Who Runs Against the Incumbent? Candidate Entry Decisions

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Abstract

This paper offers an explanation for the common observation that political incumbents not only frequently win reelection, but often face weak competition or no competition at all when running for reelection. I explain this outcome by modeling the entry decision of potential election candidates as a process of self-selection. Candidates choose either to enter a political race against a known-quality incumbent or to wait for an open election. The model predicts that the entry decision is non-monotonic in candidate quality: Both low quality and very high quality candidates choose to enter the race. The tendency of mid-quality candidates to stay out increases the ex-ante probability that the incumbent will win, suggesting an explanation for incumbency advantage, the existence of uncontested races, and of "sacrificial lambs."

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1 Introduction

In the 2006 elections, 94% of U.S. House incumbents and 92% of incumbent governors won reelection[1]. In the 2008 elections, 8% of U.S. Representatives ran uncontested and the percentage was even higher for state representatives.[2] It is also common for U.S. incumbents to be challenged by "sacrificial lambs," candidates with very little chance of winning.[3] These features of U.S. elections are difficult to explain as the result of rational decisions by politicians. Why do some politicians choose to enter an electoral race while others wait for future elections? What factors influence which challenger ends up running against an incumbent? How can incumbency advantage be explained as the outcome of a rational entry decision by prospective challengers?

I am able to answer these questions by modeling the decision of a challenger politician who first decides whether to face an incumbent now or wait for an open election in the next electoral period. If a high quality incumbent is running for reelection, it is optimal for medium quality challengers to wait for the next election, where there is a chance that they will face a lower quality opponent. On the other hand, the prospect of losing a subsequent primary election can lead low quality challengers to contest the current incumbent. Finally, the high probability of winning the current election can lead high quality challengers to run against the current incumbent.

I analyze the challenger's entry decision using a two-period model with perfect information. I model the rational decision of a challenger who decides either to run against an incumbent or to wait for an open election in the next period when the incumbent's term limit is met. Entering implies a known low probability of winning against the incumbent but certainty regarding participation in the general election. Waiting is better if the prospective general election opponent is of lower quality than the incumbent, but it

[1] In the 2010 midterm elections, 85% of U.S. House incumbents and 84% of incumbent governors were reelected.

[2] For instance, 35% of Wisconsin State Senators were uncontested in 2008's elections. In 2010 this went down to 12%.

[3] For example, in 2004's Arizona Senate elections, a teacher with no political experience ran against incumbent John McCain. Likewise, in Wisconsin's 2008 2nd District House election, a computer programmer trainer with no political experience ran against incumbent Tammy Baldwin. In 2010, John Dennis did his first run for an elective office against Nancy Pelosi, Speaker of the House and incumbent representative of the 8th Congressional District of California.

introduces uncertainty regarding the participation in the general election: the candidate has to win the primary election first.

In my model, challengers differ only in quality; there is no assumed incumbency advantage or ex-ante difference in payoffs or costs of running for candidates.[4] I show that two types of candidates choose to enter: high quality candidates and low quality candidates. High quality candidates enter due to the high probability of winning the election against the current incumbent. Low quality candidates enter because they do not have much to lose, and the probability of winning the primary election next period is slim, thus not worth waiting for. Consequently, some incumbents are contested by two types of challengers, low and really high quality, what I call "double crossing." The pattern implies that the incumbent has an electoral advantage contingent on quality, with higher quality incumbents more able to deter entrance. I provide conditions on the underlying distribution of candidate quality and on the probability of winning the election to get the "double-entry" result. I illustrate the result with specific examples.

In a further contribution of the paper, I show that the model can be embedded into richer frameworks which include strategic incumbents and competition among strategic challengers. Using a richer framework, I find that incumbent incentives to spend money are non-monotonic in incumbent quality. Mid-quality incumbents get a double benefit from the expenditure since they not only have a higher chance of winning an election but also are more able to deter entrance. This result suggests a possible explanation for why campaign spending is not an unambiguous signal of incumbent quality. I also find that if the entry decision of the prospective candidates is made at the primary election stage, then the incentives to enter the electoral race are increased, so the presence of primary elections reduces the incumbency advantage.

More importantly, my model generates predictions that match stylized facts under the assumption of rational challengers. Some high quality incumbents run uncontested, because the prospective challenger decided to wait for an open election. However, there are "sacrificial lambs" that choose to face incumbents; low quality challengers, with a low probability of winning any election choose to face the incumbent if they have the chance to do so. Waiting is too costly for these candidates due to the high probability of losing the primary election next period. The ex-ante probability that

[4] These sources of incumbency advantage exist but are ignored here.

the incumbent is reelected is thus higher than the probability that a same quality candidate will win an open-seat election. Challenger decisions thus result in an incumbency advantage.

The study of incumbency advantage in U.S. elections has a long history in political science.[5] Ansolabehere and Snyder established the existence of incumbency advantage for incumbents in almost all executive and legislative offices. A variety of explanations have been proposed, with causal mechanisms spanning the range of strategic actors in the electoral process – voters, incumbents and challengers.

