Designing a Two-Sided Matching Protocol under Asymmetric Information

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Abstract. We have developed a new two-sided matching protocol including job applicants and employers in the condition that applicants have conditional preferences and well informed applicants exist. In past research, two-sided matching has covered some assignment problems such as residency matching. However, in the case of matching on the information network, different applicants are differently informed and well informed applicants hide its information to obtain more desirable matching. That is, asymmetric information possessed by applicants causes unstable matching. To overcome this difficulty, we design a new two-sided matching protocol in which applicants are allowed to report their conditional preferences and well informed applicants generally have an incentive to share information among applicants by allowing applicants to report their conditional preferences and deciding the matching on the basis of the preferences of applicants who share information (informers). We experimentally evaluated our protocol through simulation and found that the protocol can attain more satisfactory matching.

1 Introduction

An aim of research in designing two-sided matching protocols is to formulate matchings of two distinct sets of agents such as employers and job-applicants in the labor market. It has been actively studied and applied to the real-world problems such as the National Residency Matching Program in the USA.

The beginning of this research was the seminal work by Gale and Shapley and they proposed the deferred-acceptance algorithm [1]. They defined a stable matching as one in which there are no blocking pairs, that is, each matching pair has no preferred partners in the matching, and they showed that a matching with no blocking pairs can be obtained using the deferred-acceptance algorithm if each job-applicant has a complete order of preference for employers, and vice versa. In the past research, some extensions of the problem, e.g., allowing preferences with ties or incomplete preferences, have been discussed, and efficient algorithms for solving these problems have been proposed [10].

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If we consider the scenarios of matchings in the network environment, we find new problems in formulating stable matching. Getting enough information about employers through the network may be difficult for applicants. If getting information about employers is expensive for some applicants, they must decide their order of preference based on limited information. Furthermore, an investigation about employers may still leave applicants uncertain about the employers. In this case, applicants may be able to infer additional information from the actions of others, leading them to revise their evaluations and, possibly their own actions. They, however, cannot reflect their change of preference in the ordering if they get additional information after the matching has been determined, which causes the matching to be unstable. Thus, uncertainty over one's own preferences brings a problem to two-sided matchings. Multiagent researches have been tried to deal with uncertainty in mechanism design, so they are promising to solve this problem.

It may happen that some applicants know an employer well, e.g., they have an experience working with the employer. The above problem can be mitigated if well-informed applicants disclose their information voluntarily, and we achieve more stable matching. However, this is difficult to achieve because each applicant behaves selfishly. For instance, an applicant who has favorable information about an employer may think that disclosing it will increase the number of applicants competing for jobs with that employer, so he/she will not disclose the information voluntarily. These problems, which we call problems of asymmetric information among applicants, have been considered in research on auctions, but they have not been discussed sufficiently in the area of two-sided matching.

To address these problems, we propose a new matching protocol. We assume that applicants have preferences conditioned on the information from other applicants, and we design protocols that can attain the information sharing among the applicants by inducing them to disclose there information. The matching agent decides a unique matching based on the applicants preferences and the disclosed information. In the proposed protocol, the applicants generally have an incentive to disclose their information.

Chakraborty et. al. discussed two-sided matching with interdependent values and showed that a stable matching mechanism does not generally exist [8]. On the other hand, this paper tries to find a stable matching mechanism by limiting the problem class, although keeping the problem setting realistic.

Our main contribution is the introduction of a two-sided matching model in which asymmetric information among applicants exists and the proposition of the protocol to deal with such a situation. This model enables applicants to share information voluntarily and to reveal definitive preferences under the shared information.

2 Preliminaries

2.1 Model

Consider a many-to-one matching market between applicants and employers. The set of job-applicants is denoted by $A = \{a_1, ..., a_n\}$ with typical element a and the set of employers is denoted by $E = \{e_1, ..., e_n\}$ with typical element e. We introduce a generic term $x \in X$ which collectively means all applicants and all employers. The number of the job openings of e is denoted by n_e, such that the number of applicants who are matched to the employer e does not exceed n_e.

A two-sided many-to-one matching μ is a function from $A \cup E$ to itself such that:

1. $\forall a \in A, \mu(a) \in (E \cup \{a\})$
2. $\forall e \in E, \mu(e) \subseteq (A \cup \{e\}), |\mu(e)| \leq n_e$
3. $\forall e_i, e_j \in E, \mu(e_i) \cap \mu(e_j) = \Phi$
4. $\forall a \in A, a \in \mu(\mu(a))$

In other words, the set of applicants and employers is broken into applicant-employer pairs (a, e) for whom $\mu(a) = e$ and $a \in \mu(e)$ and unmatched agents a' for whom $\mu(a') = a'$. Let M be the set of all feasible matchings.

