Understanding Portfolio Efficiency with Conditioning Information*

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Abstract

I develop two new types of portfolio efficiency when returns are predictable. The first type maximizes the unconditional Sharpe ratio of excess returns and differs from unconditional efficiency unless the safe asset return is constant over time. The second type maximizes conditional mean-variance preferences and differs from unconditional efficiency unless, additionally, the maximum conditional Sharpe ratio is constant. Using stock data, I quantify and test their performance differences with respect to unconditionally and fixed-weight efficient returns. I also show the relevance of the two new portfolio strategies to test conditional asset pricing models.

Keywords: Conditional CAPM, Dynamic portfolio strategies, Jensen's alpha, Mean-variance frontiers, Performance efficiency, Residual efficiency, Sharpe ratio.

JEL: C12, G11, G12

I Introduction

The seminal paper of Hansen and Richard (1987) analyzes the tension between the conditional implications of asset pricing theory and the use of unconditional moments in empirical work. Perhaps their best-known result is that *unconditionally efficient* (UE) returns are a subset of the *conditionally efficient* (CE) returns. For instance, this result shows that the conditional capital asset pricing model (CAPM) implies that the market portfolio is CE but not necessarily UE.

The difference between CE and UE returns is well-known in empirical asset pricing[1] but the latter represent the only subset of CE returns that has been studied. Several papers have used UE returns to guide portfolio choice. For example, Ferson and Siegel (2001) and, adding a benchmark, Chiang (2009). Other papers, such as those of Brandt and Santa-Clara (2006) and Bansal, Dahlquist, and Harvey (2004), approximate UE returns through managed portfolios. My main contribution is a comprehensive analysis of two new types of CE returns that are more relevant from an empirical and theoretical perspective.

Unconditional Sharpe ratios and Jensen's alphas of excess returns with respect to the safe asset return are commonly used in empirical work. For this reason, I characterize a new set of efficient returns that achieve the maximum unconditional Sharpe ratio or, equivalently, display zero unconditional Jensen's alphas as a pricing factor. I therefore refer to this new subset of CE returns as *performance–efficient* (PE) returns. I show that, for a given target of expected return, PE returns minimize the variance of the excess return instead of the total return variance, which is minimized by UE returns. These two variances do not coincide in the presence of a safe asset such as the Treasury-bill, whose return is conditionally riskless but unconditionally risky.[2]

Ferson and Siegel (2009) construct an efficiency test based on the maximum unconditional Sharpe ratio with conditioning information. My results indicate that such a test is actually testing if a particular return is PE instead of UE. The authors assume a constant safe asset return when they develop their test and, in this case, both types of efficiency are

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[1]Lettau and Ludvigson (2001) is an influential reference on the distinction between conditional and unconditional asset pricing models.

[2]As a by-product, I carefully describe UE returns in the presence of this safe asset. Hansen and Richard (1987) study a general set-up that may or not include a safe asset. However, they only make the role of a safe asset explicit to clarify certain ideas, such as the safe return is CE but not necessarily UE.

equivalent. In fact, I show that this is the only case where UE and PE returns coincide.

I also develop a second type of efficient returns that can be rationalized by conditional mean-variance preferences that are commonly used in finance theory. If we decompose the unconditional variance of a return as the average conditional variance plus the variance of the conditional mean, then these optimal returns minimize only the first component. I use the term *residually efficient* (RE) returns because the average conditional variance is equal to the variance of the residual from a predictive regression. RE returns achieve the maximum Sharpe ratio and display zero Jensen's alphas as a pricing factor if the required variances and covariances are based on the residuals instead of the returns themselves. I define these new performance measures as the residual Sharpe ratio and alpha, respectively.

Abhyankar, Basu, and Stremme (2012) empirically evaluate several return predictors. For this purpose, they approximate the slope of the UE frontier asymptotes by the second moment of the slope of the CE frontier, that is, the maximum conditional Sharpe ratio. My results indicate that this second moment has a exact relation with the slope of the RE frontier instead, which is given by the maximum residual Sharpe ratio. I also show that the UE and RE return frontiers are equal if and only if both the safe asset return and the maximum conditional Sharpe ratio are constant over time.

Interest rates and Sharpe ratios change over time and hence UE, PE and RE returns represent different ways of exploiting conditioning information. On the other hand, how different they are is an empirical question and I use stock data to quantify and test their performance differences. I work with the three Fama-French factors as excess returns and three prominent predictors: the dividend price ratio, the default spread, and the term spread. To study the role of the investment set, I also work with the six and 25 size and book-to-market sorted Fama-French portfolios.

I study monthly returns during two periods, 1954–1983 and 1984–2012, because some features of return predictability may differ across data periods. We can associate the first period with stronger market predictability and the second one with weaker market predictability. However, the size and value effects are relatively more predictable in the second period.

I estimate and test the differences in unconditional Sharpe ratios between PE and

UE returns. My theoretical results show that UE returns do not display a unique unconditional Sharpe ratio. This ratio may change considerably for return targets around the safe asset but, as we increase the target, this ratio converges toward its value for PE returns. For this reason, in the empirical application I study a low and a high target for UE returns, 6% and 10%, respectively, in annualized terms. The performance gaps are economically and statistically significant with the 25 portfolios and the low target 6%, with a stronger gap in the first period than in the second one.

I extend the standard econometric set-up of unconditional Sharpe ratios to residual Sharpe ratios. I estimate and test the differences in the residual Sharpe ratios of RE returns with respect to UE returns. Once again, the performance gap is especially strong for the low target and the 25 portfolios.

The return frequency is another relevant dimension to the differences across efficiency types. Annual returns show larger differences across the subsets of CE returns. In fact, UE returns are considerably different with respect to PE or RE returns for a small investment set, namely, the three Fama-French factors.

Finally, I also connect my theoretical results and empirical methodology to the testing of conditional asset pricing models. I use the same data to revisit the evidence against the CAPM, which is a classic example of efficiency and asset pricing tests that still attracts attention. For instance, there is still controversy about the relative performance of the unconditional and conditional CAPM, where the former model is the textbook one with fixed-weight efficient (FE) returns that do not exploit return predictability.

When we only use the three factors as the investment set, in the first period the market is much more inefficient with FE returns than with PE or RE returns, but the second period provides the opposite situation. Then the increases in Sharpe ratios when adding the size and value portfolios to the PE and RE strategies are much higher than those with FE strategies, which are not statistically significant. My findings also indicate that it is good empirical practice to compute the performance of both PE and RE returns when testing asset pricing models.

The rest of the paper is organized as follows. Section II describes the theoretical framework with conditioning information and defines CE returns. Section III provides a theoretical analysis of two new types of efficient returns as subsets of CE returns that

generally differ from UE returns. Next, Section IV develops the empirical application. Finally, Section V summarizes the conclusions and discusses some avenues of further research. The auxiliary results are gathered in the online appendix.

II   Conditionally Efficient (CE) Returns

The investment set is defined by a safe asset with return R_0 and some risky assets with return vector $\mathbf{R}$. If we subtract R_0 from each return in $\mathbf{R}$, then we obtain the vector of excess returns $\mathbf{r} = \mathbf{R} - R_0\mathbf{1}$, where $\mathbf{1}$ is a vector of ones. The case where no safe asset is available is studied in Appendix C. The investors' information set is given by a vector $\mathbf{z}$ of predictors that are informative about future asset payoffs.

The safe asset return is conditionally riskless, $Var(R_0|\mathbf{z}) = 0$, but it may be unconditionally risky with $Var(R_0) > 0$. For ease of exposition, I assume the conditional variance $Var(\mathbf{r}|\mathbf{z})$ is nonsingular with probability one. This implies that none of the primitive risky assets is actually conditionally riskless or redundant. I also assume that the vector of risk premia $E(\mathbf{r}|\mathbf{z})$ has at least one nonzero entry to avoid trivial efficient returns.

If an investor is endowed with positive wealth, which we can normalize to one without loss of generality, then the investor will be interested in portfolio strategies that cost one for every possible value of $\mathbf{z}$. Let $\mathbf{w}$ denote the vector of portfolio weights on the risky returns $\mathbf{R}$, where each weight is a decision variable of the investor conditional on her information set and hence a function of $\mathbf{z}$. The weight on the safe asset return R_0 must be $1 - \mathbf{w}'\mathbf{1}$ in a unit-cost portfolio. Therefore, the payoff of such a portfolio can be represented by the return

$$R = R_0(1 - \mathbf{w}'\mathbf{1}) + \mathbf{R}'\mathbf{w} = R_0 + (\mathbf{R} - R_0\mathbf{1})'\mathbf{w},$$

which can be interpreted as a unit weight on the safe return and weights $\mathbf{w}$ on the vector of excess returns

$$R = R_0 + \mathbf{r}'\mathbf{w}. \tag{1}$$

In this setting, the *conditionally efficient* (CE) returns of Hansen and Richard (1987) are the returns with minimum conditional variance for a given target of conditional ex-

pected returns, that is, the set of returns (1) that solve the optimization problem

$$\min_{R} Var\left(R|\mathbf{z}\right) \text{ for a given } E\left(R|\mathbf{z}\right), \tag{2}$$

which I denote R_C.

This problem is the conditional counterpart of the classic Markowitz (1952) portfolio problem, where the mean and variance are unconditional and the weights $\mathbf{w}$ in (1) are real numbers instead of functions of $\mathbf{z}$. Similarly, the safe asset is interpreted as unconditionally riskless. In that well-known context, the efficient returns can be represented by

$$R_0 + \omega \mathbf{r}' \boldsymbol{\varphi}, \tag{3}$$

where[3]

$$\boldsymbol{\varphi} = \left[Var\left(\mathbf{r}\right)\right]^{-1} E\left(\mathbf{r}\right) \tag{4}$$

and ω is a real number that depends on the mean target $E\left(R\right)$. I refer to these returns as *fixed-weight efficient* (FE) because investors are constrained to fixed-weight strategies.

