stresses are calculated from

$$\sigma = C\epsilon = Cbu \quad (7)$$

where C is the matrix of material properties, ϵ is the strain matrix, b is a matrix of derivatives of shape functions, and u is the matrix of nodal displacements.

Contributed by the Pressure Vessels and Piping Division and presented at the Pressure Vessels and Piping Conference, Minneapolis, Minnesota, June 19–23, 1994, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the PVP Division, March 3, 1994; revised manuscript received June 27, 1995. Technical Editor: S. Y. Zamrik.

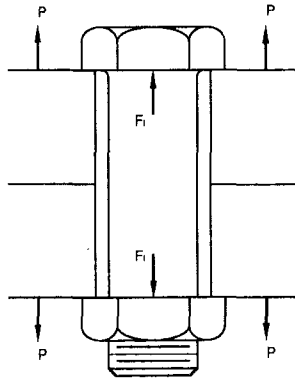


Fig. 1 Typical bolted joint

Although this relation is usually considered to be linear, it also can be nonlinear when the loads, stiffnesses and effective areas vary with the external loads, which this research shows to be true. The effective area calculations are sources of nonlinearity, especially for the members, and the nonlinearity also is impacted because the forces are affected nonlinearly by the stiffnesses which are affected by the external loads and displacements. The presence of external loads also changes the critical joint separation radius.

The Finite Element Model

Figure 2 is a schematic diagram of the axisymmetric model. The parabolic element size, type and distribution were all established in earlier studies, by Lehnhoff et al. (1992, 1993). They have been retained here to avoid any differences due to those factors. Gap elements were used at the contact surfaces and these were shown in the earlier work to give similar results to multipoint constraints in the absence of friction. The zero slope boundary conditions at the outer radius reflect the interaction of other bolts in this simulation. Figure 3 shows the finite element model with the deformed geometry superimposed on the undeformed geometry. The exaggerated deformations clearly show a physical separation of the members at the so-called critical radius, r_c . The reasons for the term critical relate to joint sealing, which will be discussed later. In this study, the bolt and nut threads were not modeled and friction was not included. Grosse and Mitchell (1990) were able to produce only phenomenological results, apparently in part because they modeled the threads and the model became quite complex. Further research on these effects is in progress.

The Nonlinear Analysis Procedure

The purpose of this analysis was to determine the extent to which the bolt and member stiffnesses, k_b and k_m , would change if calculated at increasing values of P . The effects on stress can then be inferred using the reasoning given in the introduction to this paper. A detailed discussion of the analysis procedure used to determine k_b , k_m , F_b , F_m can be found in Lehnhoff and Wistehuff (1996). Based on this discussion and the reasoning given in the introduction, it is logical to assume that since the

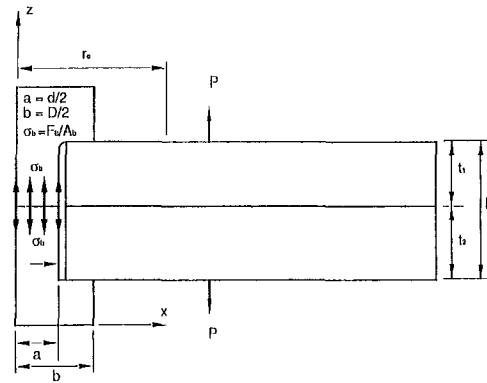


Fig. 2 Schematic of axisymmetric model

stresses in Eq. 6 depend on the bolt force, F_b , and the member force, F_m , which have been shown to be nonlinear because the stiffnesses are nonlinear, the stresses also should be nonlinear. The degree of nonlinearity should be of the same order of magnitude as that of the bolt and member stiffnesses. Even though the member stress assumption is in error, it will be of more benefit to designers to examine the actual finite element stress distributions, which are nonlinear functions of the distance from the axis of symmetry as well as of the external load. In the process of considering the stresses, it became evident that the critical separation radius, r_c , also is a nonlinear function of the external loads.