One group of models emphasizes the decisions of voters. In a world with imperfect information, the median voter chooses whether to retain the incumbent or to elect an unknown challenger. The ability of voters to precisely infer information regarding a candidate's quality has been considered one of the main factors explaining the advantage that incumbents enjoy.[6] Other explanations focus on the activities that incumbents perform both to attract voters (redistricting, publicity, promises of pork barrel legislation) and to deter prospective challengers (high levels of campaign spending).[7]

A third perspective emphasizes the decision-making problem faced by a challenger. This view suggests that incumbency advantage derives from the rational decision of strong challengers to wait for an open election rather than face an incumbent of known high quality. Some papers that build on this perspective include Banks and Kiewiert (1989), Canon (1993), Carson, Engstrom and Roberts (2006), Carson and Roberts (2007), Goodliffe (2005), Gordon, Huber and Landa (2007) Lazarus (2008), Meirowitz (2008), Romero (2004), and Stone, Maisel and Maestas (1999,2004). Most of this literature has not succeeded in reconciling rational challengers' entry decisions with the stylized facts of the electoral process.

[5]Some of the many studies discussing the subject are: Ansolabehere and Snyder (2001), Ansolabehere, Snyder and Stewart (2000), Carey, Niemi and Powel (2000), Cover (1977), Cox and Katz (1996), Erikson (1971), Fiorina (1989), Gelman and King (1990), Gowrisankaran, Mitchell and Moro (2005), Hickley (1980), Jacobson (1983,1987), Jacobson and Kernell (1981), Mayhew (1974), and Wrighton and Squire (1997).

[6]Ansolabehere et.al.(2000), Erikson (1971), Fiorina (1981) point out the relevance of voter behavior. Gordon, Huber, Landa (2007) provide a model where voters choose to become politically informed.

[7]This is a classic explanation proposed by Mayhew (1974) and analyzed from many different perspectives in the literature. Stone, Maisel and Maestas (2004) provide a good survey of this literature.

Several papers assume that challengers differ either in the payoffs obtained from being in office or in the costs incurred for being in office. Without such assumptions, the observed participation of low quality challengers (sometimes called amateur politicians) cannot be explained in a consistent way. In this paper, assuming only that candidates differ in their qualities, I am able to explain observed behavior.

This paper contributes to the challenger entry-decision literature. It extends the two-type framework of Banks and Kiewiert (1989) by modeling the rational decision of challengers with a continuum of qualities. Given their own quality and that of the incumbent, the challenger decides to face the incumbent or to wait for next period's open election. Having a wider range of qualities for the candidates allows me to capture the basic intuition of the model. Candidates who optimally decide to face the incumbent derive from two different sets, one with very high quality and one with very low quality.

The paper is organized as follows. Section 2 introduces the model. It presents the preliminary assumptions, presents the general result and provides some examples using specific functional forms. Section 3 embeds the model in a richer framework that includes strategic interactions. The exercise suggests how the approach taken in this paper may reverse prior explanations provided in the literature. Section 4 discusses the stylized facts of the electoral process matched by the model and discusses the applicability of the model to empirical analysis. Section 5 concludes and suggests some immediate extensions.

2 The Model

I model a binary decision made by a rational political challenger: to run against the incumbent or to wait for next period's open election. Politicians derive utility only from being in office. If the challenger enters, she knows with certainty that she will oppose the current incumbent. If she waits, she will face a random opponent in the general election but this is an uncertain prospect, because she needs to win the primary election of her own party first.

The assumptions presented below conform to the simplest possible model in order to focus on the study of challenger entry and its effects on incumbency advantage. Many other influences on the electoral process, such as party alliance, publicity, learning by campaigning and political vocation are relevant, but are shut down here to isolate the effect of entry due to perspective of winning.

2.1 Essential Assumptions

The challenger's decision will be embedded in a two-period election model with two parties: A and B. Politicians only derive utility from being in office, and thus, they only care about the probability of winning the elections given their own quality and the quality of their opponents. All candidates discount the future payoffs by δ. There is only one strategic decision taken at $(t = 1)$ by a first-period challenger, to run against the incumbent or to wait until the next period election.

Incumbents in this model are non-strategic. Their quality is publicly known, they seek reelection, are unopposed in the primary election and are restricted by term limits (maximum of two periods/terms in office). Second period challengers enter any election they can. Given the model time-structure, this is strictly dominant for them.

I impose some simplifying assumptions regarding the electoral rules. There are no primaries in the first period. Candidates are out of the electoral game if they lose an election (primary or general), and there are at most two candidates in each election. In each period, the out of office party picks a random quality candidate c from a distribution $F(c)$. The distribution $F(.)$ is the same for both parties (no a priori differences).

The probability that a quality-c challenger wins the election given a quality-x opponent is given by function $\phi(c - x)$. This probability is the same for primary and general elections. Some basic properties that $\phi(.)$ satisfies are:

- $\phi(0) = \frac{1}{2}$: no assumed incumbency advantage.[8]
- $\frac{\partial \phi(c-x)}{\partial (c-x)} > 0$: the probability of winning is increasing in own-quality.

[8] If $\phi(0) \neq \frac{1}{2}$, then there would be an ex-ante advantage ($\phi(0) > \frac{1}{2}$) or disadvantage ($\phi(0) < \frac{1}{2}$) for a candidate . As the function ϕ is the same for all candidates, and there has to be a winner in each election (the sum of the probability of winning has to be one), then $\phi(0) = \frac{1}{2}$ is a result of imposing these two assumptions.