In our more general set-up where investors exploit return predictability, the returns (1) that solve (2) have a similar structure

$$R_C = R_0 + \omega_C \mathbf{r}' \boldsymbol{\varphi}_C, \tag{5}$$

where

$$\boldsymbol{\varphi}_C = \left[Var\left(\mathbf{r}|\mathbf{z}\right)\right]^{-1} E\left(\mathbf{r}|\mathbf{z}\right), \tag{6}$$

and ω_C is a function of $\mathbf{z}$ that depends on the conditional mean target $E\left(R|\mathbf{z}\right)$.

We can also define Sharpe ratios and Jensen's alphas in this setting. We can translate a return R into an excess return $r = R - R_0$ and define the conditional Sharpe ratio of r as

$$S_C = E\left(r|\mathbf{z}\right) / \sqrt{Var\left(r|\mathbf{z}\right)},$$

which is a function of $\mathbf{z}$. These Sharpe ratios and other types in the following sections are

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[3]Stevens (1998) interprets the vector $\boldsymbol{\varphi}$ in terms of hedging regressions. Each entry of $\boldsymbol{\varphi}$ is equal to the ratio of the intercept and the residual variance of the regression of that excess return onto a constant and the rest of excess returns in $\mathbf{r}$.

defined for r different from zero or, equivalently, for R different from R_0. The conditional Jensen's alpha of r with respect to the pricing factor r_β is defined as

$$\alpha_C = E(r|\mathbf{z}) - \beta_C E(r_\beta|\mathbf{z}), \qquad \beta_C = Cov(r, r_\beta|\mathbf{z})/Var(r_\beta|\mathbf{z}).$$

Similarly to Sharpe ratios, Jensen's alphas are defined for r_β different from zero.

The CE returns have interesting properties in terms of the previous two measures. The excess returns of CE returns different from R_0 are characterized by achieving the maximum squared conditional Sharpe ratio, denoted $\mathcal{S}_C^2$ and given by

$$\mathcal{S}_C^2 = E(\mathbf{r}|\mathbf{z})' [Var(\mathbf{r}|\mathbf{z})]^{-1} E(\mathbf{r}|\mathbf{z}). \tag{7}$$

These excess returns are also characterized by $\alpha_C = 0$ when they are used as a factor to price any r.

These properties translate the textbook Markowitz properties into conditional moments. The CE returns lie along two straight lines in the $[\sqrt{Var(R|\mathbf{z})}, E(R|\mathbf{z})]$ space for each possible value of $\mathbf{z}$. These two lines intersect on the vertical axis at R_0 and their slope is given by $\mathcal{S}_C$. I will refer to this conditionally linear relation between the optimal $\sqrt{Var(R|\mathbf{z})}$ and $E(R|\mathbf{z})$ as the CE frontier.

III   Two New Types of CE Returns

This section develops the theory of two subsets of CE returns, each one associated with a different construction of ω_C in (5). I define these two types of efficient returns, show their properties, and study their differences with respect to a third subset of CE returns, the only one that has been studied in the literature.

A   Performance–Efficient (PE) Returns

The analysis of unconditional moments is also relevant when returns are predictable. Unconditional moments, estimated by sample averages, are often used in empirical work. The performance evaluation of a portfolio manager is another example of the use of unconditional moments because the evaluator may not have access to the manager's information

set.

In particular, unconditional Sharpe ratios and Jensen's alphas of excess returns are commonly used in empirical finance. For instance, see Ferson and Siegel (2009) and the classic references therein such as Jobson and Korkie (1982) and Gibbons, Ross, and Shanken (1989). The unconditional Sharpe ratio of an excess return r is defined as the real number

$$S_U = E\left(r\right)/\sqrt{Var\left(r\right)}$$

and the unconditional Jensen's alpha of an excess return r with respect to a pricing factor r_β is defined as

$$\alpha_U = E\left(r\right) - \beta_U E\left(r_\beta\right), \qquad \beta_U = Cov\left(r, r_\beta\right)/Var\left(r_\beta\right).$$

The Sharpe ratio above is computed from the variance of the excess return $Var\left(r\right) = Var\left(R - R_0\right)$, which is not equal to the variance of the return $Var\left(R\right)$ when the safe asset is unconditionally risky. On the other hand, even though $Var\left(R_0\right) > 0$, the safe asset is riskless in terms of $Var\left(r\right)$ because $Var\left(R_0 - R_0\right) = 0$. Following the use of $Var\left(R - R_0\right)$ in empirical finance, I define the PE returns as the returns (1) that solve the problem

$$\min_{R} Var\left(R - R_0\right) \text{ subject to } E\left(R\right) = \nu \tag{8}$$

and I denote them R_P. The following proposition analyzes the PE returns.

Proposition 1 *The representation and properties of PE returns R_P defined by (8).*

  1. *The PE returns are CE returns (5) with*

$$\omega_C = \left[\frac{\nu - E\left(R_0\right)}{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)}\right]\frac{1}{1+\mathcal{S}_C^2}, \tag{9}$$

   which is a function of $\mathbf{z}$ because it depends on $\mathcal{S}_C^2$.

  2. *The excess returns of PE returns different from R_0 are characterized by achieving*

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the maximum S_U^2, denoted $\mathcal{S}_U^2$ and given by

$$S_U^2 = \frac{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)}{1 - E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)}. \tag{10}$$

These excess returns are also characterized by $\alpha_U = 0$ when they are used as a factor to price any r.

Point 2 of Proposition 1 shows that the PE returns have similar properties to the classic Markowitz set-up with a safe asset in terms of Sharpe ratios and alphas. However, this is not the case for *unconditionally efficient* (UE) returns, the only subset of CE returns that has been studied before. Hansen and Richard (1987) define them as the returns with minimum unconditional variance $Var\,(R)$ for each target of unconditional expected return $E\,(R)$. Thus the UE returns are the set of returns (1) that solve the problem

$$\min_R Var\,(R) \text{ subject to } E\,(R) = \nu \tag{11}$$

and I denote them R_U. Adapting their results to the existence of a safe return R_0 that may be unconditionally risky, the UE returns R_U are CE returns (5) with

$$\omega_C = \left[\frac{\nu + E\left(R_0\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right) - E\,(R_0)}{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)} - R_0\right]\frac{1}{1+\mathcal{S}_C^2}, \tag{12}$$

which is a function of $\mathbf{z}$ because it depends on R_0 and $\mathcal{S}_C^2$.

From point 1 of Proposition 1, the difference between PE and UE returns with the same mean is

$$R_P - R_U = \left[R_0 - \frac{E\left(R_0\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)}{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)}\right]\frac{1}{1+\mathcal{S}_C^2}\mathbf{r}'\boldsymbol{\varphi}_C. \tag{13}$$

When the safe asset return is unconditionally riskless, there is no conflict between the criteria $Var\,(R)$ and $Var\,(R - R_0)$ and we have $R_P - R_U = 0$ in (13). The following corollary to Proposition 1 states that this is the only case where the subsets of UE and PE returns are equal.

Corollary 1 *Given the representations of UE and PE returns above, these two subsets*

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of CE returns are equal if and only if

$$Var(R_0) = 0.$$

In this special case the intercept of the CE frontier R_0 cannot change over time, but the slope $\mathcal{S}_C$ may change. In this regard, Proposition 1 provides a general relation (10) between $\mathcal{S}_U^2$ and $\mathcal{S}_C^2$. Jagannathan (1996) obtained a similar relation for the unconditional Sharpe ratio of UE returns with a constant safe asset return. My results show that PE returns are the only ones that achieve $\mathcal{S}_U^2$ independently of the behavior of R_0.

Ferson and Siegel (2001) also study UE returns under the assumption of a constant R_0. Their equation (12) provides the optimal vector of weights $\mathbf{w}$ in (1) that solves the UE problem (11), which is equal to

$$\left[\frac{\nu - R_0}{E\left[E(\mathbf{r}|\mathbf{z})' \left[E(\mathbf{rr}'|\mathbf{z}) \right]^{-1} E(\mathbf{r}|\mathbf{z}) \right]} \right] \left[E(\mathbf{rr}'|\mathbf{z}) \right]^{-1} E(\mathbf{r}|\mathbf{z}).$$

When the safe asset is such that $R_0 = E(R_0)$, it is easy to show that these optimal weights are equal to $\omega_C \boldsymbol{\varphi}_C$, with ω_C from (9), by exploiting the relation[4]

$$E(\mathbf{rr}'|\mathbf{z}) = Var(\mathbf{r}|\mathbf{z}) + E(\mathbf{r}|\mathbf{z}) E(\mathbf{r}|\mathbf{z})'.$$

The weights must be equal because Corollary 1 states that UE and PE returns coincide in this special case.

Similarly, Ferson and Siegel (2009) construct an efficiency test that is based on $\mathcal{S}_U$. My theoretical results indicate that such a test is actually testing wether a particular return is PE, not UE. The empirical application in Section IV.C shows that the performance gap between UE and PE returns can be sizeable.

I will refer to the relations between the standard deviations of UE and PE returns and the mean target as the UE and PE frontiers, respectively. Figure 1 displays the UE and PE frontiers in two different spaces, using the data and empirical methods of Section IV. In the left plot of Figure 1, the PE frontier is a straight line with slope $\mathcal{S}_U$ for positive risk

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[4]This relation implies that $\left[E(\mathbf{rr}'|\mathbf{z}) \right]^{-1} E(\mathbf{r}|\mathbf{z}) = \boldsymbol{\varphi}_C / \left(1 + \mathcal{S}_C^2 \right)$ and $E\left[E(\mathbf{r}|\mathbf{z})' \left[E(\mathbf{rr}'|\mathbf{z}) \right]^{-1} E(\mathbf{r}|\mathbf{z}) \right] = E\left[\mathcal{S}_C^2 / \left(1 + \mathcal{S}_C^2 \right) \right]$.

premia in the $[\sqrt{Var(r)}, E(r)]$ space and provides the best performance in that space.