Bolt Spacing—Critical Radius

Proper bolt spacing to achieve sealing in metal-to-metal as well as gasketed joints has long been a subject of major interest to design engineers. Gasket manufacturers recommend clamping pressures for the particular gaskets which they manufacture. This data seems to be mostly derived from experience. Blake (1986), in his book on threaded fasteners, is less than complimentary to the state of the art of joint design to prevent leakage. Most design texts offer some preliminary guidance and then direct the reader to other sources such as the manufacturer. Edwards and McKee (1991) give a good representative treatment of the subject. The American Society of Mechanical Engineers Boiler and Pressure Vessel Code in Section VIII, Division 1, Appendices 2 and Y (ASME, 1989) gives an extensive presentation of gasketed, hard and/or soft flange joint design procedures. The flange loads are based on sound principles of mechanics, although most were developed before FEA became a practical design tool. ASME is sponsoring current research on gasketed joints so this data will change or be enhanced.

Other authors, for example, Gould and Mikic (1972) and Tang and Deng (1988) have observed the separation of the connected members at a distance, r_c , such as is shown in Fig. 2. For the models studied here the figures show the separation radius. Separation of the members at a certain radial distance is a phenomena that can be detrimental to maintaining a sealed joint. The compression of the connected members in the near vicinity of the bolt results in an elastic action that causes this

Nomenclature

F_b = force in bolt, N	P_m = portion of external load carried by members, N	E_b = modulus of elasticity of bolts, GPa
F_i = preload in bolt, N		S_p = proof strength, MPa
F_m = force in connected members, N	EP = external pressure load, N/mm ²	R_c = member separation radius, mm
P = external load, N	K_b = bolt stiffness, N/mm	d = bolt diameter, mm
P_b = portion of external load carried by bolt, N	K_m = member stiffness, N/mm	x' = radial distance from axis of symmetry, mm
	δ_b = displacement in bolt, mm	x_p = location of external load, mm
	δ_m = displacement in members, mm	

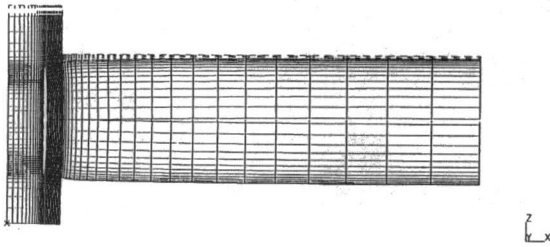


Fig. 3 Finite element model including the deformed geometry

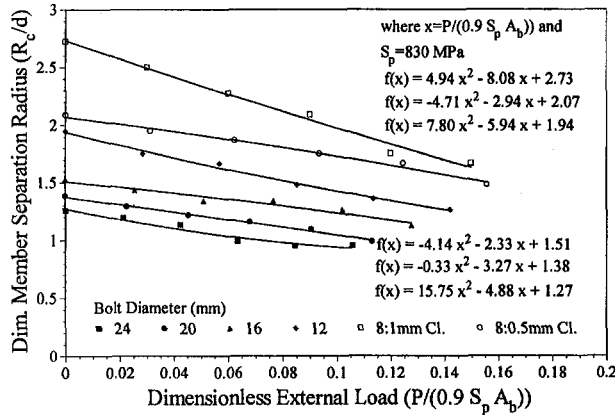


Fig. 4 Member separation radius versus external load (20/20 thickness ratio, steel)

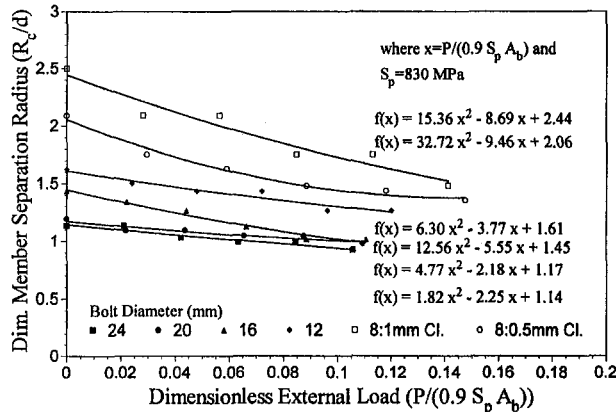


Fig. 5 Member separation radius versus external load (16/20 thickness ratio, steel)

member separation. Thus, controlling this tendency for the members to separate by proper spacing of bolts in the flange or joint group can help to prevent leakage. When the desired compressive pressure is known, the figures in this paper can be used to determine the proper bolt spacing so that the overlap of the stress distributions will produce the desired pressure for the parameters given.