If the distributions of the politician's quality have full support, then it is required that: $\phi(-\infty) = 0$ and $\phi(+\infty) = 1$.[9]

2.2 Timing of the Model

The two period model starts with an incumbent of known quality i from party A in office. At $(t = 1)$, party B, the out-of-office party, draws a challenger c from distribution $F(.)$. Challenger c is the rational challenger who decides to enter or to wait until next period election. If Challenger c enters, she faces the incumbent i in the general election. If c loses, she receives payoffs of zero and is out of the game. If c wins, she will be the period $(t = 2)$ incumbent. If, on the other hand, the challenger chose to wait, incumbent i wins the $(t = 1)$ election uncontested. This will be i's last term.

At the second election, if c entered and won, she is the incumbent (of known quality for all opponents). She seeks reelection, after winning the primary uncontested, and faces a random opponent that party-A will draw from $F(.)$. If, on the other hand, c waited, then there will be a primary in party-B, and the waiting candidate will face a random opponent drawn from $F(.)$. If c wins, she will face a random opponent from party A at the general election and, in case she wins, she will be the period-2 incumbent. If c loses any election (primary or general) she receives a payoff of zero and is out of the electoral race.

2.3 Value Functions

The value of entering the $(t = 1)$ general election is given by:

$$V^E(i, c) = \phi(c - i)\left[1 + \delta E_{x_A}\phi(c - x_A)\right].$$

[9] The function $\phi(c - x)$ does not need to depend on the difference. It could be written as $\phi(c, x)$. This is true because the result does not depend on the probability of winning being the same for any difference in quality irrespective of the quality level. It is nevertheless presented as a function of the difference for elegance, especially in the required limiting conditions. The second requirement can be rewritten as $\frac{\partial\phi(c,x)}{\partial c} > 0$ and $\frac{\partial\phi(c,x)}{\partial x} < 0$. The limiting conditions are more cumbersome, given that it has to be established that $\lim_{c\to\infty}\phi(c, x) = \lim_{x\to-\infty}\phi(c, x) = 1$ and $\lim_{c\to-\infty}\phi(c, x) = \lim_{x\to\infty}\phi(c, x) = 0$ and the case when both tend to infinity or minus infinity at the same time needs to be explicitly imposed.

Given an incumbent of quality i, if the quality-c challenger enters she gets a payoff equal to the probability of beating i in the first election $\phi(c-i)$. If she wins the election, she is the incumbent and the party-B general election candidate at $(t=2)$. Party-A will pick a random opponent from $F(.)$ who will face c in the election. If c wins, this is her last period.

The value of waiting for next period election is given by:

$$V^W(i,c) = \delta E_{x_B}\phi(c-x_B)\, E_{x_A}\phi(c-x_A)\left[1+\delta E_{x_A}\phi(c-x_A)\right].$$

If Challenger c chooses to wait, then she will face a random opponent at the primary and general elections. She needs to win both elections to become the incumbent. The assumption is that there is a continuation payoff attached to being in office, the expected value of the probability of winning a general election again. Notice that for a given candidate c, the value of waiting is independent of the first period incumbent's quality.

Given that all qualities are distributed $F(.)$ and that the probability of winning is the same in all elections, these two functions reduce to:

$$\begin{aligned}\frac{V^E(i,c)}{[1+\delta E_x\phi(c-x)]} &= \phi(c-i),\\ \frac{V^W(i,c)}{[1+\delta E_x\phi(c-x)]} &= \delta\left[E_x\phi(c-x)\right]^2.\end{aligned}$$

Given this, the challengers makes his decision according to:

$$V^E(i,c) - V^W(i,c) = \left\{\phi(c-i) - \delta\left[E_x\phi(c-x)\right]^2\right\}\left[1+\delta E_x\phi(c-x)\right].$$

The second bracketed term, $[1+\delta E_x\phi(c-x)]$, is always positive given that for all c and x the expected probability of winning is between *zero* and *one*. Therefore, in order to study the entry decision, I focus on:

$$V^{E^*}(i,c) - V^{W^*}(i,c) = \phi(c-i) - \delta\left[E_x\phi(c-x)\right]^2.$$

In this model, the challenger decides to enter or not based on the difference between the probability of beating the current incumbent i and the expected discounted probability of beating two random opponents (one in the primary election and one in the general election).

2.4 General Result

In order to characterize the challenger decision, I need to impose some structure on $\phi(c-i)$ and $F(x)$.

Remark 1 *Notice that if* $\phi(c-i)$ *is given by*:

$$\phi(c-i) = \begin{cases} 1 & if & c > i \\ \frac{1}{2} & if & c = i \\ 0 & if & c < i \end{cases},$$

then the decision is:

- *for* $c > i$: *Enter,*
- *for* $c = i$: *The decision depends on the distribution. If* $F(x)$ *is strictly continuous, then it depends on whether* $F(c)$ *is larger or lower than* $\left(1-\sqrt{\frac{1}{2\delta}}\right)$,
- *for* $c < i$: *Wait (as long as the distribution* $F(x)$ *assigns positive weight to* x *larger than* c*).*

This $\phi(c-i)$[10] implies that voters are able to recognize the best candidate with probability one, and this induces that incumbents will only be challenged by higher quality candidates. If this is the case, the incumbent's only advantage comes from the fact that sometimes he is uncontested. If contested, he loses.

To get a more interesting result I will assume that $\phi(c-i)$ does not imply perfect information regarding the underlying qualities: $\phi(.)$ is strictly increasing, so that the probability of winning is higher for higher quality candidates but it approaches *zero* and *one*.