<Figure 1>

We can relate the excess returns of UE and PE returns with the same mean ν by

$$R_U - R_0 = (R_P - R_0) - (R_P - R_U)$$

with $R_P - R_U$ displayed in (13). This term is uncorrelated with $R_P - R_0$ and hence we have the following variance decomposition

$$Var(R_U - R_0) = Var(R_P - R_0) + Var(R_P - R_U).$$

The difference between $Var(R_U - R_0)$ and $Var(R_P - R_0)$ is the same across different values of ν because $R_P - R_U$ does not depend on the chosen ν. On the other hand, as is common practice, the plots in Figure 1 are based on standard deviations instead of variances. We can see in the left plot that the unconditional Sharpe ratio of $R_U - R_0$ converges to the ratio for $R_P - R_0$ as ν increases.

In the right plot of Figure 1, we can compare the PE and UE frontiers on the $[\sqrt{Var(R)}, E(R)]$ space, where the latter is the most efficient. We can relate the PE and UE returns with the same mean ν by

$$R_P = R_U + (R_P - R_U)$$

with $R_P - R_U$ displayed in (13). This term is uncorrelated with R_U and hence the difference between both frontiers follows from

$$Var(R_P) = Var(R_U) + Var(R_P - R_U).$$

The PE returns are located on a parabola that is parallel to the parabola of UE returns in the $[Var(R), E(R)]$ space. The difference between the two parabolas is equal to $Var(R_P - R_U)$. However, the right plot of Figure 1 is based on standard deviations instead of variances. In this plot, the UE and PE frontiers share the location of the minimum and the asymptotes, being more different for lower values of ν.

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For comparison, the location of the Markowitz frontier is also displayed in Figure 1. This frontier is defined by the FE returns in (3). In the left plot, the FE frontier is also represented by a straight line and there is a region where FE returns perform better than UE returns.

Finally, we can also compare the unconditional alphas of PE and UE returns. The former yield zero α_U as a factor when pricing any excess return, but α_U is not generally zero when we use UE returns as pricing factors. In fact, the value of α_U depends on the particular mean target of the UE return.

Nevertheless, Hansen and Richard (1987) show that UE returns satisfy a different beta pricing equation. In a general set-up that may not include a safe asset, a return R_β different from the minimum unconditional variance one is UE if and only if, for every R,

$$E\left(R\right) - E_U = \frac{Cov\left(R, R_\beta\right)}{Var\left(R_\beta\right)}\left[E\left(R_\beta\right) - E_U\right] \tag{14}$$

where E_U is a real number. This number is interpreted as the unconditional mean of the corresponding zero-beta return and depends on the chosen UE pricing factor R_β.

B Residually Efficient (RE) Returns

Appendix A analyzes the link between mean-variance preferences and frontiers with conditioning information. Ferson and Siegel (2001) show that UE returns can be rationalized by preferences $E\left(R|\mathbf{z}\right) - \left(b/2\right)E\left(R^2|\mathbf{z}\right)$ for some positive real number b. However, the most common conditional mean-variance preferences in finance theory are probably

$$E\left(R|\mathbf{z}\right) - \left(\theta/2\right)Var\left(R|\mathbf{z}\right)$$

for some positive real number θ. Areas such as market microstructure and rational expectations equilibria often rely on these preferences. See, e.g., Easly and O'Hara (2004) or Brunnermeier (2001) for a survey of asset pricing theory under asymmetric information.[5]

If we average these preferences over $\mathbf{z}$ then we obtain $E\left(R\right) - \left(\theta/2\right)E\left[Var\left(R|\mathbf{z}\right)\right]$ and hence these preferences penalize only a component of the unconditional variance.

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[5]They are also used in continuous time asset allocation (Basak and Chabakauri (2010) and the references therein) and to study the complexity of the performance evaluation of an informed manager by an uninformed agent (Dybvig and Ross (1985)).

Specifically, we can decompose the unconditional variance of a return as

$$Var\left(R\right)=E\left[Var\left(R|\mathbf{z}\right)\right]+Var\left[E\left(R|\mathbf{z}\right)\right].$$

I refer to the first component $E\left[Var\left(R|\mathbf{z}\right)\right]$ as the residual variance because it is the variance of the residual $R-E\left(R|\mathbf{z}\right)$ from a predictive regression,

$$Var\left(R-E\left(R|\mathbf{z}\right)\right)=E\left[\left(R-E\left(R|\mathbf{z}\right)\right)^{2}\right]=E\left[Var\left(R|\mathbf{z}\right)\right].$$

Note that the safe asset return may be risky from the perspective of $Var\left(R_0\right)>0$, but not from the perspective of $E\left[Var\left(R_0|\mathbf{z}\right)\right]=0$.

Following the link between conditional mean-variance preferences and $E\left[Var\left(R|\mathbf{z}\right)\right]$, I define the RE returns as the set of returns (1) that solve the problem

$$\min_{R} E\left[Var\left(R|\mathbf{z}\right)\right]\ \text{subject to}\ E\left(R\right)=\nu, \tag{15}$$

which I denote R_R. This link also motivates the use of residual Sharpe ratios and alphas, where the required variances and covariances are based on the residuals of predictive regressions instead of the returns themselves. I define the residual Sharpe ratio of an excess return r as the real number

$$S_R = E\left(r\right)/\sqrt{E\left[Var\left(r|\mathbf{z}\right)\right]}$$

and the residual Jensen's alpha of an excess return r with respect to a pricing factor r_β as

$$\alpha_R = E\left(r\right)-\beta_R E\left(r_\beta\right),\qquad \beta_R = E\left[Cov\left(r,r_\beta|\mathbf{z}\right)\right]/E\left[Var\left(r_\beta|\mathbf{z}\right)\right].$$

The following proposition analyzes RE returns.

Proposition 2 *The representation and properties of RE returns R_R defined by (15).*

1. *The RE returns are CE returns (5) with*

$$\omega_C = \frac{\nu-E\left(R_0\right)}{E\left(\mathcal{S}_C^2\right)}, \tag{16}$$

which is a real number.

2. *The excess returns of RE returns different from R_0 are characterized by achieving the maximum S_R^2, denoted $\mathcal{S}_R^2$ and given by*

$$\mathcal{S}_R^2 = E\left(\mathcal{S}_C^2\right). \tag{17}$$

These excess returns are also characterized by $\alpha_R = 0$ when they are used as a factor to price any r.

Point 2 of Proposition 2 shows that RE returns have standard properties in terms of residual Sharpe ratios and alphas. Once again, this is not the case for UE returns. We can decompose the difference between RE and UE returns with the same mean as

$$R_R - R_U = (R_R - R_P) + (R_P - R_U),$$

where R_P is the PE return with the same mean. From point 1 of Propositions 1 and 2, the second component $R_P - R_U$ is computed in (13) and the first component is

$$R_R - R_P = [\nu - E\left(R_0\right)]\left[\frac{1}{E\left(\mathcal{S}_C^2\right)} - \frac{1}{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)\left(1+\mathcal{S}_C^2\right)}\right]\mathbf{r}'\boldsymbol{\varphi}_C. \tag{18}$$

The next corollary characterizes the special cases where the subset of RE returns is equal to the subsets of PE or UE returns.[6]

Corollary 2 *Given the representations of RE, UE, and PE returns above,*

1. *The RE and PE returns are equal if and only if*

$$Var\left(\mathcal{S}_C\right) = 0.$$

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[6]There are other theoretically possible cases. The RE and UE returns share one element if and only if there are two real numbers (a, b) such that

$$R_0 = a + b\mathcal{S}_C^2,$$

in which case the shared return has a constant conditional mean a. A simple example is a constant R_0, since then this return is also UE.

2. The RE and UE returns are equal if and only if

$$Var\left(R_0\right) = Var\left(\mathcal{S}_C\right) = 0.$$

In this case, these two subsets of CE returns are also equal to the PE returns.

Corollary 2 indicates the relevance of the Sharpe ratio $\mathcal{S}_C$, i.e., the slope of the CE frontier, in the relation between RE returns and the other two efficiency types. For instance, the difference $R_R - R_P$ in (18) is driven by the fact that the RE returns are constructed by scaling $\mathbf{r}'\boldsymbol{\varphi}_C$ with real numbers, while the PE returns are constructed by scaling $\mathbf{r}'\boldsymbol{\varphi}_C/\left(1 + \mathcal{S}_C^2\right)$ instead.

Abhyankar, Basu and Stremme (2012) approximate the slope of the UE frontier asymptotes by the second moment of the slope of the CE frontier in their analysis of ex -ante gains from return predictability.[7] That is, they approximate $\mathcal{S}_U$ in (10) by the second moment of $\mathcal{S}_C$. However, equation (17) in Proposition 2 shows that the second moment of $\mathcal{S}_C$ has a exact relation with the slope of the RE frontier $\mathcal{S}_R$. The relation between $\mathcal{S}_R$ and $\mathcal{S}_C$ is more direct than the relation between $\mathcal{S}_U$ and $\mathcal{S}_C$. Moreover, point 2 in Corollary 2 states that the UE and RE returns are different subsets of CE returns unless both R_0 and $\mathcal{S}_C$ are constant over time. The empirical application in Section IV.C shows that the performance gap between UE and RE returns can be sizeable.

The RE frontier is given by two straight lines in the $\left[\sqrt{E\left[Var\left(R|\mathbf{z}\right)\right]}, E\left(R\right)\right]$ space with zero residual variance at $E\left(R\right) = E\left(R_0\right)$, as displayed in the left plot of Figure 2. The PE frontier is also linear in that space and shares the safe asset return with the RE frontier.

