A Structured Search for Novel Manufacturing Processes Leading to a Periodic Table of Ring Rolling Machines

Julian M. Allwood
Institute for Manufacturing,
University of Cambridge,
Mill Lane,
Cambridge CB2 1RX, UK
e-mail: jma42@cam.ac.uk

Manufacturing processes based on cutting have been extensively automated over the past 30–40 years leading to greatly increased flexibility of operation. In contrast, processes based on ductile forming have largely remained dependent on fixed tooling and lack flexibility. Recent innovations have shown that forming can also be made flexible, by new process configurations typically using simpler and smaller tools with increased (and controllable) freedom of motion. In order to facilitate development of such flexible forming processes, this paper examines the possibility that all such processes can be predicted and organized so that subsequent process development may be based on selection rather than invention. The approach taken is based on Zwicky's "morphological analysis," in which the features of a design are parameterized and an exhaustive search is conducted, with appropriate constraints used to reject infeasible designs. As an example of this approach, the process of ring rolling is explored, and a "periodic table" of 102 "elemental" ring rolling machines is presented. The combination of elements into compounds is described, and the use of the table for development of practical flexible machines is discussed. Having applied this approach to the example of ring rolling, its likely value in exploring other processes is discussed. [DOI: 10.1115/1.2712217]

Keywords: *innovation, manufacturing processes, morphological analysis, ring rolling*

1 Introduction

Is it possible to give a structured classification for all manufacturing processes? The periodic table of elements provides such a classification for chemists. Physical properties of the atom were used to organize known elements, predict undiscovered elements, and inform the formation of compounds. Could a similar fundamental classification of manufacturing processes be identified?

The rise of mass production has given a particular direction to the development of manufacturing processes, promoting technologies which support rapid repetitive operations to allow high volume production of standardized components. Yet companies are increasingly aware that higher profits can be made from customized goods rather than standard ones, so require more flexible means of production able to respond rapidly to customer needs. One area in which such flexibility is well developed is in the development of computer numerically controlled (CNC) machines for material removal. In the past 10 years, considerable effort has also been applied to development of flexible additive technologies, generally referred to as "rapid prototyping" processes.

In contrast to these two areas, the development of flexibility in forming processes has to date received less attention. Some development has occurred, particularly in Japan, with a recent review given by Allwood and Utsunomiya [1]. Outside Japan, a few flexible processes are attracting interest—for instance work on "incremental sheet forming" is reviewed by Jeswiet et al. [2], and Ziegelmayer et al. [3] describe work on forging with a workpiece manipulated by a robot. While such development of flexible forming processes is in its early stages, a structured classification of processes might help to identify important novel classes of machine.

In this paper, the process of ring rolling will be used to demonstrate the potential development of a classification of flexible processes. Ring rolling is the process by which forged and pierced "preformed" rings are deformed by an inner mandrel and outer forming roll so that the ring wall thickness is reduced and diameter increased. A schematic of the process is shown in Fig. 1. The process is widely used in the manufacture of bearings and other axisymmetric metal components, and mainly used to form rings with rectangular cross section. Bearing rings typically have a complex cross section, to accommodate a "race" for the balls transmitting force, and various grooves required for assembly and mounting of oil seals. Thus a typical manufacturing route would follow ring rolling with various machining and grinding stages to create a specified ring cross section. If a flexible ring rolling process could be created, able to form a programmable cross section to the ring, material waste and the extra manufacturing cost of these post-rolling processes could be avoided. In preparation for this paper, a review of 170 articles on ring rolling in English and German has been completed [4,5] showing that very little innovation has occurred in the design of ring rolling processes since 1856. The two important exceptions are Omori et al. [6] who describe a controllable process for creating railway wheels from disk shaped blanks, and Onoda and Nakagawa [7] who describe a process with three mandrels used to form the ring into an external die to form the "tyre" of a gear. An examination of a controllable or "incremental" ring rolling process is described in Allwood et al. [8] and shows that a narrow mandrel moving both radially and axially may be used to form a range of ring cross sections. However this demonstrates only one possible change to the design of the process, and many more options exist. An initial description of an attempt to classify ring rolling machines has been presented [9] and is here described fully and set in the context of a general search for new process designs. Section 2 describes a general approach to organizing such a search which is applied to ring rolling machine design in Secs. 3–5.

Contributed by the Design for Manufacturing of ASME for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received October 27, 2005; final manuscript received April 20, 2006. Review conducted by David Kazmer.

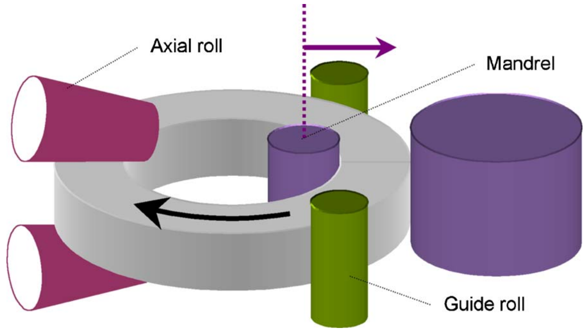


Fig. 1 A schematic of the conventional ring rolling process

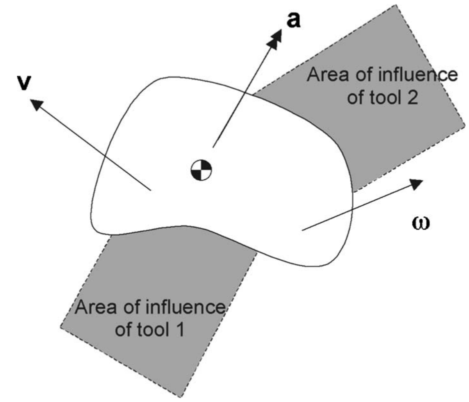


Fig. 2 Generic characterization of a simple forming process

2 A Structured Search for New Processes

A structured search for new manufacturing processes has two attractions: it allows confidence that new machine designs have been selected from a broad range of feasible alternatives; it promotes a fundamental examination of the design options for the machine, and should ideally allow identification of all possibilities. Porter et al. [10] reporting on the activities of a "Technology Futures Analysis Methods Working Group" present a catalogue of techniques for anticipating future technology innovations and their consequences. Many of the 51 techniques are focused more on needs and responses than the process of technology innovation, but their catalogue suggests that the approach known as "morphological analysis" is most suited to the structured search required here.

2.1 Morphological Analysis. The idea of morphological analysis is that a finite number of key features of a problem can be parameterized and a small number of settings chosen for each parameter. An exhaustive search through every possible combination of parameter settings is then conducted and appropriate criteria used to select favorable combinations. This approach has a long history. Coyle [11] quotes Aristotle [12] introducing a discussion on the varieties of government with an example of exhaustive search. In the 20th century interest in this approach was promoted by the Swiss–American Physicist, Zwicky [13,14], who defined three categories or morphological analysis: systematic field coverage, in which all possible solutions to a problem are examined; the morphological box, in which combinations of solutions are examined; and the method of negation and construction in which each in turn of a set of axioms which are assumed true in existing work, are negated and the consequences explored. Coyle [11] provides an excellent introduction to these methods, of which the method of the morphological box is most relevant to the work of this paper. According to Zwicky [14], a five stage procedure is required:

- The problem to be solved must be exactly formulated;
- All parameters which might enter the solution of the problem must be characterized;
- The morphological box which contains all solutions of the problem is constructed;
- All solutions in the box are evaluated with respect to the purposes to be achieved; and
- The best solutions are selected and implemented.