Proposition 1 *Existence of "double crossing"*

If the quality of challengers is distributed $x \sim F(x)$; $F(x)$ has full support; and the probability of winning an election $\phi(c-x)$ is a function satisfying $\phi(0) = \frac{1}{2}$, $\frac{\partial\phi(c-x)}{\partial(c-x)} > 0$, $\phi(-\infty) = 0$, and $\phi(+\infty) = 1$ and $\frac{\partial^2\phi(c-x)}{\partial(c-x)^2} > 0$ for $(c-x) < 0$, then:

[10] This can be written as $\phi(c,i)$ with no changes.

1. Some challengers wait. For every challenger c, there is an incumbent i^* such that the challenger c prefers to wait for next period election:

$$\left[V^{E^*}(i^*,c) - V^{W^*}(i^*,c)\right] < 0.$$

2. Double crossing. Given an i^* for whom some challenger waits, low quality challengers and high quality challengers enter against i^*:

$$\begin{aligned}\lim_{c\to-\infty}\left[V^{E^*}(i^*,c) - V^{W^*}(i^*,c)\right] &> 0,\\ \lim_{c\to\infty}\left[V^{E^*}(i^*,c) - V^{W^*}(i^*,c)\right] &> 0.\end{aligned}$$

   Against an incumbent i^*, low quality challengers enter, medium quality challengers wait, and high quality challengers enter.

The proof is in the Appendix 6.1.

This proposition implies that there are at least two groups of challengers who enter, those with low qualities and those with really high qualities.

If $\phi(c-i)$ is such that it is convex for $(c-i) < 0$ and concave for $(c-i) > 0$, then there are only two crossings.

This main result implies that in this model the non-strategic incumbent will not face competition from middle quality challengers. Thus, the incumbent benefits from the strategic choice of a prospective challenger. The possibility of an open seat next period lures some potential candidates away, resulting in unchallenged incumbents. It also results in rational entry by low quality challengers. Since it is better to face an incumbent than to risk the opportunity of being a candidate by entering a primary election.

2.5 Examples

2.5.1 Normal Distribution

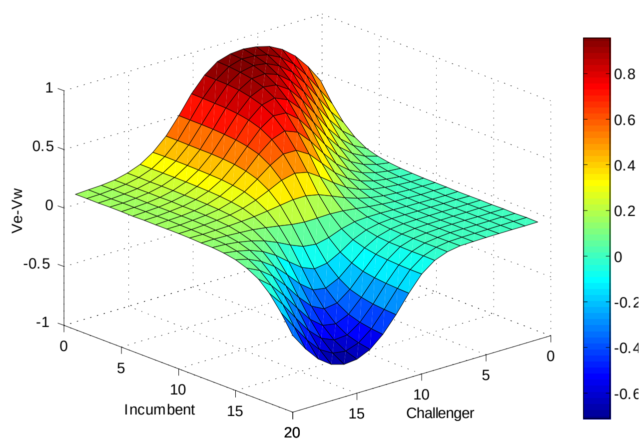
As an example., if a normal distribution $x \sim N(0,.5)$ is used, with a logistic function

$$\phi(c-x) = \frac{1}{2}[1+\tanh(2(c-x))],$$

the double crossing result is obtained.

The result is shown in the figure below. It shows the candidates' qualities from zero to twenty. The x-axis shows incumbent quality and the y-axis the challenger quality. The vertical axis shows the difference between the value of entering and the value of waiting. The color scale from red to blue goes from positive to negative numbers. The double crossing occurs in the deepening of the figure, going from positive to negative numbers and back to positive for the same incumbent. Notice that if the challenger's quality is fixed, the figure decreases with respect to the incumbent quality. The incentives to enter the race decrease for a challenger when the opponent is of higher quality.

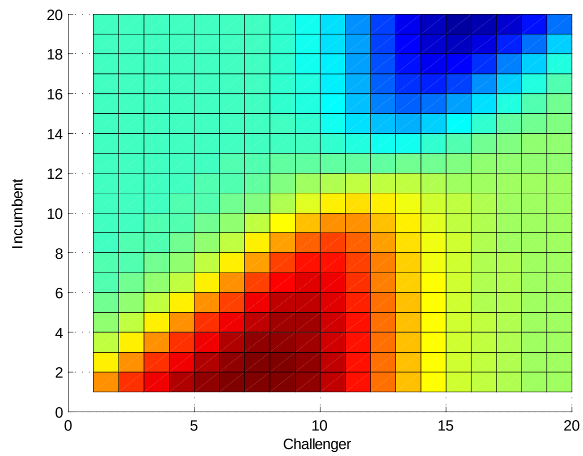
In this example, everybody enters against low quality incumbents (quality below 10), and entering is specially appealing for medium quality challengers.[11] There is double crossing for higher quality incumbents, with challengers of medium-high quality choosing to wait. For instance, against an incumbent of quality 14, challengers from quality 8 to 16 choose to wait. Really low quality challengers (below 8) and really high quality (above 16) run against the incumbent.



The figure below shows the topographic view of the above graph. The Challenger Quality is in the x-axis and the Incumbent Quality in the right

[11] Medium quality challengers are the ones that risk more if they wait: low quality challengers have a low probability of winning either way, and high quality challengers will win with high probability in any case. The opportunity of running against a low quality incumbent is more valuable for medium quality challengers.

axis. The waiting area is the deep blue area (at the northeast quadrant). Everybody enters for incumbent qualities below 10.