Discussion of Results

Figures 4, 5, and 6 show the relationship between the member separation radius and the magnitude of the external load for steel members. The separation radius is the point where the two members separate due to the loading. The separation radius is a function of bolt size (Grade 10.9, $S_p = 830$ MPa), external load magnitude and location, and connected material thickness ratio. Larger bolts have larger separation radii. This is partly

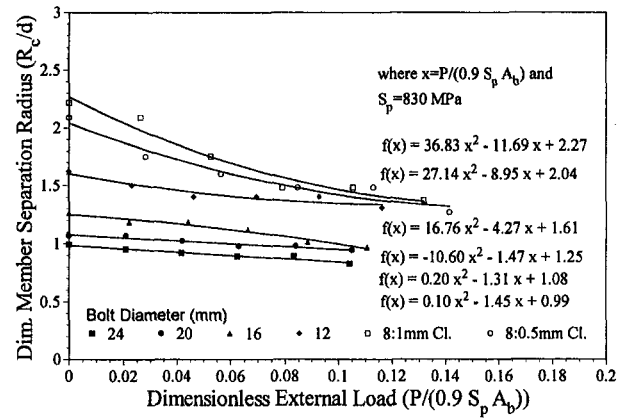


Fig. 6 Member separation radius versus external load (12/20 thickness ratio, steel)

due to the larger contact area between the bolt head and the member. It was found that for a constant load, the separation radius decreased in proportion to the decrease in bolt size. The 20/20 thickness ratio models also had higher separation radii, followed by the 16/20 and then the 12/20. The higher the external load, the smaller the separation radius. A 27-percent decrease for 24-mm bolts to 39-percent decrease for 8-mm bolts in the separation radius resulted with changes in the load magnitude from zero external load to the maximum external load that could be sustained, before joint separation, for steel members.

Figures 7, 8, and 9 show the relationship between the member separation radius and the magnitude of the external load for other member material combinations and 20-mm and 24-mm bolts. The materials chosen were aluminum, cast iron and a combination aluminum over cast iron. The member separation radius decreased by 2–10 percent for all bolt sizes and thickness ratios. The small decrease is attributed to the decreased member and bolt stiffness resulting in a decreased maximum external load. The data exhibited some scatter in that the separation radius was approximated to be the last node found to be in compression. The actual critical radius could have fallen anywhere between that node and the next node farther out. Further refinement of the finite element mesh would have helped reduce the scatter in the data. A second-order polynomial was fit to the curves to clearly show the trend.

Figures 10 and 11 show how the stress distribution in the members is a function of the position of the externally applied load. The farther out radially on the member the load is applied, the larger the stress in the member near the bolt. A closer

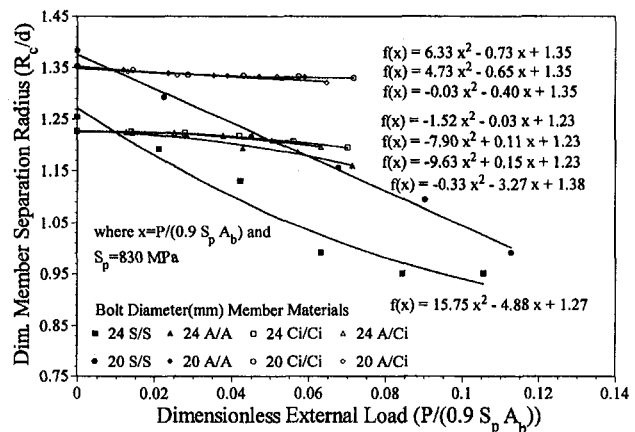


Fig. 7 Member separation radius versus external load (20/20 thickness ratio, various materials)