Zwicky applied this approach to a variety of technical and nontechnical problems. Most famously, he explored the range of possible designs for jet engines activated by chemical energy, and anticipated the design of the aeroturbojet. He also applied the method to diverse areas including problems in astrophysics, restocking libraries, and creating a legal framework for space exploration. Ayres [15] provides a mechanism to examine the likelihood that options in the morphological box might become useful through a measure of "morphological distance." The approaches developed by Zwicky are promoted by the organization Swemorph [16] which gives links to current projects in the area.

Morphological analysis has had a wide variety of applications within engineering design. Pahl and Beitz [17], Sec. 4.1, refer to Zwicky's morphological boxes as "design catalogues" and give examples related to coating the backs of carpets and connecting a shaft to a hub. Diekhöner and Lohkamp [18] give a catalogue of transmissions and Raab and Schneider [19] give one for mechanisms. The approach has apparently had little application in the area of innovation in manufacturing process design, although Pahl and Beitz refer to work by Ersoy [20] on a design catalogue for casting, and by Roth [21] on a catalogue of forging processes.

2.2 Characteristics of Manufacturing Processes. Figure 2 shows a generic characterization of a simple forming process. The workpiece is shown as a contiguous volume of ductile material, acted on by one or more tools whose function is to apply force, displacement, or stiffness to the boundary of the workpiece. Body forces, for instance those due to gravity or electromagnetic fields, may also apply. Heat may also be applied locally by a tool or globally—by preheating, or by operating the process in a controlled atmosphere.

The word "tool" is used carefully here to indicate any means of constraining the workpiece boundary through application of force, displacement, or a stiffness relationship, whether driven or passive. Four characteristics are required to distinguish the action of a tool on a workpiece as follows:

1. Forces acting at the boundary may act continuously or intermittently, and may or may not be dependent on the relative motion of the tool and the workpiece. The catalogue of forces provided by Pahl and Beitz [17] p. 87 has been used to create examples of such forces according to these two distinctions in Table 1.
2. The relative geometry of tool and workpiece may influence the deformation arising from their interaction. In the case of a rigid press-tool used in forging this is obvious. For a water-jet used to deform an unsupported sheet, the interaction will depend on the distribution of massflow rate across the jet, and the distance of the workpiece from the jet.
3. The relative motion of tool and workpiece must be specified in order to characterize their interaction. In the design of CNC machine tools, this relative motion is generally characterized by the number of axes of the machine.
4. The tool may or may not exchange heat with the workpiece. In some cases, a tool may be used to exchange heat without applying any significant mechanical force.

A manufacturing process must comprise at least one tool and is subject to four constraints:

Table 1 Sources of force classified by time and motion dependence

	Time dependence	
Motion dependence	Continuous	Intermittent
Does not require relative motion	Gravitational force (n.b. this is a body force) with associated reaction forces (surface forces) Magnetic force Electrostatic force Hydrostatic force Aerostatic force	Thermal (or biological) expansion Piezo-electric force
Requires relative motion	Rolling contact Sliding contact Electrodynamic force Hydrodynamic force Aerodynamic force	Inertial force (n.b. this is a body force) with associated reaction forces (surface forces) Impulsive force from stream of particles Hammering force

1. The interactions within the machine must obey Newton's laws of motion. Often this leads to the requirement of equilibrium, but some processes make use of the inertia of the workpiece;
2. The deformation of the workpiece must conform to the constitutive law of the material of which it is made and be controlled so that unintended deformations (burrs, cracks, or even failure) do not arise;
3. The configuration of workpiece and tools must be geometrically possible—without mutual penetration; and
4. The tools must be sufficiently strong to survive process operation without damage.

Figure 3 gives a summary of the definition and parameterization of a manufacturing process presented in this section: the machine comprises a workpiece with one or more tools; the tools are defined by four characteristics; and the machine is subject to four constraints. Such a definition makes possible the third step of Zwicky's method given in Sec. 2.1—the creation of a morphological box of all manufacturing processes. However, the range of processes is so great that the resulting box would be difficult to use. Instead, the approach given in general form here can be applied to a particular process, where the characteristics of the workpiece, required deformation, and perhaps also material range, can be used to limit the number of options considered in Fig. 3 to give a useful "box" of options. The next three sections of this paper demonstrate this by examining the characteristics of machines for ring rolling of metal.

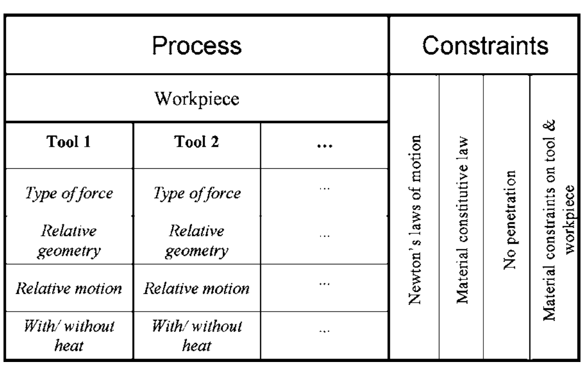


Fig. 3 Summary of the definition and parameterization of a manufacturing process

3 Definition and Parameterization of Ring Rolling Machines

The process of ring rolling leads to the mechanical deformation of a ductile ring under the action of solid rolling tools. Table 2 lists a set of features that define the ring rolling process considered here, and five assumptions about the process behavior used to simplify the analysis. The aim of this section is to deduce the simplest complete parameterization of ring rolling machines.

3.1 Parameterization of Tools for Ring Rolling. Three forms of variety can be identified in the tools in a ring rolling machine: the average roll diameter; the cone angle of the roll; and the profile of the tool surface. The roll diameter can have any positive value, from "very small" to "very large" and an appropriate number of settings can be defined relative to the size of the ring being produced. The cone angle of rolls could have any value between zero and π, as illustrated in Fig. 4(*a*), with values greater than $\pi/2$ indicating a tool external to the ring, with the cone of angle π being an external die.