Regarding the specific example shown in the graphs above, it is worth noting that for those incumbents that are able to deter entrance, the expected quality conditional on entrance is lower than the unconditional expected quality of the opponents.

2.5.2 Uniform Distribution

If the distribution of candidates' quality is uniform, $x \sim U(0,1)$, then $\phi(c-x)$ needs to have more structure in order to get "double crossing." In particular, the condition on the upper bound needs to be satisfied.

For example, let:

$$\phi(c-i) = \begin{cases} \frac{1}{2}\left(1+(c-i)^{\frac{1}{3}}\right) & if \quad c-i \geq 0 \\ \frac{1}{2}\left(1-(i-c)^{\frac{1}{3}}\right) & if \quad c-i < 0. \end{cases}$$

Thus, for a quality$-c$ candidate, the value of entering the electoral race is given by:

$$V^{E^*}(i,c) = \begin{cases} \frac{1}{2}\left(1+(c-i)^{\frac{1}{3}}\right) & if \quad c-i \geq 0 \\ \frac{1}{2}\left(1-(i-c)^{\frac{1}{3}}\right) & if \quad c-i < 0. \end{cases}$$

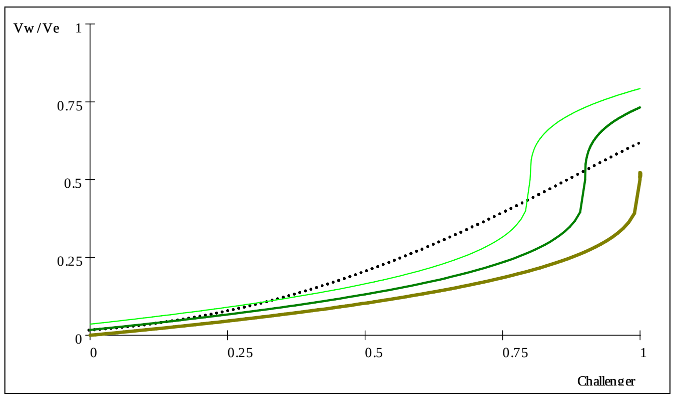
The expected value of the probability of winning an election is given by:

$$E_x \phi\left(c - x\right) = \frac{1}{2} + \frac{3}{8}\left[\left(c\right)^{\frac{4}{3}} - \left(1 - c\right)^{\frac{4}{3}}\right].$$

The value of waiting is:

$$V^{W^*}\left(i, c\right) = \delta \frac{1}{4}\left(1 + \frac{3}{4}\left[\left(c\right)^{\frac{4}{3}} - \left(1 - c\right)^{\frac{4}{3}}\right]\right)^2.$$

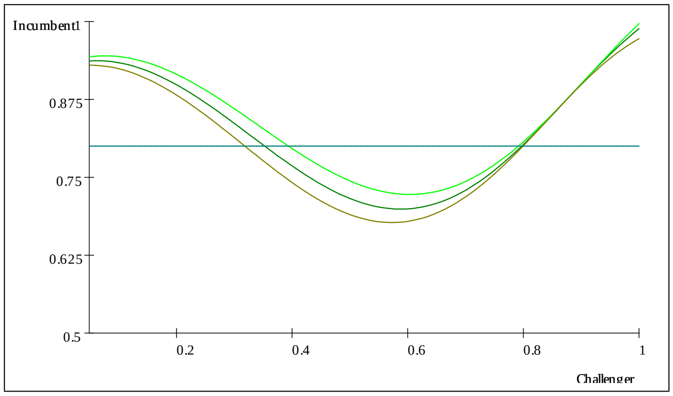
Notice that for a challenger, if the discount factor $\delta = .8$:



The dotted black line represents the value of waiting, which is independent of the incumbent quality. In different shades of green and different thickness is the value of entering against different incumbents. The lower thicker line is the value of entering against a $i = 1$, the middle line is the value of entering against $i = .9$, and the upper thinner line is the value of entering against $i = .8$.

When the two values are equal, the boundary between the value of waiting and the value of entering can be determined.

$$i = c - \left\{\delta \frac{1}{2}\left(1 + \frac{3}{4}\left[\left(c\right)^{\frac{4}{3}} - \left(1 - c\right)^{\frac{4}{3}}\right]\right)^2 - 1\right\}^3 \tag{1}$$



The graph above shows the boundary between the quality needed for waiting and entering the electoral race. For a given quality of the incumbent, if the quality of the candidate lies below the line, then the candidate enters the election. If, on the other hand, the quality of the candidate lies above the line, then the candidate waits for next period's open election.[12] The difference between the lines is the value of δ. When δ increases, the waiting area (above the line) increases.

Notice that if the incumbent is of quality .8, then challengers of quality $c \in (.35, .80)$ choose to wait for the next period. Hence the expected quality of those who face the incumbent $E\left(c \mid c \quad enters\right) = .44$ is lower than the unconditional expected quality. The magnitude of the incumbency advantage for this example, measured by the increase in the ex-ante probability of winning, is provided in Appendix 6.2.

3 Extensions

I present two examples of how the basic setup can be embedded in settings with more strategic agents. The aim is to suggest the effects that the underlying challenger entry-decision has on richer settings.