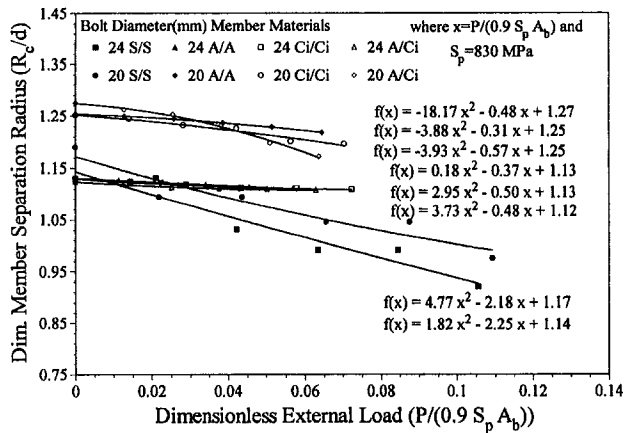


Fig. 8 Member separation radius versus external load (16/20 thickness ratio, various materials)

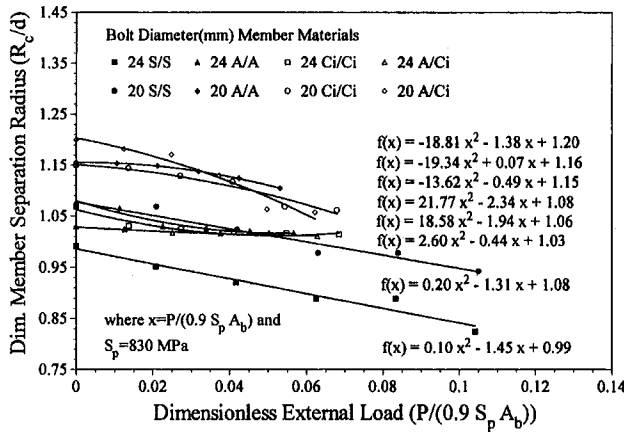


Fig. 9 Member separation radius versus external load (12/20 thickness ratio, various materials)

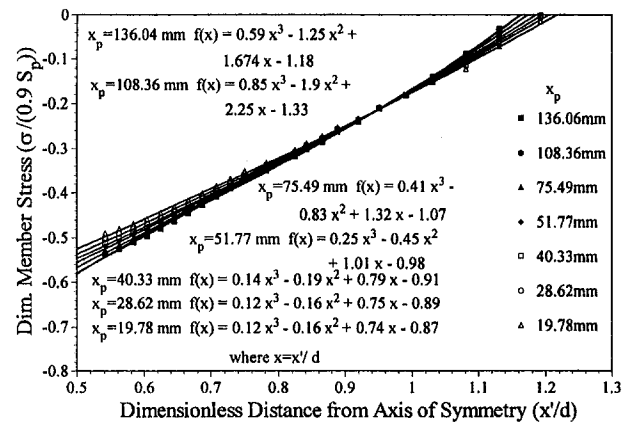


Fig. 10 Member stress versus distance from axis of symmetry—constant load $P = 9610.4$ N, variable position, 24-mm bolt, 20/20 thickness ratio, steel

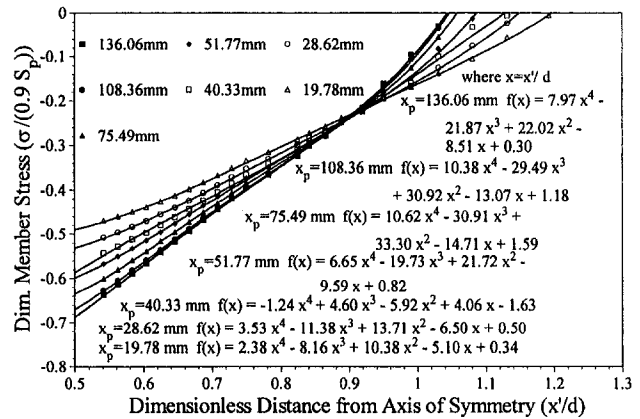


Fig. 11 Member stress versus distance from axis of symmetry—constant load $P = 31256.5$ N, variable position, 24-mm bolt, 20/20 thickness ratio, steel