The tools of Fig. 4(*a*) have flat surfaces, but these could be modified. Such surface modifications could be characterized for instance by a basis-function expansion such as that of the Chebyshev polynomial series. This would lead to an infinite variety of

Table 2 Definitions and assumptions used in defining the periodic table of ring rolling machines

Statements of definition	The ring is made of metal which could be hot or cold The tools do not exchange heat with the ring The ring is initially circular and remains so during and after deformation All tools are stiff solid rollers The ring remains in equilibrium throughout the process The axis of the ring remains stationary The ring and tools remain in constant nonadhesive contact through the deformation The tools do not penetrate each other or the ring
Assumptions about the process	Inertial forces may be discounted At one point in their region of contact the ring and tool do not slip The relative geometry of the tool surface is adequately characterized by "flat," "impulse," and "shoulders" Relative motion of tool and ring is disregarded in creating the periodic table, but can be included during the machine realization phase The ring initially has a square cross section

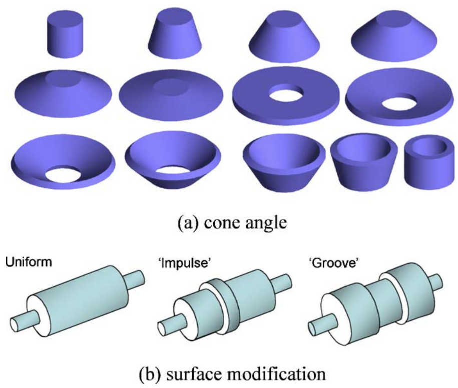


Fig. 4 Parameterization of solid rolls: (*a*) cone angle; (*b*) surface modification

modifications, but only a few will be significant in leading to distinctive ring rolling processes. Figure 4(*b*) illustrates the three possibilities that will be used in the parameterization of this paper: no modification; addition of "shoulders" that constrain the tangential growth of the face of the ring in contact with the tool; and a spatial "impulse" or incremental tool, with only partial contact with the ring. The "impulse" is a limiting case of all possible forms where the roll is not in full contact with the ring—and has the possibility that, once deformation has begun, tangential motion of the tool relative to the ring can lead to much higher tangential forces than possible with just frictional effects.

The conversion of continuous variation into discrete settings described above carries the danger that interesting possible machine designs may not be identified. This is unsatisfactory, but is inherent in Zwicky's approach—where a discrete set of choices must be identified. Without such pragmatic discretization, the search space—or the "morphological box"—would be continuous and infinite so unusable, unless a cost function could be provided to direct the selection of parameters. This paper assumes that the discrete choices made above is sufficiently broad to allow identification of all interesting options, but the results of the search must be interpreted in the knowledge of the arbitrary discrete settings that have been selected.

Following the requirements of Fig. 3, the type of tool, and its relative geometry have been described. The relative motion of tool and workpiece remains. Given the assumptions of Table 2, the influence of the tool on the ring must comprise a non-negative normal force and a tangential force. The location of the equivalent force must be within the region of contact, and its magnitude will be limited either by friction considerations, or by whatever prior deformation has occurred that allows the tool to "grip" the ring. This must be considered when the equilibrium constraint is applied, but otherwise the relative motion does not influence the design, so can be disregarded when searching for a basic set of distinct machines.

3.2 Simplification Related to Workpiece Geometry. The previous section has shown that a ring rolling machine comprises a number of tools arranged in continuous contact around a deforming ring with a stationary axis. The relative distribution of rolls around the ring has not yet been specified. However, the second of the four constraints in Sec. 2.2—on the constitutive law of deformation—usefully restricts the relative locations of the tools, as inappropriate arrangement of tools will lead primarily to bending deformation by which the ring will lose circularity. If the ring is to remain circular then, in any location where it deforms permanently, it must experience circumferential (or hoop) strains that increase linearly with radial distance from the ring axis. Exploring this requirement via a lower bound approach gives a useful insight into the necessary arrangement of tools around the ring.

When loads are applied to the ring, its deformation is initially elastic. By Hooke's law, the elastic hoop strain in the ring is related to the three components of direct stress by

$$E\varepsilon_\theta = \sigma_\theta - \nu\sigma_r - \nu\sigma_z \quad (1)$$

As the applied loads increase, the ring begins to deform permanently. If the hoop strains do not remain approximately constant across the ring cross section, a "plastic hinge" will form, and the ring will lose circularity. It will therefore be assumed that a "lower bound" criterion for maintaining circularity is that the hoop strain must be (nearly) uniform across the cross section as the loads on the ring increase and the stresses in the ring first approach a yield locus. As this is an elastic, and therefore linear, criterion, the in-plane and normal components of deformation may be treated separately.

Figure 5 shows a plane segment of the ring, with a region of permanent deformation shown at $\theta=0$, and three possible loading conditions—with zero, one, or two rolls contacting the ring in this region. No other tools contact the segment in the region shown, but other tools acting on the ring may lead to boundary forces and moments as shown. (The dimensions of the ring wall are assumed to be small compared to the ring radius so that approximation of the stress state by a shear force, axial force, and bending moment is acceptable.) If the only permanent deformation to occur in this segment is in the indicated region of deformation, then as the applied loading increases from zero, the equivalent stress must be a maximum in this region just prior to plastic deformation occurring. For the loading of Figs. 5(*a*) and 5(*b*), this cannot occur, and if the loading is increased to the point that plastic deformation occurs in the intended region, plastic deformation will already have occurred elsewhere. Therefore, only the loading of Fig. 5(*c*) can lead to permanent deformation in which the ring can remain circular. A proof is given in the Appendix.

It is straightforward to extend this analysis to three dimensions,

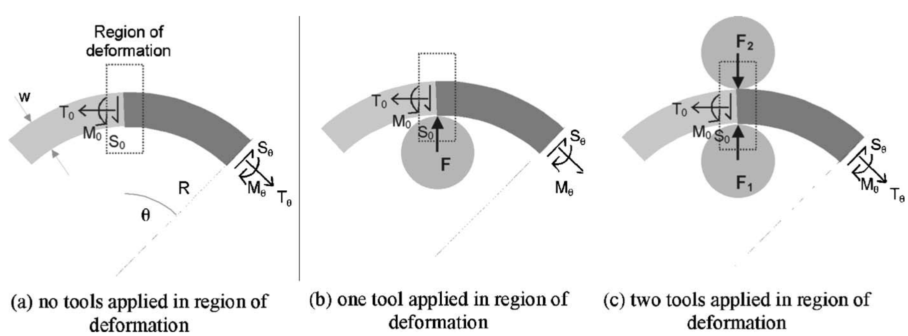


Fig. 5 Possible loadings of a segment of a plane ring: (*a*) none; (*b*) one; and (*c*) two tools acting in the region of deformation

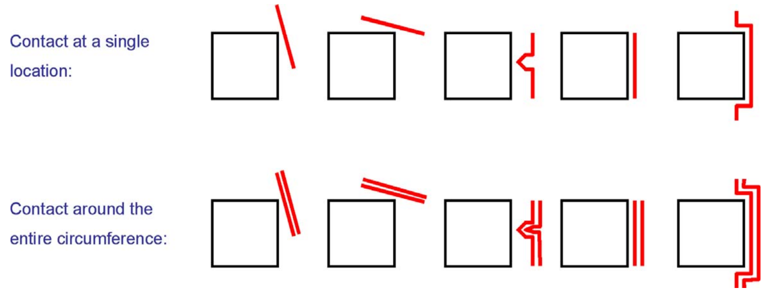


Fig. 6 All possible contacts that may occur with the outer face, or upper–outer edge of a ring cross section through application of the tools in Fig. 4

with the same result. It is thus possible to deduce that permanent deformation which allows the ring to remain circular can only be achieved by a set of opposed rollers acting within the region of deformation, primarily causing deformation through compressive radial stress. Within the deformation zones, the forces created by the rolls need not actually lead to equilibrium of the ring: it is possible for the deformation to be influenced by elastic forces outside the region of deformation—as occurs with the conventional guide rolls shown in Fig. 1—but such forces must be constrained so that the consequent state of stress outside the region of deformation does not approach a plastic limit. As shown in the Appendix, this can only occur if the tools within the region of deformation are organized in such a way that they can impose an equilibriate set of forces on the ring.