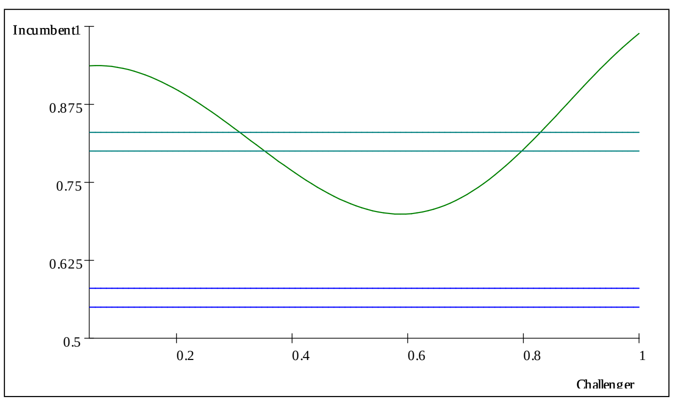
3.1 Strategic Incumbent

The incumbent may choose to spend $\$M$ to improve his probability of winning. The assumption is that after this expenditure, the incumbent is perceived by voters as being a higher quality politician, and thus has a higher

[12] An open election is one where there are no incumbents; both contesting candidates are "new."

probability of winning. The incumbent lowers his probability of winning by $k(i)$ when he spends $\$M$.[13]

Challengers have perfect information regarding the true quality of the incumbent, but their entry decision depends only on the probability of winning; on voter's perceptions shown by $\phi(c-i)$. The graph below shows an example of how perceptions change when the incumbent spends $\$M$ in the uniform distribution case. In this example, incumbents are perceived as having an increase of .03 in their quality after the expenditure. The green curve shows the limit between the waiting and the entering area (above and below the curve respectively). In this example, the dark blue solid line indicates an incumbent of quality $i = .8$. The parallel dark blue dashed line shows voter's perception when he spends $\$M$. He is perceived as higher quality and thus is able to deter more challengers. The bright blue solid line indicates an incumbent of quality $i = .55$. In this case, when the incumbent expends $\$M$ he is perceived as a higher quality (as shown by the bright blue dashed line) but he is still unable to deter challenger's entrance.



The main question in this setting[14] is which incumbents choose to spend $\$M$ dollars? The gain from expenditure is due to two forces: higher probability of winning against those who enter and increased entry deterrence. The deterrence effect depends on the quality of the incumbent, in that:

[13] One explanation for this cost is that he is perceived as corrupt for making this expenditure.

[14] Where only incumbents decide whether to spend money or not, just to provide the intuition.

- low quality are still not deterring entrance.
- medium quality are able to deter more challengers.

This double-source deterrence effect implies that the gain from the expenditure is not monotonic in incumbent quality; those incumbents who are able to increase deterrence have a higher gain.

In the graph below, I show in red (solid line) the gain from spending $\$M$ and in different shades of green (dashes) different profiles of the cost $k(i)$. The dotted dark green line represents costs that are decreasing in the incumbent quality. Given this cost structure, only the medium high quality incumbents will choose to incur the expenditure. This is also true with a constant and high cost profile, as shown by the dashed medium green line. In contrast, if the cost is constant but low (dashed bright green line), all incumbents except for the extremely high quality will spend $\$M$.

This extension suggests that the expenditure decision is also not-monotonic in the quality of the incumbent. Depending on the costs it can be the case that medium-high quality incumbents are the ones who choose to spend the money. These incumbents gain through the interaction of the higher deterrence effect and the higher probability of winning against those who still face them. Thus, the expenditure might not be a univocal signal of quality in the presence of "double crossing" in the entry decision, as has been suggested by the literature.[15]

3.2 Primary Election Competition

I will now modify the first model by including a primary election at $(t = 1)$ for the out-of-office party. Candidates choose whether to enter the primary election or not, and the primary winner has to participate in the general election.[16] In the second period, the out-of-office party holds a primary. This primary might involve the first period candidates who waited or new ones. In the first period, challengers a and b from the out-off-office party are randomly chosen. At $(t = 1)$ each challenger decides to enter the primary election or to wait. If both enter, then primaries take place. The winner faces the incumbent in the general election. In this setting, the assumptions that the incumbent win the primaries uncontested and that losers are out of the game still hold.

If a waits, the following cases with b may result:

- if b also waits, they compete at $(t = 2)$ primary.
- if b enters and loses, a competes against a random opponent at $(t = 2)$ primaries.
- if b enters and wins, b is $(t = 2)$ incumbent, and a is out of the race.

The decision of both candidates is simultaneous, and they do not know the quality of their opponent at the time of the primary. As in the previous model, the quality of the incumbent is known by all prospective challengers.

In period-2 primaries, given the time structure of the model, all candidates will choose to enter the primary election. The expected quality of the candidate who wins a primary and runs for the general election is higher

[15] It has been suggested that more expenditure implies lower quality (because those are the ones who benefit the most from being perceived as higher quality candidates) or that it implies high quality, because those are the one benefiting from the deterrence effect.

[16] This is consistent with US electoral law; the winner of the primary election has to participate in the general election and thus, the entry decision is made at the primary stage.

than in the previous model, being the best of two random draws. There are two possible scenarios at $(t = 2)$. If the incumbent won reelection in period-1 there is an open election, if the incumbent lost, then there is an incumbent running for reelection from period-1 out-off-office party. In any case, the winner of a primary election in period-2 is the best of two draws, whereas in the previous model this only occurred when a candidate waited in the first period.

The value of entering the first period primary election for candidate a is:

$$\begin{aligned} V^E(i,a) &= [P(b \quad entered)\,\phi(a-b) + P(b \quad waited)] * \\ &\quad \phi(a-i)\,(1+\delta E_x \phi(a-x \mid x \quad won)). \end{aligned}$$

Thus, if Challenger b entered, a needs to beat b and then the incumbent i to be the next period incumbent. If b waited, a needs only to beat i.