<Figure 2>

The RE returns provide the best risk-return trade-off in that space, while UE returns provide the best frontier in the right plot for the $\left[\sqrt{Var\left(R\right)}, E\left(R\right)\right]$ space. We can relate the RE and UE returns with the same mean ν by

$$R_R = R_U + \left(R_R - R_U\right),$$

---

[7]They also evaluate the ex- post unconditional performance of UE returns and claim that these returns improve the performance with respect to a constant ω_C in (5). My results show that such CE returns are actually RE returns and hence they are efficient in terms of $\mathcal{S}_R$, but not $\mathcal{S}_U$.

with $R_R - R_U$ equal to the sum of (13) and (18) and thus dependent on the chosen ν. The relative location of the RE and UE frontiers in the left plot of Figure 2 follows from the residual variance decomposition

$$E\left[Var\left(R_U|\mathbf{z}\right)\right] = E\left[Var\left(R_R|\mathbf{z}\right)\right] + E\left[Var\left(R_R - R_U|\mathbf{z}\right)\right],$$

which follows itself from the zero residual covariance between the term $R_R - R_U$ and R_R. On the other hand, the lack of correlation between the term $R_R - R_U$ and R_U yields the variance decomposition

$$Var\left(R_R\right) = Var\left(R_U\right) + Var\left(R_R - R_U\right)$$

and thus the location of RE returns to the right of UE returns in the right plot.

We can also see the location of the FE frontier in Figure 2. In this example, in some regions the performance of FE returns is better than that of some CE returns. In fact, RE returns look very similar to FE returns in the right plot, even though they are the optimal returns in the left plot.

Finally, Appendix B derives the implications of the properties of PE and RE returns for testing mean-variance efficiency and asset pricing models. In particular, we can test conditional asset pricing models by comparing the maximum unconditional or residual Sharpe ratios from two investment sets. We can test the validity of models with both traded and non-traded factors. The conditional CAPM is an example of a traded factor because in this model the market portfolio must be CE in equilibrium.[8] In the case of non-traded factors, we would work with the associated mimicking portfolios.

IV Empirical Application

This section develops an econometric framework for the estimation of unconditional and residual Sharpe ratios. This framework is applied to testing the performance gaps between the different efficiency types that are studied in Section III, and to testing asset pricing models.

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[8]The CAPM was originally developed by Sharpe (1964), Lintner (1965), and Mossin (1966) in a context without conditioning information.

A   Unconditional and Residual Sharpe Ratios

I extend the standard set-up in empirical finance for testing the difference between the unconditional Sharpe ratios of two investment strategies (see Ledoit and Wolf (2008) and the references therein) to residual Sharpe ratios. For some excess return r_i and a vector of predictors $\mathbf{z}$, I define the influence functions

$$
\mathbf{h}\left(r_i, \mathbf{z}; \mu_i, \mathbf{b}'_i, \sigma_i^2\right) = \left(
\begin{array}{c}
\begin{pmatrix} 1 \\ \mathbf{z} \end{pmatrix} \left(r_i - \mu_i - \mathbf{b}'_i \mathbf{z}\right) \\
\left(r_i - \mu_i - \mathbf{b}'_i \mathbf{z}\right)^2 - \sigma_i^2
\end{array}
\right)
$$

with parameters $(\mu_i, \mathbf{b}'_i, \sigma_i^2)$. The first block of influence functions pins down the conditional mean of r_i with $\mu_i + \mathbf{b}'_i \mathbf{z}$, while the last influence function pins down the residual variance of r_i. The linearity of the conditional mean is not really a constraint because we could add powers of $\mathbf{z}$ or other functions of the predictors. Given time series data on $(r_i, \mathbf{z})$, we can estimate the residual Sharpe ratio of r_i by means of the moment conditions

$$
E\left[\mathbf{h}\left(r_i, \mathbf{z}; \mu_i, \mathbf{b}'_i, \sigma_i^2\right)\right] = \mathbf{0},
$$

which identify the ratio as $\mu_i / \left(\sigma_i^2\right)^{1/2}$ if we have subtracted the mean from the predictors $\mathbf{z}$ and thus μ_i identifies the expectation of r_i.

We can jointly estimate the residual Sharpe ratio of two excess returns r_1 and r_2 by means of the moment conditions

$$
E\left[
\begin{array}{c}
\mathbf{h}\left(r_1, \mathbf{z}; \mu_1, \mathbf{b}'_1, \sigma_1^2\right) \\
\mathbf{h}\left(r_2, \mathbf{z}; \mu_2, \mathbf{b}'_2, \sigma_2^2\right)
\end{array}
\right] = \mathbf{0},
\tag{19}
$$

with parameters to estimate $(\mu_1, \mathbf{b}'_1, \sigma_1^2, \mu_2, \mathbf{b}'_2, \sigma_2^2)$. If we have subtracted the mean from the predictors $\mathbf{z}$, then we can identify the difference in residual Sharpe ratios by

$$
\frac{\mu_2}{\left(\sigma_2^2\right)^{1/2}} - \frac{\mu_1}{\left(\sigma_1^2\right)^{1/2}}
$$

and we can test if its true value is zero with a Wald test.

If we are interested in the difference in unconditional Sharpe ratios instead, then we

can simply change the influence functions in (19) to

$$\mathbf{h}\left(r_i, \mathbf{z}; \mu_i, \mathbf{b}_i', \sigma_i^2\right) = \left(\begin{array}{c} \begin{pmatrix} 1 \\ \mathbf{z} \end{pmatrix} (r_i - \mu_i - \mathbf{b}_i'\mathbf{z}) \\ (r_i - \mu_i)^2 - \sigma_i^2 \end{array} \right).$$

We could also simplify the upper block to $r_i - \mu_i$ and hence we would not use $\mathbf{z}$ in the influence functions. This would be the standard set-up in empirical finance.[9]

I use the generalized method of moments (GMM) of Hansen (1982) with the system of moment conditions (19) to develop the corresponding inference. We can compute standard errors of estimators and p-values of tests without assuming Gaussian or independent and identically distributed returns. In particular, I use Newey-West (1987) standard errors and p-values, which are robust to heteroskedasticity and autocorrelation.

In case the GMM asymptotic p-values do not provide a good approximation in these tests, we can complement them with block bootstrap p-values. For that purpose, I sample Wald statistics divided by their standard errors as follows. I run a first-order vector autoregression (VAR(1)) of $(r_1, r_2, \mathbf{z})$ and store the residuals. This VAR is my data generating process (DGP) after changing the value of the estimated μ_2 to satisfy the null hypothesis of equal Sharpe ratios. I impose $\mu_2 = (\sigma_2^2)^{1/2} \mu_1 / (\sigma_1^2)^{1/2}$ in the DGP, which only changes the level of r_2 without altering its variance or dynamics. I sample blocks of residuals with replacement to consider additional nonlinear dynamics in the data, such as generalized autoregressive conditional heteroskedasticity (GARCH) effects. For each sample of $(r_1, r_2, \mathbf{z})$ generated in this way, I store the associated Wald statistic divided by its standard error. I repeat this process several times, which provides a sample of t-statistics. Then I compute the corresponding p-value of the t-statistic obtained from the actual data.

This block bootstrap on the residuals of a VAR(1) is similar to that of Ledoit and Wolf (2008). On the other hand, these authors only consider unconditional Sharpe ratios and therefore resample (r_1, r_2) instead of $(r_1, r_2, \mathbf{z})$. They do not impose the null hypothesis in the DGP either. Another difference is that they consider automatic approaches to pin

---

[9]We need a good approximation to the true conditional mean of r_1 and r_2 to identify the residual Sharpe ratios, which is not required for the computation of unconditional ratios. However, neither of the two ratios requires a good approximation of the true conditional variances of r_1 and r_2.

down the number of lags in the Newey-West standard errors and the number of blocks in the bootstrap. I consider several combinations of values instead to check the robustness of the computations.

B   Data Description

I use monthly returns from 1954 to 2012 available from Ken French's data library. In particular, I use the Treasury bill return as R_0 and different combinations of the Fama-French size and book-to market sorted portfolios as the vector $\mathbf{r}$. To make the analysis more transparent, I often focus on the three Fama-French factors, whose first factor is the excess return on the market, while the two additional excess returns are associated with the size and value effects. These are called SMB (long in small capitalization stocks and short in big ones) and HML (long in high book-to-market stocks and short in low ones) respectively. Given their widespread use in empirical work, I also consider the six and 25 Fama-French portfolios to study the role of the investment set in the differences across types of efficient returns. I subtract the safe asset return from their returns to obtain the corresponding excess returns. See Ken French's web page and Fama and French (1993) for further details.

I use three predictors to define the vector $\mathbf{z}$: the US dividend price ratio, the default spread, and the term spread. The first predictor is taken from Robert Shiller's web page, while the other two predictors are constructed from the Federal Reserve Economic Data. The default spread uses yields on AAA- and BAA-rated bonds and the term spread uses 10- and one-year constant maturity Treasury bond yields. These predictors are widely used in empirical finance (e.g., see Ferson and Siegel (2009)).

As a first description of the data, Table 1 analyzes the mean predictability in each one of the three Fama-French factors. I divide the full sample into two halves (1954–1983 and 1984–2012) because some features of return predictability may differ across different periods. On the other hand, a formal empirical analysis of predictability changes across time is beyond the scope of this paper.

<Table 1>

Panel A of Table 1 reports the coefficient of determination R^2 from each predictive

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regression as a statistical measure of predictability. In the case of the market factor, we can associate the first period with stronger predictability and the second one with weaker predictability. See Ferson and Siegel (2009) and the references therein for similar evidence up to the early 2000s, with weaker predictability in the 1990s. In particular, these authors study the periods 1963–1994 and 1995–2002 and find lower predictability in the second period. However, the weaker market predictability in the second period does not necessarily mean that predictability is disappearing, since SMB and HML do not follow the same pattern. The predictive regressions of SMB and HML show a stable or even increasing R^2.

Panel B of Table 1 displays an economic measure of return predictability for each Fama-French factor. We can test the gains from conditioning information by using FE returns as r_1 and PE or RE returns as r_2 in the econometric framework of Section IV.A. If we apply the representation of FE returns (3) to a single Fama-French factor r then the excess returns of FE returns are spanned by scaling $r\varphi$ with real numbers, or equivalently by scaling r because φ is also a real number. The unconditional and residual Sharpe ratios do not depend on such scaling as far as it does not change the sign of the ratio. Therefore we can simplify the computation of Sharpe ratios of FE returns by using $r_1 = r$ directly, i.e., we associate r_1 with holding the factor. Regarding r_2, if we apply the representation of PE returns in (5) and (9) to a single Fama-French factor r, then the excess returns of PE returns are spanned by scaling $r\varphi_C/(1 + \mathcal{S}_C^2)$ with real numbers. Therefore, we can use $r_2 = r\varphi_C/(1 + \mathcal{S}_C^2)$ to compute the unconditional and residual Sharpe ratios of PE returns. Following the representation of RE returns in (5) and (16), we can choose $r_2 = r\varphi_C$ to compute the Sharpe ratios of RE returns.