3.3 Parameterization of Ring Rolling Machines. The analysis of the previous section has demonstrated that in ring rolling, deformation must occur in discrete regions, and must primarily be caused by opposed rollers creating compressive radial stress in these regions. Each such deformation must maintain the circularity of the ring, so it is possible to describe the tools required to create any such deformation independently of those involved in any other deformation. This greatly simplifies the task of defining all possible machines, as it is now possible to consider only "elemental" ring rolling machines in which tools act at a single location (in θ) around the ring. An elemental machine is characterized by the arrangement of rolls around a single cross section of the ring, and must be capable of applying sufficient compressive forces to cause permanent deformation of the ring while maintaining ring equilibrium. As the rolls and ring are mechanically separate, their interaction is defined by the region of contact between them, so elemental machines can be characterized by showing on a cross section of the ring, a set of lines indicating where the tools contact the ring.

It is now possible to parameterize all possible elemental ring rolling machines. Figure 6 shows a ring cross section and all the possible contacts that may occur with the outer face, or upper–outer edge of the ring through application of the tools in Fig. 4. Strictly, the angle between tool face and ring axis, required when contact is at a ring edge, could have any value. The angles 15 deg and 75 deg have been used as indicative of all likely candidates that would avoid burring defects.

The contacts of Fig. 6 could be replicated by further tools acting at the other three faces and three edges of the cross section. With six possible contact types (or none) on each face and four (or none) on each vertex, this suggests a total of $(7^4 \times 5^4) = 1{,}500{,}625$ elemental ring rolling machines. (Each such machine has many "realizations" once the line of contact is replaced by a tool with given diameter and cone angle.) However, most of these will violate some of the constraints of Fig. 3.

4 Exhaustive Search Subject to Constraints

The argument of Sec. 3 has shown that it is possible to simplify the search for all possible ring rolling machines to a search for all possible "elemental" machines, which could create equilibriate deformation of the ring at a single cross section. However, following the constraints of Fig. 3, the tools in a candidate machine must not intersect each other, and the ring must be in equilibrium, and these two constraints significantly reduce the number of candidates. This section provides a parameterization of these two constraints, and shows how a structured search through all candidate machines can be executed to provide a set of feasible elemental machines which can be organized in a "periodic table." The periodic table describes only the contact between tools and ring at a single cross section, so a further set of combinations, to realize the contacts through particular tool designs, is explored in the next section.

4.1 Constraint to Avoid Tool Collision. The possible contacts of Fig. 6 arise when the candidate tool is on the outer face or upper–outer edge of a ring cross section, and each tool could equally act at three other faces or edges. Some combinations of tools are not possible—for instance any two of the ten tools shown in Fig. 6 could not coexist as the tools would collide. A simple parameterization of this constraint can be achieved by dividing the area outside the cross section into 12 zones as shown in Fig. 7(*a*). Any tool will occupy a certain number of these zones as shown in Figs. 7(*b*) and 7(*c*). If any zone contains more than one tool—as in Fig. 7(*c*)—the candidate machine may be rejected.

This collision test depends solely on the regions of contact between tools and ring. A further collision test will be required once a given machine design is realized with particular tool geometries as the tools and ring may also collide away from the deformation zone.

4.2 Constraint of Workpiece Equilibrium. It was shown in Sec. 3.2 that the forces applied to the workpiece by the tools in an

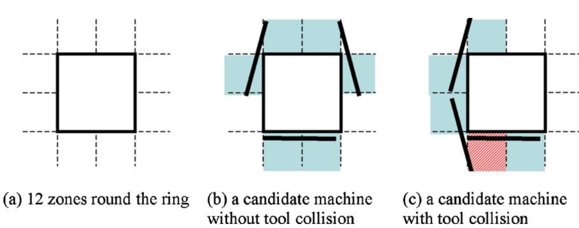


Fig. 7 Test for tool collision: (*a*) 12 zones around the ring; (*b*) a candidate machine without tool collision; and (*c*) a candidate machine with tool collision

elemental machine must be equilibriate. For a machine with N_{tools} tools, each applying an equivalent (vector) force $\mathbf{F}_i$, at location $\mathbf{r}_i$, equilibrium of forces and moments requires that

$$\sum_{i=1}^{N_{\text{tools}}} \mathbf{F}_i = \mathbf{0} \tag{2}$$

and

$$\sum_{i=1}^{N_{\text{tools}}} \mathbf{F}_i \times \mathbf{r}_i = \mathbf{0} \tag{3}$$

For each of the ten types of contact between tool and workpiece shown in Fig. 6, the direction of the equivalent force may vary within some limits, and the location of the equivalent force may be anywhere within the region of contact. The magnitude of the tool force must always be positive—acting away from the tool—as the contact is nonadhesive. Testing whether a candidate machine is acceptable depends on a parameterization of the tool force and its location, and an algorithm to determine whether the combination of forces provided by a given candidate machine can satisfy Eqs. (2) and (3). At first sight it is simple to parameterize the force and location but, due to the cross product, Eq. (3) is nonlinear, and this leads to a difficult test for feasibility. Instead, the parameterization used here is chosen to ensure that one or other components of each term in the cross product is known, so that a linear form of Eq. (3) can be used, and the test for feasibility reduces to a test for a feasible solution to a linear programming problem.

Figures 8(*a*)–8(*d*) show how the tool force direction and location may vary for each of the types of contact shown in Fig. 6. Figures 8(*e*)–8(*g*) show a parameterization of the unknown components of force and location for the tool with line contact of Fig. 8(*b*). For any candidate machine, the unit vectors $\mathbf{e}_n$ and $\mathbf{e}_t$ indicating the directions of the normal and tangential components of tool force are known from geometry, and the location of the center of the contact region, $\mathbf{c}$, is also known. The actual location of the equivalent force $\mathbf{r}$ can then be found as $\mathbf{r}=\mathbf{c}+x\mathbf{e}_t$, where x is a scalar variable subject to constraints determined by the type of tool and the ring geometry which can be written as $x^l \leqslant x \leqslant x^u$. The tool force $\mathbf{F}$ can be written as $\mathbf{F}=f_n\mathbf{e}_n+f_t\mathbf{e}_t$, where f_n and f_t are scalar unknowns (also subject to constraints). Expanding the cross product of Eq. (3) gives

$$\mathbf{F} \times \mathbf{r} = (f_n\mathbf{e}_n + f_t\mathbf{e}_t) \times (\mathbf{c} + x\mathbf{e}_t) = (f_n\mathbf{e}_n + f_t\mathbf{e}_t) \times \mathbf{c} + (f_n x)\mathbf{e}_n \times \mathbf{e}_t \tag{4}$$

The right-hand side of Eq. (4) contains a product of two variables $(f_n x)$ so is nonlinear. But, using the constraints on x, the cross product of Eq. (4) can be rewritten as

$$\mathbf{F} \times \mathbf{r} = (f_n\mathbf{e}_n + f_t\mathbf{e}_t) \times \mathbf{c} + m\mathbf{e}_n \times \mathbf{e}_t$$

$$\text{subject to } m \geqslant x^i f_n \text{ and } m \leqslant x^a f_n \tag{5}$$

which is a linear equation subject to linear constraints.