The value of waiting is given by:

$$\begin{aligned} V^W(i,a) &= \delta[P(b \quad entered)\,\phi(i-b)\,E_x\phi(a-x) + P(b \quad waited)\,\phi(a-b)] \\ &\quad E_x\phi(a-x \mid x \quad won)\,(1+\delta E_x\phi(a-x \mid x \quad won)). \end{aligned}$$

Challenger a is out of the race if b is the incumbent in period-2, because it is assumed that incumbents win primaries unopposed.

The only pertinent factors in making a decision are:

$$\begin{aligned} \frac{V^E(i,a)}{(1+\delta E_x\phi(a-x \mid x \quad won))} &= [P(b \quad entered)\,\phi(a-b) + P(b \quad waited)] \\ &\quad \phi(a-i) \\ \frac{V^W(i,a) =}{(1+\delta E_x\phi(a-x \mid x \quad won))} &= \delta[P(b \quad entered)\,\phi(i-b)\,E_x\phi(a-x) + P(b \quad waited) \\ &\quad \phi(a-b)]E_x\phi(a-x \mid x \quad won) \end{aligned}$$

In this setting both the value of entering and of waiting have decreased compared to the previous setting. The value of waiting now depends on the quality of the incumbent, given than it will affect the entry decision of the own party opponent.

Consider for simplicity a discrete case example: Candidates can be low, medium or high quality with equal probability $(\frac{1}{3})$. The probability of winning the election is given by $\phi(c-i)$.

In\Ch	L	M	H
h	.02	.05	.50
m	.15	.50	.95
l	.50	.85	.98

For the information above, in a Subgame Perfect Equilibrium;

- All candidates enter against low quality incumbents.
- All candidates enter against medium quality incumbents.
- Against high quality incumbents we get double crossing. High quality candidates enter no matter what their opponents do. Given that high quality candidates enter, medium quality candidates strictly prefer to wait for the next election, and, given this, low quality candidates prefer to enter.

Low qualities enter when there is a high quality incumbent because they have a better chance of winning the primary election when the medium qualities are not running. Medium qualities wait, because the chances of winning the primary and the election today are too slim. They wait to face an unknown opponent next time. Notice that the problem they have is that the high quality challenger in the primary might leave them out of the race. The high qualities will enter today no matter what.

It is worth comparing the entry decision of the candidates in this setting, given the described primary election interaction with the original model with no first-period primary. In the example above adding the primary election is actually pushing people into the race. This primary election game among candidates results in the incumbent reducing his advantage, instead of increasing it.

4 Empirical Implications: Incumbency Advantage, "Sacrificial Lambs" and Uncontested Races

Some implications of the model match stylized facts of the electoral process already studied by the literature. The existence of an incumbency advantage is a well established fact that Mayhew (1974) studied thoroughly for U.S. House elections, and Ansolabehere and Snyder (2001) evaluated for almost all executive and legislative positions. In the model, an incumbent has an advantage because his ex-ante probability of being reelected is higher than the probability of winning an open election. This is due to two forces. Sometimes the incumbent runs for reelection uncontested, or when contested, the expected quality of his opponents is lower than the unconditional expected quality. Of course, this advantage depends on the quality of the incumbent. Low quality politicians do not enjoy the same privilege.

The model also implies that some politicians with a really low probability of winning the election choose to run against an incumbent. In particular, it proves that there is a challenger of low enough quality that is willing to run against any incumbent. This is also a common feature of U.S. elections that has been established by many[17] before but explained as either non-rational behavior or as one driven by non-political goals. Another common explanation for the existence of "sacrificial lambs" is that some politicians run against an incumbent due to their party loyalty[18]. In the model politicians of all qualities participate for the same reason, their chances of winning. With this motivations, it is able to explain both participation by independents and by party-members.

The existence of uncontested incumbents is especially common in House elections, as Hinckley (1980), Jacobson (1990) and Wrighton and Squire (1997) established. This is also a common feature of lower level elections. The model presented above results in higher quality incumbents running uncontested often. This deterrence effect is not a result of any incumbent activity but just a rational decision of challengers. An uncontested incumbent in this model is a sign of strategic entrance and not of a corrupted election process.

[17] Among others Banks and Kiewiert (1989), Canon (1993), Jacobson and Kernell (1981).
[18] As in Fowler 1977.

4.1 Gubernatorial Elections

The assumptions of the model apply directly to most gubernatorial elections. In the United States, 34 governors out of 50 states are limited to two terms.[19] There are only a few studies that analyze the competitiveness of gubernatorial elections and even fewer that address the issue of entry to gubernatorial races.[20]

Gubernatorial election data can be used to test the existence of "double entry" and to analyze the effects that this entry pattern has on both incumbent and challengers behavior.

5 Conclusions and Further Research

In this paper, I have presented a model of candidate entry and shown that challengers face a trade-off when deciding to enter the political race, either representing their party with certainty against a known incumbent or taking the chance of their own-party primaries to face an unknown opponent in next period's general election. This trade-off results in a non-monotonic entry decision. Low-quality challengers choose to enter and really high-quality challengers also choose to enter. Thus, even when incumbents are non-strategic, higher quality incumbents are able to deter middle quality challengers. In addition, I have shown how this underlying entry decision impacts the result of richer strategic frameworks.