The computation of r_2 depends on objects like φ_C and $\mathcal{S}_C^2$ (defined in (6) and (7), respectively) and hence it requires the conditional mean and variance of the Fama-French factors. If we use the true conditional mean and variance, then we identify the true maximum Sharpe ratios. If this is not the case, r_2 still exploits conditioning information, albeit not optimally. Following the spirit of predictive regressions, I model the conditional mean vector as linear in the predictors and the conditional variance matrix as constant. We can identify the former vector by running least squares of excess returns on a constant and the predictors and the latter matrix by the unconditional variance of the residuals.

Here I focus on mean predictability, but I consider GARCH effects in the next sections.

Statistical significance with asymptotic p-values is denoted by the symbol *, while +
is used to denote bootstrap p-values based on 1000 bootstrap samples.[10] I compute the
p-values for six and 12 lags in the Newey-West standard errors (the cubic root of the
sample size is around seven) and, similarly, blocks of size six and 12 in the bootstrap. To
conserve space, in Table 1 I only report the statistical significance for six lags, and blocks
of size six in the case of bootstrap p-values. I do the same in the following tables. The
significance is similar for other combinations of lags and block sizes, which are available
upon request.

The findings in Panel B of Table 1 follow the patterns of R^2 in Panel A. In terms
of both unconditional and residual Sharpe ratios, we find high and statistically signifi-
cant gains from conditioning information for the market in the first period and high and
statistically significant gains for SMB in the second period. The increase in annualized
Sharpe ratios is around 0.6 in both cases. The low unconditional and residual Sharpe ra-
tios of holding SMB in the second period are due to a sharp decrease in its unconditional
risk premium. The gains for HML are also higher in the second period, but we find no
statistical significance.

C   Sharpe Ratios of Efficient Returns

This section studies the Sharpe ratios of efficient returns constructed from the three
Fama-French factors and the associated six and 25 sorted portfolios. I keep the conditional
mean of the corresponding vector $\mathbf{r}$ linear in the predictors, but I study both a constant
and a time-varying conditional variance of $\mathbf{r}$. In the second case, that random matrix
should be positive semidefinite at every realization. For this purpose, I follow the spirit of
Bollerslev's (1990) constant conditional correlation model. I proceed in two steps: First I
fit a GARCH(1,1) to each residual from the predictive regressions to obtain the conditional
variance of each excess return in $\mathbf{r}$.[11] Second, I compute the conditional correlations of $\mathbf{r}$

---

[10]The econometric framework is the standard one in which the econometrician compares the Sharpe
ratios of two time series of excess returns, r_1 and r_2. The econometrician does not have information
on or simply does not consider how these returns were constructed. To consider the estimation of their
portfolio weights in our inference, we should add the corresponding moments to the system (19). This
would complicate the computations, especially the bootstrap.

[11]I compute the unconditional variance of the residuals and use it as the steady state of the conditional
variance. I estimate the remaining two parameters of the GARCH(1,1), usually denoted α and β, by

as the unconditional correlations of the residuals scaled by their GARCH volatilities.

Table 2 compares the unconditional performance of PE returns against FE and UE returns. The statistical significance of the differences in Sharpe ratios follows the econometric framework of Section IV.A. For instance, when we compare the Sharpe ratios of FE and PE returns, we can use $r_1 = \mathbf{r}'\boldsymbol{\varphi}$ from the representation of FE returns in (3) and $r_2 = \mathbf{r}'\boldsymbol{\varphi}_C/\left(1 + \mathcal{S}_C^2\right)$ from the representation of PE returns in (5) and (9). Panel A displays the results when PE returns do not exploit GARCH effects and Panel B displays the results when they do.

<Table 2>

In Panel A of Table 2 the performance gap of PE returns with respect to FE returns is high and statistically significant in the first period and increases with the number of assets. This pattern does not hold in the second period, where the gap becomes lower than in the first period for the six and 25 portfolios. In fact, the performance gap is not statistically significant for the six portfolios. Similarly, Ferson and Siegel (2009) reject the lack of gains from conditioning information in the period 1963–1994, but not in the period 1995–2002. Considering GARCH effects in Panel B increases the performance of PE returns with the six portfolios and the 25 portfolios, both economically and statistically.

Corollary 1 states that UE and PE returns are equivalent if and only if the intercept of the CE frontier is constant. However, the unconditional variance of R_0 is not zero. The rate of return of the safe asset has a mean of 0.430 and a standard deviation of 0.264 in the first period. Both statistics decrease in the second period to 0.340 and 0.216, respectively. Table 2 quantifies the difference between PE and UE returns in terms of Sharpe ratios. It reports the performance gap for two mean targets $E\left(R\right) = \nu$, a low one of 6% and a high one of 10% in annualized terms, because the Sharpe ratio of UE returns depends on the chosen target. Following the representation of UE returns in (5) and (12), we can use $r_1 = \omega_C \mathbf{r}'\boldsymbol{\varphi}_C$ in the econometric framework of Section IV.A. The scaling ω_C depends on the target ν and some expectations that are estimated by their sample counterparts.

Gaussian quasi-maximum likelihood. I explored the joint estimation of the conditional mean and variance and the estimation of the steady state variance without variance targeting. The results are similar, but the computations are much more time-consuming.

In the left plot of Figure 1 the unconditional Sharpe ratios of UE returns converge to the ratio for PE returns as ν increases. Therefore, the performance gap should be more relevant for targets closer to the safe asset. Panel A of Table 1 shows that the gaps are economically and statistically significant with the 25 portfolios and the low target 6%, with a stronger gap in the first period than in the second. For that target, the gap is higher than 0.1 for the six portfolios, but only statistically significant in the second period, with asymptotic p-values. In that period, we also find a statistically significant difference for the 10% target and the 25 portfolios, but only with the asymptotic p-value. In the first period, some differences for that target become slightly negative, but this simply reflects how similar these UE returns are to PE returns.

The main patterns from Panel A still hold when we consider GARCH effects in Panel B. PE and UE returns are especially different for the low mean target and the 25 portfolios, with a larger gap in the first period. This investment set is widely used in empirical work (e.g., Ferson and Siegel (2009)).

Table 3 studies the residual performance of RE returns against FE and UE returns. In the econometric framework of Section IV.A, we can use $r_2 = \mathbf{r}'\boldsymbol{\varphi}_C$ to compute the Sharpe ratios of RE returns following their representation in (5) and (16). Similarly to PE returns in Table 2, the performance gap with respect to FE strategies is especially large in the first period. Interestingly, Panel B of Table 3 shows that considering GARCH effects clearly improves RE returns with the six and 25 portfolios, especially in the second period. This translates into large and significant performance gaps with respect to FE returns in the second period, which are not found without considering the GARCH effects in Panel A.

<Table 3>

Corollary 2 states that UE and RE returns are equivalent if and only if both the intercept and slope of the CE frontier are constant, but none of these variables has a zero unconditional variance. In particular, when we work with the three Fama-French factors and we do not consider GARCH effects, the slope of the CE frontier (whose square is given by (7)) has a mean of 0.373 and a standard deviation of 0.172 in the first period. Both statistics decrease in the second period to 0.285 and 0.117, respectively.

Since UE and RE returns are not equivalent for these data, Table 3 shows the estimates and tests of their performance differences. The performance gap is particularly large for the low target and the 25 portfolios, as in Table 2. In Panel A the gap is not statistically significant in the second period, but this changes when we consider GARCH effects in Panel B. The performance gaps with respect to UE returns increase considerably, becoming strongly significant with the 25 portfolios and the low target in both periods.

Tables 2 and 3 focus on monthly returns, but the return frequency is another relevant dimension for the differences across efficiency types. Appendix D reports the counterpart of these tables with annual returns of the three Fama-French factors. Table D1 shows that the differences across the subsets of CE returns are larger for annual returns. In fact, UE returns are clearly different with respect to PE or RE returns for this small investment set. In this regard, Figures 2 and 3, which illustrate Section III, correspond to these annual returns in the first period, so that the different efficiency types are easier to distinguish.

Appendix D also reports an out-of-sample exercise on the gains from conditioning information during the second period. Table D2 confirms the weaker market predictability and stronger SMB predictability in that period. We can also find gains of dynamic strategies with respect to the equally weighted portfolios of the three factors and the six and 25 portfolios.

Finally, the previous tables and the discussion focus on Sharpe ratios, but we can run an equivalent analysis in terms of Jensen's alphas. These computations are available upon request. For instance, when we use UE returns as pricing factors, the value of α_U depends on the particular mean target and can be very different from zero.[12] Similarly, the zero-beta returns of UE returns, E_U in the beta-pricing equation (14), can be very different from the average safe asset return.

D Testing Asset Pricing Models

We can also apply the econometric methodology of Section IV.A to test conditional asset pricing models. In particular, I use the previous data to test if the conditional

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[12]Lewellen and Nagel (2006) analyze the unconditional alphas of CE returns. These alphas are generally not zero, even though the conditional alphas of CE returns must be zero. However, they do not study UE returns in particular.

CAPM holds for a given vector $\mathbf{r}$. The main implication of this model is that the market return is CE, but we can use PE and RE returns. If we want to test the conditional CAPM with RE returns, then we apply the representation of RE returns in (5) and (16) to the market excess return r and the vector $\mathbf{r}$ to obtain $r_1 = r\varphi_C$ and $r_2 = \mathbf{r}'\boldsymbol{\varphi}_C$, respectively. If we want to test the conditional CAPM with PE returns, which are represented in (5) and (9), then the computations are similar but using excess returns like $\mathbf{r}'\boldsymbol{\varphi}_C/(1 + \mathcal{S}_C^2)$ instead. I do not use UE returns because they do not have a unique unconditional or residual Sharpe ratio. On the other hand, we can test the unconditional CAPM by means of the FE returns in (3), using $r_1 = r$ and $r_2 = \mathbf{r}'\boldsymbol{\varphi}$.