Thus using Eq. (5) to convert the product of unknowns in Eq. (3) to a linear formulation it is possible to test whether a candidate machine can satisfy the equilibrium conditions of Eqs. (2) and (3) by creating a linear program with $3 \times N_{\text{tools}}$ variables (f_n, f_t, and x for each tool), with equality constraints from Eqs. (2) and (3) and appropriate limits constraining the values of f_n, f_t, and x applied via slack variables. The linear program needs no objective function and the first phase of a conventional two-phase simplex solver can be used to test whether a feasible solution exists.

The design of the elemental ring rolling machines assumes that all tools are in continuous nonadhesive contact so, given the sign convention of Fig. 8, constraints must be applied to the normal force f_n such that $0 < f^{\min} \leqslant f_n \leqslant f^{\max}$. For the purposes of this analysis, the values of $f^{\min}$ and $f^{\max}$ are arbitrary—as any equilibriate set of forces could be scaled linearly. The constraints on the tangential force and offset x vary for each of the tools of Fig. 6, and are given in Table 3. For flat tools, the tangential force is limited by friction—simple Coulomb friction is used here—where for the incremental tool and that with shoulders, the magnitude of the tangential force could potentially reach $f^{\max}$. The offset x allows the location of the effective force to vary over the region of contact. For tools in complete circumferential contact, this allows a wider range of options, as the effective location of the force could be on either side of the ring, or in some cases, anywhere across its diameter.

For each of the candidate elemental machines generated in Sec. 3, it is now possible to apply two tests. The test of Sec. 4.1 ensures that a physical realization of the machine would be possible without collision between the tools. The test of this section establishes whether the candidate machine could apply a set of equilibriate forces to the ring—which could be scaled up to the point at which ring deformation occurs.

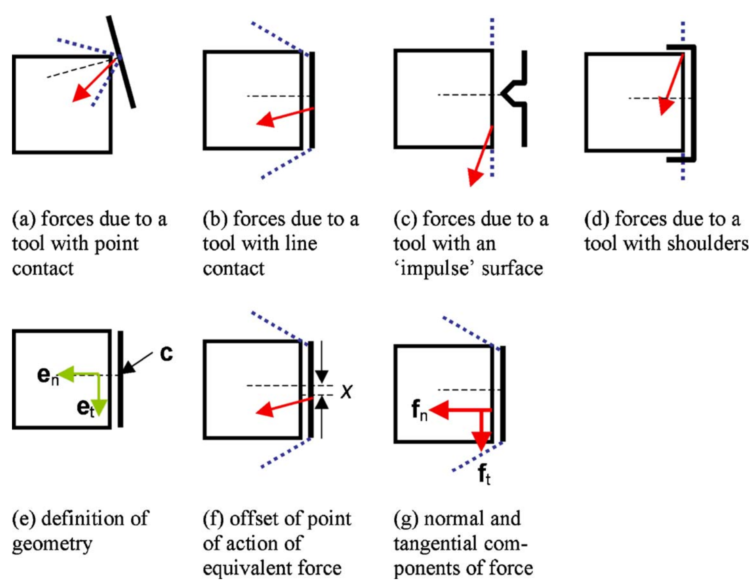
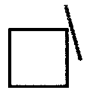
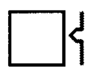
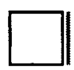
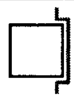
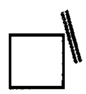
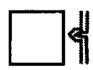
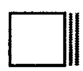
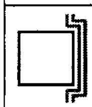


Fig. 8 Forces acting on a cross section of the ring due to a single tool

Table 3 Constraints on the tangential force component and force location for each tool contact[a]

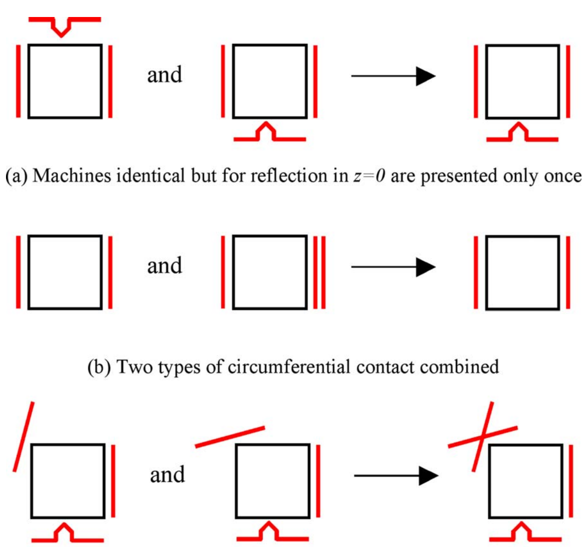
Tool contact	f_t lower	f_t upper	x lower	x upper	Comments
	$-\mu f_n$	μf_n	0	0	
	$-\alpha \times f^{max}$	$\alpha \times f^{max}$	$\pm h/4$	$\pm h/4$	As the incremental tool moves across the face, it must be tested in two locations – x=±$h/4$ –, and equilibrium must be possible with both options
	$-\mu f_n$	μf_n	$-h/2$	$+h/2$	
	$-\alpha \times f^{max}$	$\alpha \times f^{max}$	$-h/2$	$+h/2$	
	$-\mu f_n$	μf_n	$\pm r_o$	$\pm r_o$	The ring is encased in an external cone, so the equivalent force could act at either side of the ring – both must be tested.
	$-\alpha \times f^{max}$	$\alpha \times f^{max}$	$\pm h/4$	$\pm h/4$	As above, the equivalent force could act at either side of the ring – both must be tested
	$-\mu f_n$	μf_n	$-h/2$	$+h/2$	As above
	$-\alpha \times f^{max}$	$\alpha \times f^{max}$	$-h/2$	$+h/2$	As above

[a]Where the ring has wall height and width h, inner radius r_i, outer radius r_o and α is a scale factor $0<\alpha<1$ indicating the potential size of the tangential force for the impulsive tool.

4.3 Periodic Table of Ring Rolling Machines. A computer program has been written to allow application of the two constraints of Secs. 4.1 and 4.2 to the 1,500,625 candidate machines from Sec. 3. This leads to a set of 1507 elemental machines that could be built subject to the assumptions of Table 2. This set includes a number of nearly identical machines, so three simplifications will be used to allow a clearer presentation of the options: machines which are identical but for reflection in the $z=0$ plane will be presented only once; for any elemental machine having a tool which contacts the ring at a single circumferential location, an identical machine will exist in which the same tool contacts the ring around the circumference, so only the simpler machine will be shown; in Fig. 6, two tools having point contact at a ring edge were shown—angled at 15 deg and 75 deg. If two machines differ only by the angle of a tool of this type at the same location, they will be superimposed. These three simplifications are illustrated in Fig. 9, and when applied to the set of 1507 valid elemental machines, reduce the set to 102 distinct elemental ring rolling machines.