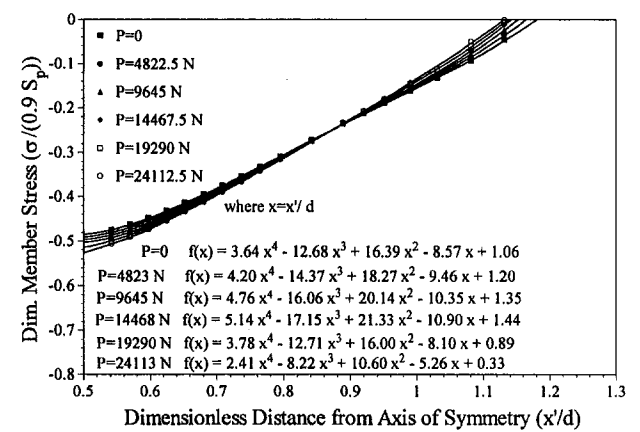


Fig. 12 Member stress versus distance from axis of symmetry—variable load, constant position, 24-mm bolt, 20/20 thickness ratio, aluminum

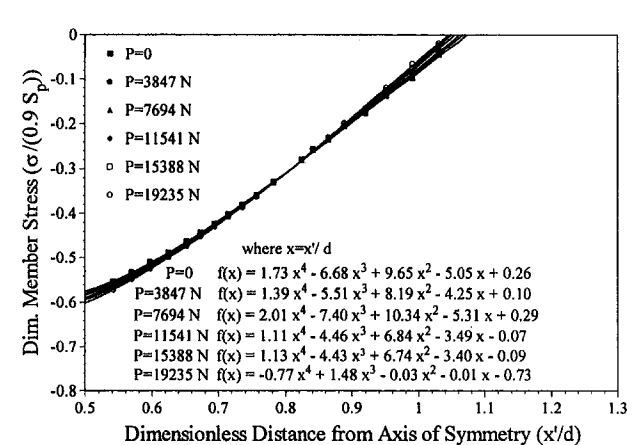


Fig. 13 Member stress versus distance from axis of symmetry—variable load, constant position, 24-mm bolt, 16/20 thickness ratio, aluminum

approximation of the separation radius can be found by extrapolation of the curves to the point where the stress becomes zero. For the minimum and maximum external load, the separation radius is reduced by 5 and 12 percent, respectively, with an increase in radial position of one to five bolt diameters for the 24-mm bolt models. The stress also decreased with radial distance at a faster rate both for the maximum external load and the maximum radial position of the external load, resulting in

a decreased separation radius. The stress plots were fitted with either third or fourth-order polynomials, depending on which showed the closest correlation.

Figures 12, 13, and 14 are for a 24-mm bolt with aluminum members. Figures 15, 16, and 17 are for a 20-mm bolt with aluminum members. The material properties are listed in Table 1. These figures show the stress distribution in the member as

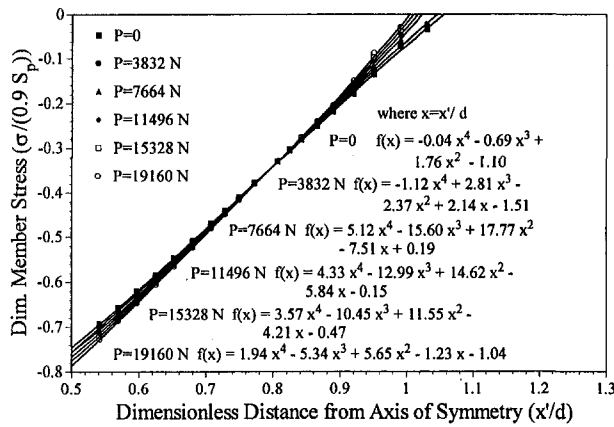


Fig. 14 Member stress versus distance from axis of symmetry—variable load, constant position, 24-mm bolt, 12/20 thickness ratio, aluminum