If we use the true conditional mean and variance of r and $\mathbf{r}$ in the previous computations, then we identify the true efficient returns and test the conditional CAPM. If this is not the case, then we are not strictly testing the CAPM but we are still testing the efficiency of the market portfolio for some (not necessarily optimal) portfolio strategy. Even with misspecified conditional moments, finding significant improvements in performance should cast some doubts on the validity of the CAPM.

Table 4 reports the tests of the CAPM,[13] comparing the performance of the market portfolio against the three investment sets of Section IV.C. Panel A displays the tests with unconditional Sharpe ratios, while Panel B displays the tests with residual ratios. In each panel, PE and RE returns are constructed for both a constant and a time-varying conditional variance (denoted C and V, respectively).

<Table 4>

Let us start with the first period and the three Fama-French factors in both panels of Table 4. The market is more inefficient with FE returns in that period. Both the unconditional and residual Sharpe ratios of the FE returns with the three factors are more than double those with the market only and this increase is statistically significant. But the increase for PE and RE returns is not that strong. Table 1 shows that the market return is relatively more predictable than the other factors in this period and hence the latter may not add value in a dynamic strategy. We find some statistical significance with

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[13]If we want to consider the estimation of portfolio weights in the test, then we should add the corresponding moments to the system (19). Similarly, if we want to test an asset pricing model with non-traded factors, then we should add the moments that estimate the mimicking portfolios.

RE returns in Panel B, however, which suggests that it would be good empirical practice to report tests for both unconditional and residual Sharpe ratios.

Importantly, the second period represents the opposite situation. Then the increases in Sharpe ratios when the SMB and HML returns are added to the PE and RE strategies are much higher than those with FE strategies, the latter not being statistically significant. Table 1 shows that the size effect is more predictable than the market in this period and hence it may add value in a dynamic strategy.

If we extend the investment set to the six or 25 sorted portfolios, then the CAPM is rejected with any of the strategies in both periods. Nevertheless, we can still find a similar pattern between the two periods with the six portfolios. The market's performance gap is greater with FE strategies than with PE or RE strategies in the first period, but the gap is higher with the latter strategies in the second period if we consider GARCH effects. If we use the 25 portfolios, then the PE and RE strategies also provide stronger evidence against the efficiency of the market portfolio than FE strategies in the first period.

Ferson and Siegel (2009) also reject the conditional CAPM in the periods 1963–1994 and 1995–2002 using the 25 Fama-French portfolios. Lewellen and Nagel (2006) use average alphas from short-window regressions to conclude that the conditional CAPM and the classic unconditional CAPM yield a similar bad performance in the period 1964–2001. My empirical evidence shows that such a conclusion depends on the investment set and the time period.

V Conclusions and Further Research

My main contribution is a thorough theoretical and empirical analysis of two new types of efficient returns when returns are predictable. The first type maximizes the unconditional Sharpe ratio that is commonly used in empirical work. Equivalently, these efficient returns display zero unconditional Jensen's alphas when they are used as a pricing factor. The second type maximizes the conditional mean-variance preferences that are commonly used in theoretical work. These efficient returns motivate the definition of residual Sharpe ratios and alphas, where the required variances and covariances are based on the residuals of predictive regressions instead of the returns themselves.

I show that the two efficiency types differ from unconditional efficiency, which is the only type that has been studied in the literature, when the safe asset return and the maximum conditional Sharpe ratio change over time. Since interest rates and Sharpe ratios change over time, we have three different ways of exploiting conditioning information, but the magnitude of their differences is an empirical question. I therefore develop an econometric framework to quantify and test differences in both unconditional and residual Sharpe ratios.

In particular, I study monthly stock returns during two periods, 1954–1983 and 1984–2012. These data show that the differences between unconditionally efficient returns and the two new types of efficient returns are more relevant the more assets we consider in the investment set and the lower the mean target, with a larger gap in the first period than in the second. These differences are stronger for annual returns, which suggests that the return frequency is another relevant dimension.

My theoretical results and empirical methodology can also be used to test conditional asset pricing models. These data show that, if we test the CAPM with the three Fama-French factors, then there is more evidence against the unconditional CAPM in the first period and against the conditional CAPM in the second period. This change in the relative evidence against the two variants of the model can be associated with the change in the relative predictability across the Fama-French factors. We can associate the first period with stronger market predictability and the second one with weaker market predictability. However, the size and value effects are relatively more predictable in the second period.

Finally, there are some interesting avenues for further research. I derive the efficiency properties for an investment set that includes a safe asset, which is common in empirical work. If such an asset is not available to investors, then we cannot work with Sharpe ratios as in my empirical application. The relevant tests are studied by Peñaranda and Sentana (2011, 2012) in a framework that does not explicitly take into account information and I plan to develop the corresponding extensions. In this regard, Appendix C studies the relation between mean-variance frontiers with and without a safe asset for the different types of efficient returns.

My analysis has provided new efficiency measures that can be useful in performance evaluation and model selection. Similarly, we could use the new efficient returns to com-

pute mimicking portfolios following Ferson, Siegel, and Xu (2006). The analysis of more general preferences that consider higher-order moments and intertemporal efficiency are additional topics of further research. Appendix D provides an out-of-sample analysis, but not a formal real-time Bayesian framework such as the one of Avramov and Chordia (2006) or Johannes, Korteweg, and Polson (2013). It would be interesting to study the new types of efficient returns as a practical portfolio strategy in such settings.

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Proofs

The weight vector $\boldsymbol{\varphi}_C$ and the squared Sharpe ratio $\mathcal{S}_C^2$ are defined in (6) and (7), respectively. We can use them to construct two excess returns

$$r_v = \mathbf{r}' \left[Var\left(\mathbf{r}|\mathbf{z}\right) \right]^{-1} E\left(\mathbf{r}|\mathbf{z}\right) = \mathbf{r}'\boldsymbol{\varphi}_C, \qquad r_e = \mathbf{r}' \left[E\left(\mathbf{r}\mathbf{r}'|\mathbf{z}\right) \right]^{-1} E\left(\mathbf{r}|\mathbf{z}\right) = \frac{1}{1+\mathcal{S}_C^2}\mathbf{r}'\boldsymbol{\varphi}_C \quad (20)$$

that will simplify the proofs. Importantly, r_e is the unique excess return that satisfies $E\left(r_e r|\mathbf{z}\right) = E\left(r|\mathbf{z}\right)$ and r_v is the unique excess return that satisfies $Cov(r_v, r|\mathbf{z}) = E\left(r|\mathbf{z}\right)$ for every excess return r. The means of these excess returns are related to $\mathcal{S}_C^2$ as follows

$$E\left(r_v\right) = E\left(\mathcal{S}_C^2\right), \qquad E\left(r_e\right) = E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2} \right). \qquad (21)$$

Proposition 1

1) For any return R, we can decompose the excess return $r = R - R_0$ into two components

$$r = \eta r_e + \epsilon, \qquad \eta = \frac{E\left(r_e r\right)}{E\left(r_e^2\right)} = \frac{E\left(r\right)}{E\left(r_e\right)},$$

where the first component is the unconditional projection of r onto the unconditional span of r_e and ϵ is the projection error. The error satisfies $E\left(\epsilon\right) = 0$ because $E\left(r_e \epsilon\right) = 0$ and thus the error does not affect $E\left(r\right)$ but increases $E\left(r^2\right)$.

The returns that solve problem (8) cannot have an error term in the previous projection and hence the solution can be represented as

$$R_P - R_0 = \eta r_e, \qquad \eta = \frac{\nu - E\left(R_0\right)}{E\left(r_e\right)}.$$

Substituting from (20) and (21), we have the equivalent expression

$$R_P = R_0 + \left[\frac{\nu - E\left(R_0\right)}{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2} \right)} \right] \frac{1}{1+\mathcal{S}_C^2}\mathbf{r}'\boldsymbol{\varphi}_C.$$

2) First, any PE return $R_P \neq R_0$ satisfies these properties. From the proof of point 1, the excess returns of a PE return $R_P \neq R_0$ can be expressed as $R_P - R_0 = \eta r_e$ with

$\eta \neq 0$. The squared unconditional Sharpe ratio S_U^2 of these excess returns is

$$\frac{E^2\left(R_P - R_0\right)}{Var\left(R_P - R_0\right)} = \frac{\eta^2 E^2\left(r_e\right)}{\eta^2 E\left(r_e\right)\left(1 - E\left(r_e\right)\right)} = \frac{E\left(r_e\right)}{1 - E\left(r_e\right)}$$

and, as a pricing factor, they provide $\alpha_U = 0$ for any excess return r because

$$\frac{Cov\left(r, R_P - R_0\right)}{Var\left(R_P - R_0\right)} E\left(R_P - R_0\right) = \frac{\eta E\left(r\right)\left(1 - E\left(r_e\right)\right)}{\eta^2 E\left(r_e\right)\left(1 - E\left(r_e\right)\right)} \eta E\left(r_e\right) = E\left(r\right).$$

Second, any return that satisfies these properties must be a PE return. From the decomposition of excess returns in the proof of point 1, we can decompose any excess return $r_\beta \neq 0$ into an underlying $R_P - R_0 = \eta r_e$ plus an error ϵ. Thus the Sharpe ratio of r_β satisfies

$$\frac{E^2\left(r_\beta\right)}{Var\left(r_\beta\right)} = \frac{E^2\left(R_P - R_0\right)}{Var\left(R_P - R_0\right) + Var\left(\epsilon\right)}$$

and the beta-pricing of the underlying $R_P - R_0$ with r_β as the pricing factor yields

$$\frac{Cov\left(R_P - R_0, r_\beta\right)}{Var\left(r_\beta\right)} E\left(r_\beta\right) = \frac{Var\left(R_P - R_0\right)}{Var\left(R_P - R_0\right) + Var\left(\epsilon\right)} E\left(R_P - R_0\right).$$