The set of 102 possible ring rolling machines are presented in Fig. 10. The table is organized into groups according to the number of faces and vertices of the ring cross section that are constrained by the machine: groups at the top of the table have four constraints while those at the bottom have none; the left side of the table shows constraints only on faces, and the right side shows constraints only on vertices. The labels of the form "F-V" indicate groups of machines with F faces and V vertices constrained. Within each group, the machines are organized by similarity, measured by the number of common faces or vertices constrained. The figure is thus more than a design catalogue of all options, as neighboring machines are more similar than distant ones, and the possibility of combining elemental machines into compounds as described in Sec. 5.1 below depends on the group membership of the elements. The figure is thus described as a "periodic table" of ring rolling machines.

Section 3 began by assuming that the ring cross section was



(a) Machines identical but for reflection in $z=0$ are presented only once

(b) Two types of circumferential contact combined

(c) Contacts at the same edge, but with different angles, combined.

Fig. 9 Simplifications used in presenting elemental machines

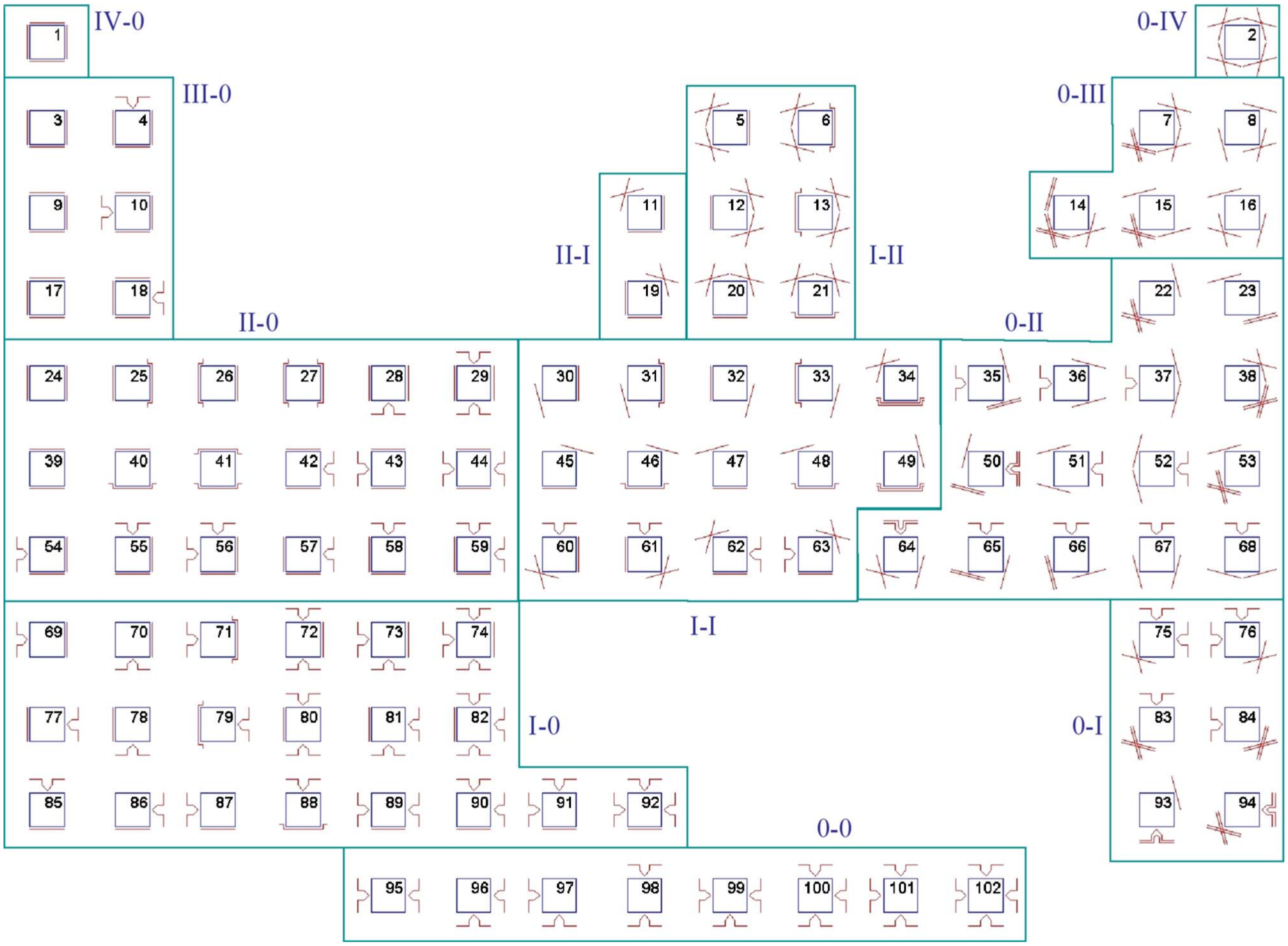


Fig. 10 The periodic table of ring rolling machines

initially square and in Sec. 3.1, a set of tools was identified to represent all possible rigid rollers. If these assumptions are correct, and the logical deductions of Secs. 3 and 4 are complete, Fig. 10 shows all possible ring rolling machines.

5 Machine Realization

The periodic table of Fig. 10 describes elemental machines acting at a single circumferential location, and indicates the extent of contact between the tools and the workpiece, so the actual shape of the tools is undefined. This section describes the realization of physically possible machines from the simplified representation of the table.

5.1 Compound Machines. A real machine may comprise any number of elemental machines distributed around the circumference of the ring, and will be referred to here as a "compound machine." As an example, the typical radial–axial machine used in hot ring rolling and illustrated in Fig. 1 comprises a compound of machines 24 and 39 from Fig. 10. All innovations in ring rolling machine design from Sec. 1 are included in the table: the two designs of railway wheel machine of Omori et al. [6] are identified in the table as 24+98, 12+29; the gear rolling machine of Onoda and Nakagawa [7] is a compound of three of elemental machine 24; and the two designs of incremental ring rolling machine examined by Allwood et al. [8] are 71 and 54+39. The fact that all known innovations in ring rolling machine design exist within the table acts as a weak form of validation as the approach. It is surprising that the compound 24+24 has not received more commercial attention, as it would have double the production speed of existing machines.

Physically, it would be possible to build machines formed from any compound of elemental machines in Fig. 10, provided that any tools having complete circumferential contact appeared identically in all elements in the compound. However, a major source of defects in ring rolling occurs when the edges of the ring cross section experience contrary deformations leading to cyclic fatigue, so it is reasonable to consider as useful only the compounds where any face or edge of the ring experiences only consistent deformations. Thus, in forming a compound with elemental machine 24, for instance, only elemental machines applying the same deformation, or none, to the radial faces should be considered. Thus machine 24, which is in group II-0 can be combined only with machines in groups IV-0, III-0, II-0, I-0, and 0-0—as all other groups require deformation of at least one ring edge, which would be counter to the flat face achieved by machine 24.