The model is able to match multiple stylized facts of the electoral process. In particular, it implies the existence of incumbency advantage, uncontested incumbents, and the presence of "sacrificial lambs." This is achieved in a setting with rational challengers that only differ in quality. Some studies have explained the entry of low quality challengers by assuming ex-ante different career objectives when running for an election.

This paper provides an explanation of the entry decision of challengers when an incumbent will be out of office in the next period. The effect of

[19] 14 have unlimited terms, 1 is restricted to only one (consecutive) reelection and 1 is restricted to two reelections.

[20] Ansolabehere and Snyder (2002) show that incumbent governors enjoy an advantage that has been growing. Squire (1992) analysis gubernatorial elections and the deterrence activities performed by incumbent governors.

eliminating term limits remains to be studied. A natural extension of the model would be to analyze an overlapping generation of candidates that chooses when to enter. Given this extension, the impact of having different institutional arrangements (regarding term limits) will be addressed, and the gubernatorial election data can be used to test the resulting hypothesis.

6 Appendix

6.1 Existence of Double Crossing

Proof of proposition (1) :

1. For any c^*, there exists a pair i^* such that:

$$\phi(c^* - i^*) < \delta [E_x \phi(c^* - x)]^2$$

For any c^* the, right hand side is fixed. As, $\phi(c^* - i^*)$ takes any value between *zero* and *one* and is decreasing in i, there exist an i^* large enough such that the above inequality holds. For a fix c^* if the inequality holds for an i^*, it also holds for any i larger than i^*, thus it can be assumed that i^* is larger than $E(x)$.

2. For i^*:

$$\lim_{c \to \infty} \phi(c - i^*) > \lim_{c \to \infty} \delta [E_x \phi(c - x)]^2$$

Both $\phi(c - i^*)$ and $E_x \phi(c - x)$ converge to one as c goes to infinity, having $\delta < 1$ is enough for this inequality to hold.

3. For i^*:

$$\lim_{c \to -\infty} \phi(c - i^*) > \lim_{c \to -\infty} \delta [E_x \phi(c - x)]^2$$

It would be sufficient to have $\phi(.)$ bounded below by ε, given that in the limit the expected value of the probability of winning goes to the lower bound, but having the squared and the discount factor assures that the inequality holds.

The strict convexity condition bounds the ratio of $\phi((c - x))$ to $E_x \phi((c - x))$ when both are close to zero.

A sufficient condition for this proof is to have $\lim_{c \to -\infty} \frac{\phi((c-x))}{[E_x \phi(c-x)]^2} > 1$. This is needed to ensure that for low c, there exists a $\delta \in (0, 1)$ such that the inequality is reversed given i^*. A function $\phi((c-x))$ strictly convex when converging to zero satisfies this condition.

Having $\lim_{c\to-\infty} \frac{\phi((c-x))}{[E_x\phi(c-x)]^2} > 1$ is sufficient for this condition to hold, given that:

$$\lim_{c\to-\infty} \frac{\phi(c-i^*)}{[E_x\phi(c-x)]^2} > 1 \Longrightarrow \lim_{c\to-\infty} \frac{\phi(c-i^*)}{\delta[E_x\phi(c-x)]^2} > 1$$

The sufficient condition is a lower bound of the distance between $\phi(c-i^*)$ and $[E_x\phi(c-x)]^2$. It can be satisfied even when $\lim_{c\to-\infty} \frac{\phi(c-i^*)}{E_x\phi(c-x)} < 1$, which holds for high enough i^*.

6.2 The measure of the incumbency advantage

Given the entry decision described in section 2.5.2, I can calculate the incumbent's advantage. The expected probability of winning an open election for a politician of quality $q_p = .8$ is:

$$E_{q_{c'}} P(win \mid q_p = .8, q_{c'}) = \frac{1}{2} + \frac{3}{8}\left[(.8)^{\frac{4}{3}} - (.2)^{\frac{4}{3}}\right] = .73464$$

When there is an incumbent politician of known quality $q_i = .8$, the expected value of the probability of winning the election depends on the entry-decision of the candidate. This decision is summarized by two cutoff values: all candidates with qualities below $\underline{q_c}$ and above $\overline{q_c}$ will enter the electoral race. The equation defining those cutoffs when $q_i = .8$ is the following:

$$.8 = q_c - \left\{\delta\frac{1}{2}\left[1 + \frac{3}{4}\left((q_c)^{\frac{4}{3}} - (1-q_c)^{\frac{4}{3}}\right)\right]^2 - 1\right\}^3$$

For $\delta = .9$, the cutoffs take the following values:

$$\begin{aligned} \underline{q_c} &= .2892 \\ \overline{q_c} &= .8 \end{aligned}$$

Given this information, the probability that the incumbent wins the election can be calculated for different settings:

- If the incumbent runs uncontested when the prospective candidate does not enter the election:

$$E_{q_{c'}} P\left(win \mid q_i = .8, q_{c'}\right) = \int_0^{\underline{q_c}} \frac{1}{2}\left(1 + (.8 - q_c)^{\frac{1}{3}}\right) dq_c + \int_{\underline{q_c}}^{\overline{q_c}} 1 dq_c + \int_{\overline{q_c}}^{1} \frac{1}{2}\left(1 - (q_c - .8)^{\frac{1}{3}}\right) dq_c$$

$$= .83691$$

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