Therefore, an excess return r_β such that

$$\frac{E^2\left(r_\beta\right)}{Var\left(r_\beta\right)} = \frac{E\left(r_e\right)}{1 - E\left(r_e\right)}$$

or such that

$$\frac{Cov\left(r, r_\beta\right)}{Var\left(r_\beta\right)} E\left(r_\beta\right) = E\left(r\right)$$

for any excess return r must satisfy $Var\left(\epsilon\right) = 0$, which translates into $\epsilon = E\left(\epsilon\right) = 0$. We conclude that r_β must be equal to a particular $R_P - R_0$. $\square$

Proposition 2

1) This proof relies on the residual inner product $E\left[Cov(x, y|\mathbf{z})\right]$ between random variables x and y and its corresponding residual norm $\sqrt{E\left[Var(x|\mathbf{z})\right]}$. This may not be a proper norm in the sense that $Var(x|\mathbf{z}) = 0$ implies $x = E\left(x|\mathbf{z}\right)$ but not necessarily $x = 0$. However, this is not a concern in our setting because in the space of excess

returns we must have $x = 0$ under the usual assumption of no arbitrage opportunities. As commented in Section III.B, I use the term residual because $E[Var(x|\mathbf{z})]$ is equal to the variance of the residual $x - E(x|\mathbf{z})$ from a predictive regression.

For any return R, we can decompose the excess return $r = R - R_0$ into two components:

$$r = \lambda r_v + \varepsilon, \qquad \lambda = \frac{E[Cov(r, r_v|\mathbf{z})]}{E[Var(r_v|\mathbf{z})]} = \frac{E(r)}{E(r_v)},$$

where the first component is the residual projection of r onto the unconditional span of r_v and ε is the projection error. The error satisfies $E(\varepsilon) = 0$ because, by construction, $E[Cov(r_v, \varepsilon|\mathbf{z})] = 0$. The error does not affect $E(r)$ but increases $E[Var(r|\mathbf{z})] = E[Var(R|\mathbf{z})]$.

Therefore, the optimal returns that solve problem (15) cannot have an error term in the previous projection and they can be represented as

$$R_R - R_0 = \lambda r_v, \qquad \lambda = \frac{\nu - E(R_0)}{E(r_v)}.$$

Substituting from (20) and (21), we have the equivalent expression

$$R_R = R_0 + \frac{\nu - E(R_0)}{E(\mathcal{S}_C^2)} \mathbf{r}' \varphi_C.$$

2) First, any RE return $R_R \neq R_0$ satisfies these properties. From the proof of point 1, the excess return of an RE return $R_R \neq R_0$ can be expressed as $R_R - R_0 = \lambda r_v$ with $\lambda \neq 0$. The squared residual Sharpe ratio S_R^2 of these excess returns is

$$\frac{E^2(R_R - R_0)}{E[Var(R_R - R_0|\mathbf{z})]} = \frac{\lambda^2 E^2(r_v)}{\lambda^2 E(r_v)} = E(r_v)$$

and, as a pricing factor, they yield $\alpha_R = 0$ for any excess return r because

$$\frac{E[Cov(r, R_R - R_0|\mathbf{z})]}{E[Var(R_R - R_0|\mathbf{z})]} E(R_R - R_0) = \frac{\lambda E(r)}{\lambda^2 E(r_v)} \lambda E(r_v) = E(r).$$

Second, any return that satisfies these properties must be an RE return. From the decomposition of excess returns in the proof of point 1, we can decompose any excess return $r_\beta \neq 0$ into an underlying $R_R - R_0 = \lambda r_v$ plus an error term. Following a similar

argument to the proof of point 2 of Proposition 1, we can show that if r_β satisfies

$$E\left(r_v\right) = \frac{E^2\left(r_\beta\right)}{E\left[Var\left(r_\beta|\mathbf{z}\right)\right]}$$

or

$$E\left(r\right) = \frac{E\left[Cov\left(r, r_\beta|\mathbf{z}\right)\right]}{E\left[Var\left(r_\beta|\mathbf{z}\right)\right]}E\left(r_\beta\right)$$

for any excess return r, then the error term must be zero and thus r_β must be equal to a particular $R_R - R_0$. $\square$

Table 1
Predictability in the Fama-French factors

| | 1954–1983 | | | 1984–2012 | | |
	MMR	SMB	HML	MMR	SMB	HML
Panel A. Predictive regressions						
R^2	0.071	0.024	0.001	0.008	0.023	0.011
Panel B. Unconditional and residual Sharpe ratios (S_U and S_R)						
S_U						
PE	1.023	0.644	0.693	0.617	0.651	0.464
FE	0.433	0.419	0.678	0.444	0.075	0.331
PE-FE	0.590	0.225	0.015	0.174	0.576	0.134
	***, ++			***, +++		
S_R						
RE	1.073	0.701	0.695	0.620	0.647	0.471
FE	0.450	0.424	0.678	0.445	0.076	0.333
RE-FE	0.623	0.276	0.017	0.175	0.571	0.138
	***, ++			***, +++		

Note: This table shows the predictability in the monthly excess returns on the Fama-French factors, which are the market portfolio (MMR) and the portfolios that capture the size and value effects (SMB and HML respectively). Panel A reports the R^2 of the predictive regression of each factor on the predictors, which are the dividend price ratio, the default spread and the term spread. Panel B displays the unconditional Sharpe ratios of PE and FE returns on each factor, and the residual Sharpe ratios of RE and FE returns. The PE and RE returns are constructed with the conditional mean of excess returns given by the predictive regressions and a constant conditional variance. The differences in Sharpe ratios and their statistical significance with Newey-West standard errors are also displayed (*, ** and *** indicate significance with asymptotic *p*-values at 10%, 5%, and 1%, respectively; +, ++ and +++ indicate significance with block bootstrap *p*-values at 10%, 5%, and 1%, respectively).

Table 2
Unconditional Sharpe ratios

	1954–1983			1984–2012		
	FF3	FF6	FF25	FF3	FF6	FF25

Panel A. Constant conditional variance of excess returns

	FF3	FF6	FF25	FF3	FF6	FF25
PE	1.288	1.436	2.256	1.047	1.508	2.517
FE	0.955	1.092	1.484	0.670	1.390	2.211
PE-FE	0.334	0.344	0.772	0.376	0.118	0.306
	**, ++	**, ++	***, +++	*, +		**, ++
UE6	1.203	1.325	1.281	1.005	1.366	1.909
PE-UE6	0.085	0.111	0.975	0.042	0.142	0.608
			***, +++		**	***, ++
UE10	1.300	1.460	2.220	1.035	1.472	2.375
PE-UE10	-0.011	-0.025	0.036	0.012	0.036	0.142
						**

Panel B. Time-varying conditional variance of excess returns

	FF3	FF6	FF25	FF3	FF6	FF25
PE	1.264	1.450	2.308	1.032	1.659	2.614
FE	0.955	1.092	1.484	0.670	1.390	2.211
PE-FE	0.309	0.358	0.824	0.362	0.270	0.403
	**, +	**, +	***, +++		**, +	**, ++
UE6	1.148	1.304	1.162	1.004	1.525	2.026
PE-UE6	0.116	0.145	1.147	0.028	0.134	0.588
			***, +++		*	***, ++
UE10	1.264	1.479	2.205	1.025	1.628	2.498
PE-UE10	0.000	-0.029	0.104	0.007	0.031	0.116
						*

Note: This table shows unconditional Sharpe ratios from the monthly excess returns on three investment sets: the three Fama-French factors (FF3) and the six and 25 Fama-French portfolios (FF6 and FF25, respectively). In particular, the table reports the unconditional Sharpe ratios of PE, FE, and UE returns. The UE returns are reported for annualized mean targets of 6% and 10%. Panel A displays the results when PE and UE returns are constructed with the conditional mean of excess returns given by the predictive regressions and a constant conditional variance, while Panel B considers a time-varying variance. The differences in Sharpe ratios and their statistical significance with Newey-West standard errors are also displayed (*, ** and *** indicate significance with asymptotic *p*-values at 10%, 5%, and 1%, respectively; +, ++ and +++ indicate significance with block bootstrap *p*-values at 10%, 5%, and 1%, respectively).

Table 3
Residual Sharpe ratios

	1954–1983			1984–2012		
	FF3	FF6	FF25	FF3	FF6	FF25

Panel A. Constant conditional variance of excess returns

RE	1.382	1.515	2.298	0.675	1.391	2.226
FE	0.969	1.112	1.494	1.046	1.491	2.316
RE-FE	0.413	0.403	0.805	0.371	0.100	0.090
	**, ++	**, ++	***, +++	*, +		
UE6	1.281	1.416	1.443	1.019	1.408	2.102
RE-UE6	0.100	0.099	0.855	0.027	0.083	0.214
			***, +++			
UE10	1.348	1.513	2.259	1.046	1.496	2.462
RE-UE10	0.034	0.002	0.039	0.000	-0.005	-0.146