5.2 Realization of Machines From Definition of Contacts. For a given compound or elemental machine to be built, the tool contacts described by the periodic table of Fig. 10 must be "realized" by tools from the set shown in Fig. 4. Any cone angle and tool radius may be used to create each contact, and the different angles and radii will lead to small differences in ring deformation—with more or less tendency to retain circularity. However, the resulting choice of tools must not collide with each other or with the ring.

It is possible to search through a set of possible realizations by examining each design with each tool having each cone angle and radius in turn. This search will involve a vast range of options, and a limited range of radii and angles should be used, along with

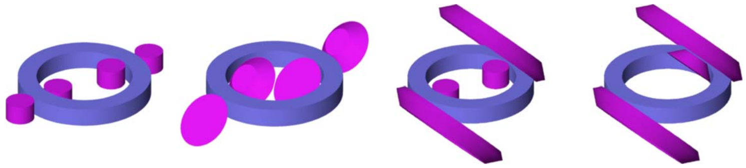
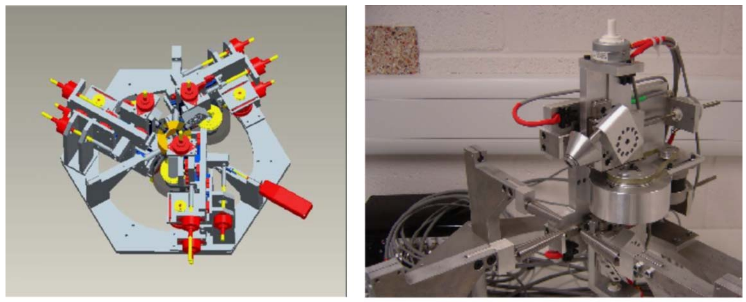


Fig. 11 Example realizations of the compound machine 24+24



(a) design of three module machine (b) photograph of one module machine

Fig. 12 Modular flexible ring rolling machine being built in Cambridge: (*a*) design of three module machine; and (*b*) photograph of one module machine

any other intelligence to limit the number of combinational choices. However, a collision test must be provided to ensure that the tools and ring do not intersect. This is a standard problem in computational geometric algebra, and appropriate algorithms are described by Ericson [22].

As an example of the process or realization, Fig. 11 shows four possible novel ring rolling machines, all formed from the compound 24+24 comprising two opposed radial roll bites.

6 Discussion

This paper has proposed a broad approach for applying the method of morphological analysis to the search for novel forming processes. The application of this approach to the design of ring rolling machines has shown that careful analysis of the required deformation and of the interaction of the tools and workpiece leads to a catalogue of manageable size. It is reasonable to hope that a similar approach would lead also to a tractable table of options for forging and extrusion, and possibly for sheet forming operations—although in the latter case, a much broader class of tools must be considered.

The success of the search for ring rolling machine designs is strongly dependent on the assumptions made in Table 2. The more assumptions made, the smaller the resulting table of options, but the greater the danger that an important possible design has been missed. It is possible to envisage machines which have no "neutral point" of zero slip between tool and ring. For instance, helical tube rolling is a process related to ring rolling used to extrude reduced diameter tubes, and the need to have the workpiece move along its axis relative to the tools ensures that some slip will occur at all contacts. The initial workpiece cross section may not be square, but the table of Fig. 10 should indicate most possibilities, although if the workpiece is more like a tube or disk, the number of tools allowed on each face of the cross section could be adjusted.

In Sec. 3, the discretization of design parameters that are inherently continuous was discussed. This is a weakness of Zwicky's approach. It is possible to envisage a wider range of roll surface profiles than the three that have been used but, provided the "impulse" shaped roll is considered to represent any tool having partial contact with a face of the ring, the range of equilibriate machines should not be affected.

The methodology could be extended in a number of ways. The linear programming formulation used to test the feasibility of designs, could also be used to indicate their mechanical stability (ability to maintain equilibrium in the face of forces and moments applied elsewhere to the ring). The elemental machine designs could be related to the geometry of ring cross sections they can produce. The periodic table could be embedded within a wider design process, thus completing the five point definition given by Zwicky and quoted in Sec. 2.1: given a specified ring preform, and a range of target ring geometries, which elemental machines could be used, and what forming limits does each impose?

It was shown above that all existing designs of ring rolling equipment are present in the periodic table. Which elements or combinations from the table would provide the most useful innovations today? In order to explore this question, a model flexible ring rolling machine has been designed and is being built in Cambridge [23]. The machine is modular, and can have up to three modules around the circumference of the ring. Each module comprises a constant radius outer forming roll to allow transmission of torque to the ring and one or two further tools which may have any geometry from Fig. 4 and can be configured to act on any other face or the two inner vertices, and move with two degrees of freedom. Figure 12 shows a model of the completed machine with three modules and a photograph of the machine with one module. Approximately half of the elemental machines of the periodic table can be implemented with this design, and the benefit and capability of a range of element combinations will be reported in future work.

Acknowledgments

The work of this paper has benefited greatly from an ongoing collaboration with Professor Erman Tekkaya of Atilim University in Ankara and Professor Hirt and Professor Kopp in Aachen, and also from discussions with Dr. Tim Stanistreet, Thomas Counsell, and Professor Ian Hutchings at Cambridge.

Appendix

Theorem. *If a segment of a thin walled ring is loaded elastically and in plane stress in the configuration of Figs. 5(a) or 5(b), and the hoop stress at location* $\theta=0$ *is uniform across the ring, the* *magnitude of the hoop stress cannot be a maximum at that location.*

Proof. For the loading of Fig. 5(*a*), in order to ensure equilibrium, the axial tension, shear force and bending moment must vary around the ring as

$$T_\theta = T_0 \cos\theta - S_0 \sin\theta$$

$$S_\theta = T_0 \sin\theta + S_0 \cos\theta$$

$$M_\theta = M_0 + T_0 R(1 - \cos\theta) + S_0 R \sin\theta \qquad \text{(A1)}$$

For a ring with rectangular cross section (ring width w, area A), the hoop stress at distance Δr from the neutral axis of the ring is calculated as,

$$A\sigma_\theta(\Delta r) = T + \frac{AM_\theta}{I}\Delta r, \quad \text{where } \left(I = \frac{Aw^2}{12}\right) \qquad \text{(A2)}$$

From Eq. (A2), in order to have uniform hoop stress across the ring at location 0, $M_0=0$. Expanding Eq. (A2) by (A1) and evaluating at the extremes of the ring (i.e., $\Delta r = \pm w/2$)

$$A\sigma_\theta\left(\frac{\pm w}{2}\right) = (T_0 \cos\theta - S_0 \sin\theta) \pm \frac{6R}{w}[T_0(1 - \cos\theta) + S_0 \sin\theta] \qquad \text{(A3)}$$

For a thin walled ring, $R \gg w$, so the bending terms in Eq. (A3) will dominate those due to tension, for all angles except $\theta \approx 0$. Thus if the tension T_0 dominates, the hoop stress will reach a maximum with $(1 - \cos\theta)$ at $\theta = \pi$, and if the shear force S_0 dominates, the magnitude of the hoop stress will reach a maximum with $\sin\theta$ at $\theta = \pm\pi/2$. In either case, the hoop stress cannot have maximum magnitude at $\theta = 0$.