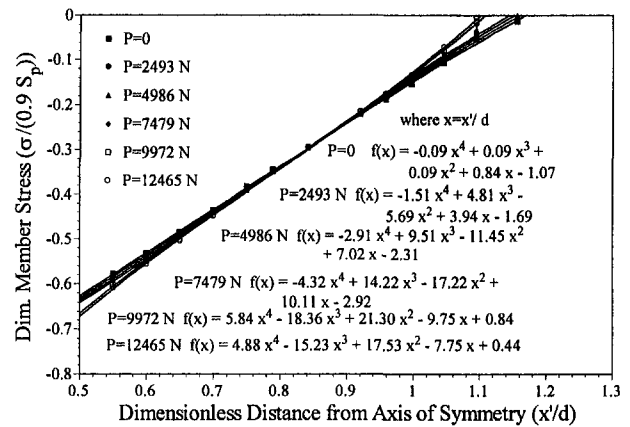


Fig. 17 Member stress versus distance from axis of symmetry—variable load, constant position, 20-mm bolt, 12/20 thickness ratio, aluminum

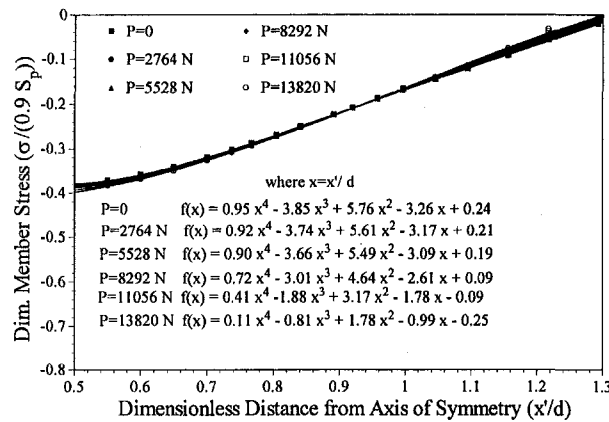


Fig. 15 Member stress versus distance from axis of symmetry—variable load, constant position, 20-mm bolt, 20/20 thickness ratio, aluminum

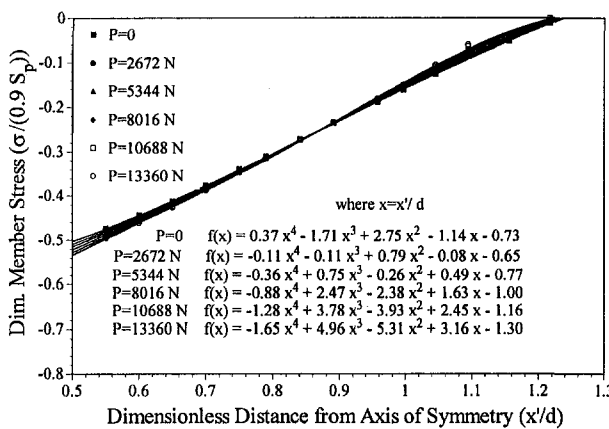


Fig. 16 Member stress versus distance from axis of symmetry—variable load, constant position, 20-mm bolt, 16/20 thickness ratio, aluminum

a function of distance from the bolt and as a function of the magnitude of external load. They show that the stress in the member is maximum closest to the bolt for the highest external load. The separation radius can be found by extrapolation to the point where the stress is zero. This separation radius value

Table 1 Mechanical properties of material

Material	Modulus of elasticity (N/mm ²)	Poisson's ratio	Density (kg/mm ³)
Steel	20.678 E+4	0.30	7.83 E-6
Cast iron	10.290 E+4	0.25	7.20 E-6
Aluminum	6.860 E+4	0.333	2.63 E-6

will also be an approximation in that the finite element mesh was not refined in the area found to contain the separation point. However, refinement of the mesh should only result in small changes in the separation radius. The separation radius is smallest for the maximum external load.

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