Panel B. Time-varying conditional variance of excess returns

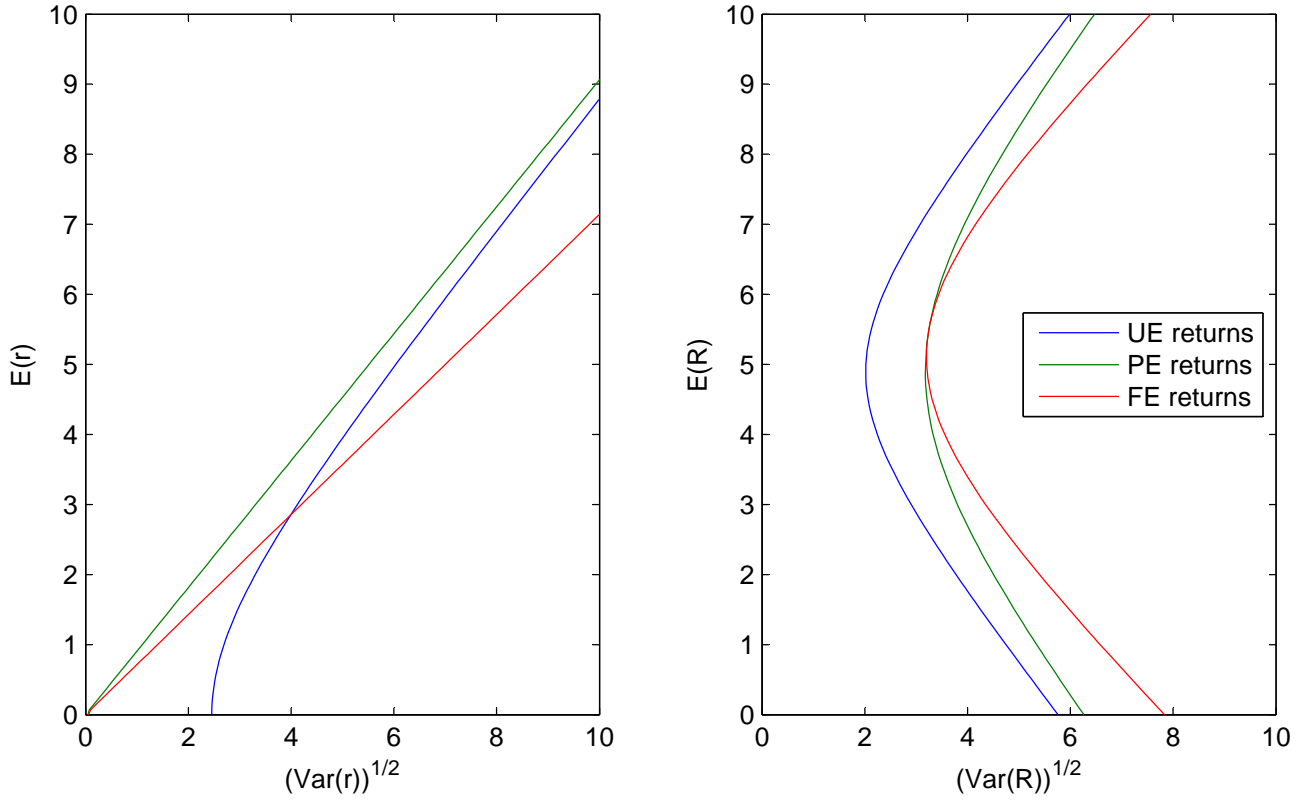|						
RE	1.353	1.542	2.354	1.011	1.656	2.652
FE	0.969	1.112	1.494	0.675	1.391	2.226
RE-FE	0.384	0.430	0.860	0.336	0.264	0.426
	**, ++	**, ++	***, +++		*, +	***, ++
UE6	1.219	1.382	1.327	1.01	1.543	2.164
RE-UE6	0.133	0.160	1.026	0.001	0.113	0.488
			***, ++			***, ++
UE10	1.306	1.534	2.285	1.030	1.634	2.535
RE-UE10	0.047	0.008	0.068	-0.020	0.022	0.118

Note: This table shows residual Sharpe ratios from the monthly excess returns on three investment sets: the three Fama-French factors (FF3) and the six and 25 Fama-French portfolios (FF6 and FF25, respectively). In particular, the table reports the residual Sharpe ratios of RE, FE, and UE returns. The UE returns are reported for annualized mean targets of 6% and 10%. Panel A displays the results when RE and UE returns are constructed with the conditional mean of excess returns given by the predictive regressions and a constant conditional variance, while Panel B considers a time-varying variance. The differences in Sharpe ratios and their statistical significance with Newey-West standard errors are also displayed (*, ** and *** indicate significance with asymptotic p-values at 10%, 5%, and 1%, respectively; +, ++ and +++ indicate significance with block bootstrap p-values at 10%, 5%, and 1%, respectively).

Table 4
Efficiency of the market portfolio against the Fama-French portfolios

	1954–1983			1984–2012		

Panel A. Unconditional Sharpe ratios

	FE	PE C	PE V	FE	PE C	PE V
MMR	0.433	1.023	1.000	0.444	0.617	0.574
FF3	0.955	1.288	1.264	0.670	1.047	1.032
FF3-MMR	0.521	0.265	0.264	0.227	0.429	0.458
	**, ++				**, ++	**, ++
FF6	1.092	1.436	1.450	1.390	1.508	1.659
FF6-MMR	0.659	0.412	0.450	0.946	0.891	1.086
	***, +++	**, +	**, +	***, +++	***, +++	***, +++
FF25	1.484	2.256	2.308	2.211	2.517	2.614
FF25-MMR	1.051	1.232	1.309	1.768	1.900	2.040
	***, +++	***, +++	***, +++	***, +++	***, +++	***, +++

Panel B. Residual Sharpe ratios

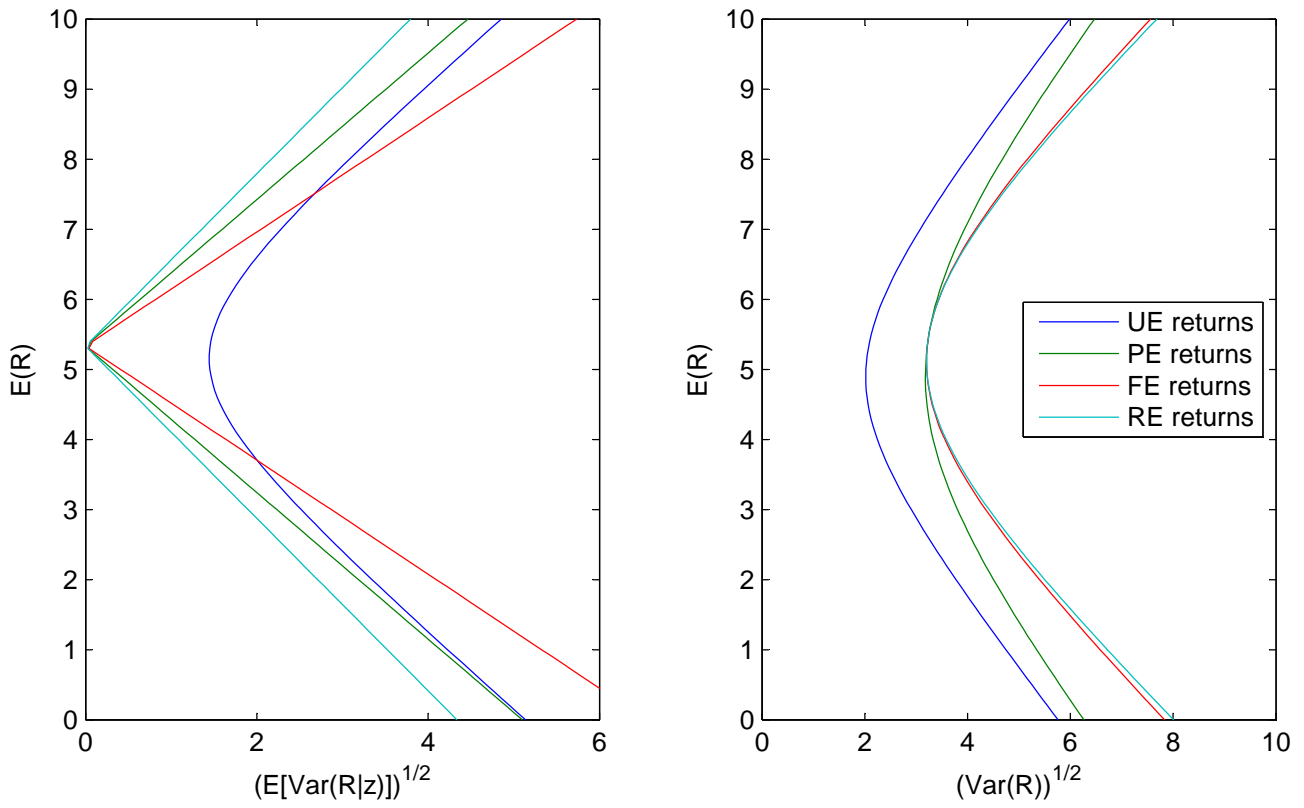|	FE	RE C	RE V	FE	RE C	RE V
MMR	0.450	1.073	1.029	0.445	0.620	0.577
FF3	0.969	1.382	1.353	0.675	1.046	1.011
FF3-MMR	0.519	0.309	0.323	0.230	0.426	0.434
	**, ++	*	*, +		**, +	**, ++
FF6	1.112	1.515	1.542	1.391	1.491	1.656
FF6-MMR	0.662	0.442	0.513	0.946	0.871	1.079
	***, +++	**, +	**, ++	***, +++	***, +++	***, +++
FF25	1.494	2.298	2.354	2.226	2.316	2.652
FF25-MMR	1.044	1.225	1.324	1.781	1.696	2.075
	***, +++	***, +++	***, +++	***, +++	***, +++	***, +++

Note: This table shows efficiency tests from the monthly excess returns on the market portfolio (MMR) and three investment sets: the three Fama-French factors (FF3) and the six and 25 Fama-French portfolios (FF6 and FF25, respectively). Panel A displays the unconditional Sharpe ratios of PE and FE returns and Panel B the residual Sharpe ratios of RE and FE returns. Here PE C and RE C are constructed with the conditional mean of excess returns given by the predictive regressions and a constant conditional variance, while PE V and RE V consider a time-varying variance. The differences in Sharpe ratios and their statistical significance with Newey-West standard errors are also displayed (*, ** and *** indicate significance with asymptotic *p*-values at 10%, 5%, and 1%, respectively; +, ++ and +++ indicate significance with block bootstrap *p*-values at 10%, 5%, and 1%, respectively).

Figure 1: UE and PE returns

Note: Means and standard deviations are measured in annual %.

Figure 2: UE, PE, and RE returns

Note: Means and standard deviations are measured in annual %.