For the loading of Fig. 5(*b*), with one tool applying a compressive force F to the inner face of the ring at $\theta = 0$, the equivalent expression to Eq. (A3) is

$$A\sigma_\theta\left(\frac{\pm w}{2}\right) = [T_0 \cos\theta - (S_0 - F)\sin\theta] \pm \frac{6R}{w}\left[\frac{M_0}{R} + T_0(1 - \cos\theta) + (S_0 - F)\sin\theta\right] \qquad \text{(A4)}$$

In this case, the applied force leads to a distribution of radial stress in the deformation region of the ring, falling from a maximum compressive value at the inner surface, to zero at the outer surface. In order to obtain uniform hoop strain, a distribution of tensile hoop stress is required, rising from zero at the inner face to a maximum at the outer face. Thus at $\theta = 0$, M_0 must dominate T_0, be positive and of sufficient magnitude to enforce yield in the outer surface of the ring. If $T_0 = 0$ and $S_0 = F$, yield will also occur throughout the segment. If T_0 is nonzero, or $S_0 \neq F$, the hoop stress for $\theta \neq 0$ must have greater magnitude on either outer or inner face of the ring than for $\theta = 0$. ■

Corollary. *The only means to generate uniform hoop strain in the deformation region of the ring segment in Fig. 5 is to apply two opposed compressive forces, as in Fig. 5(c), creating a (nearly) uniform field of radial stress in the region. This distribution may be influenced by axial force, shear force and bending forces applied to the segment by imbalanced forces acting elsewhere on the ring, but their influence must be constrained so that the peak hoop stress outside the deformation region is less than that which would cause yield.*

Comment. The approach here can be used to find the hoop stress anywhere in a ring subject to an equilibriate set of discrete radial forces. The ring can be divided into a set of segments of arc ϕ_i each having the form of Fig. 5(*b*), and with equilibrium described by Eq. (A1) (with S_0 replaced by S_0-F). At the interface of two segments, $T^i_{\phi_i} = T^{i+1}_0$ etc., so provided the applied loads are equilibriate, the hoop stress can be calculated anywhere in the ring. By inspection of Eq. (A4), the maximum hoop stress will always occur either at the point of loading, or at an angle $\pi/2$ or π from this point, provided that the segment is larger than this angle.

References

[1] Allwood, J. M., and Utsunomiya, H., 2006, "A Survey of Flexible Forming Processes in Japan," Int. J. Mach. Tools Manuf., **46**(15), pp. 1939–1960.

[2] Jeswiet, J., Micari, F., Hirt, G., Bramley, A., Duflou, J., and Allwood, J. M., 2005, "Asymmetric Single Point Incremental Forming of Sheet Metal," CIRP Ann., **54**(2), 623–650.

[3] Ziegelmayer, O., Schneider, V., Kopp, R., and Durr, O., 2004, "Analyses for the Production of Plane-Curved Work Pieces by the Assist of a Robot in Open-Die Forging," Materialwiss. Werkstofftech., **35**(7), pp. 447–453.

[4] Allwood, J. M., Tekkaya, A. E., and Stanistreet, T. F., 2005, "The Development of Ring Rolling Technology," Steel Res., **76**(2/3), pp. 111–120.

[5] Allwood, J. M., Tekkaya, A. E., and Stanistreet, T. F., 2005, "The Development of Ring Rolling Technology—Part 2: Investigation of Process Behaviour and Production Equipment," Steel Res., **76**(7), pp. 491–507.

[6] Omori, S., et al., 1984, "Experiment on the Roll Forming for Disk-Shaped Blanks," *Proceedings 3rd International Conference on Rotary Metalworking Processes*, Kyoto, Japan, 8–10 Sept. 1984, pp. 229–238.

[7] Onoda, Y., and Nakagawa, T., 1984, "Spline and Gear Forming by Expanding Type Ring Rolling," *Proceedings 3rd International Conference on Rotary Metalworking Processes*, Kyoto, Japan, 8–10 Sept. 1984, pp. 291–300.

[8] Allwood, J. M., Kopp, R., Michels, D., Music, O., Oztop, M., Stanistreet, T. F., Tekkaya, A. E., and Tiedemman, I., 2005, "The Technical and Commercial Potential of an Incremental Ring Rolling Process," CIRP Ann., **54**(1), pp. 233–236.

[9] Allwood, J. M., 2005, "A Periodic Table of Ring Rolling Processes," *Proceedings International Conference on Technology of Plasticity*, Verona, Italy, 9–13 Oct. 2005.

[10] Porter, A. L., 2004, "Technology Futures Analysis: Toward Integration of the Field and New Methods," Technol. Forecast. Soc. Change, **71**, pp. 287–303.

[11] Coyle, G., 2003, "Techniques for Exploring the Possibilities, Relevance and Impacts of Innovation," Def. Sci. J., **8**(3), pp. 191–201.

[12] Aristotle, 350 BC, "Politics," Book 4 Part IV, B. Jowett, trans., http://classics.mit.edu/Aristotle/politics.html

[13] Zwicky, F., 1969, *Discovery, Invention, Research—Through the Morphological Approach*, Macmillian, Toronto, Canada.

[14] Zwicky, F., 1967, "Prologue," *New Methods of Thought and Procedure*, F. Zwicky and A. G. Wilson, eds., Pasadena, CA 22–24 May, Springer, New York.

[15] Ayres, R. U., 1969, "Morphological Analysis," in *Technology Forecasting and Long Range Planning*, McGraw–Hill, New York, pp. 72–93.

[16] Swemorph, 2005, "The Swedish Morphological Society," www.swemorph.com.

[17] Pahl, G., and Beitz, W., 1995, *Engineering Design: A Systematic Approach*, K. Wallace, translator, L. Blessing and F. Bauert, eds., Springer, London, UK.

[18] Diekhöner, G., and Lohkamp, F., 1976, "Objektkataloge—Hilfsmittel beim Methodischen Konstruieren," Konstruktion, **28**, pp. 359–364.

[19] Raab, W., and Schneider, J., 1982, "Gliederungssystematik für Getriebetechnische Konstruktionskataloge," Antriebstechnik, **21**, pp. 603–607.

[20] Ersoy, M., 1976, "GieBtechnische Fertigungsverfahren-Konstruktionskatalog für Fertigungsverfahren," Wt-Z. in der Fertigung, **66**, pp. 211–217.

[21] Roth, K., 1982, *Konstruieren mit Knostruktionskatalogen*, Springer, Berlin, Germany.

[22] Ericson, C., 2005, *Real-Time Collision Detection*, Morgan Kaufmann, San Francisco, CA.

[23] Stanistreet, T. F., Allwood, J. M., and Willoughby, A. M., 2006, "The Design of a Flexible Model Ring Rolling Machine," *Proceedings Metal Forming 2006*, Birmingham, UK, Sept. 11–13, J. Mat. Proc. Tech. **177**(1–3), pp. 630–633.