Each employer e is characterized by an unobserved quality $q_e \in Q$, where Q is a finite set. Applicant a may receive a private signal $s_{a,e} \in S$, where S is a finite set. This paper assumes each applicant receives at most one private signal but more than one applicants may receive the same signal about employer e, i.e., $s_{i,e} = s_{j,e}$. Signals and qualities are positively correlated. Thus, applicants can obtain more accurate preferences by learning signals. The set of signals about e is denoted by $S_e = \{s_{i,e} | i = 1, ..., n\}$. Each employer e has preference $p_e = \{a_i, a_j, ...\}$ over their matches, which means e prefers a_i to a_j and so on. Similarly, each applicant a also has preference $p_a = \{e_i, e_j, ...\}$. The notation $p_e = \{..., e, a_k, ...\}$ means that the employer e does not accept matching with the applicant a_k and vice versa. We introduce a decision rules about applicant $a_k$fs preference $d_k(s_{a,e} \cup \{y\})$ which returns the preference over employers, where $\{y\}$ denotes the information obtained from other applicants. If other applicants publicly announce their private signals, all the applicants can share the information $\{y\}$.

The preferences of applicants in the initial stage ($t = 0$) are determined by only their private information $p_k^{(0)} \leftarrow d_k(s_{a,e})$. After hearing other applicantsf announcements, the preferences of applicants are revised to those given by their decision rules based on their private information and information obtained from other applicants. When applicant a obtains information $\{y\}$ from other applicants, his/her preference is updated to be $p_k^{(t)} \leftarrow d_k(s_{a,e} \cup \{y\})$.

2.2 Metrics

For discussions about the evaluation of the matching results in the later section, we define two metrics of the matching stability.

At first, agent x's matching rank is denoted by $r_x(\mu(x))$, which means preference order of $\mu(x)$. We deal with the asymmetric information among applicants, so the first one is the sum of applicants' utilities. We assume that the utility of applicant a_k is determined by $r_{a_k}(\mu(a_k)) = r_k^a(e, d_k)$, that is, the rank of the matching partner e on a_k's decision rule d_k.

Definition 1. *We define the sum of applicants' utility $U(\mu)$ as follows:*

$$U(\mu) = \sum_{a_k \in A} \{|E| + 1 - r_k^a(\mu(a_k), d_k(S_{\mu(a_k)}))\}$$

The pair (a, e) satisfying $e \succ_a \mu(a)$ and $\forall a' \in \mu(e), a \succ_e a'$ is called a blocking pair. The classic concept of stability is defined as that no blocking pair exists in the matching. In contrast, under asymmetric information, a particular matching may be stable after one mechanism and unstable after another. Therefore, we introduce quantitative evaluations about the matchings obtained by our mechanisms on the basis of blocking pairs.

At first, we define the blocking partner $b(\mu : x)$ as follows: If x's blocking pairs exist in matching μ, then $b(\mu : x)$ is the top-ranked partner among x's blocking pairs, else $b(\mu : x)$ is the matching partner. The larger the difference between $r_x(\mu(x))$ and $r_x(b(\mu : x))$ become, the more agent x is unpleased with the matching.

Definition 2. *We define the stability of matching $S(\mu)$ as follows:*

$$S(\mu) = -\sum_{x \in A \cup E} \{r_x(\mu(x)) - r_x(b(\mu : x))\}$$

$S(\mu)$ can have a zero or negative values. In fact, $S(\mu) = 0$ if μ is stable in the classic concept of stability.

2.3 Gale-Shapley Algorithm

The preferences of applicants and employers are given in Table 1. For example, the first choice of applicant 1 is A, the second choice is B, and so on.

Each agents report their preference orders to the matching designer and the matching designer execute the deferred-acceptace algorithm (which we call GS algorithm). The GS algorithm proceeds as follows.

1. The first applicant is temporarily assigned to the employer of his/her first choice.
2. The kth applicant selects the employer of his/her first choice.

Table 1. Preferences of applicants and employers

Applicants	Employers
1: A B C D E	A: 3 2 5 1 4
2: A C B E D	B: 4 5 2 1 3
3: B A C D E	C: 3 2 1 5 4
4: B C A D E	D: 4 2 3 1 5
5: A D C E B	E: 2 3 1 4 5

3. When applicant a selects e,
   (a) if e has a remaining job opening, a is temporarily assigned to e.
   (b) if e has already been assigned with someone, e chooses higher-ranked applicant a', and matches with a'. Then, the rejected applicant relabeled as a chooses his/her preferred employer from employers that a has not yet selected, and selects the employer.
4. Process 3 is repeated until all applicants have been assigned to an employer or have been rejected by all employers.
5. $k \leftarrow k+1$ and back to process 2 until all applicants have selected.

Table 2 shows the results for applying the GS algorithm to Table 1.

Table 2. The matching result obtained by GS algorithm

Applicants	Employers
1: A B C **D** E	A: **3** 2 5 1 4
2: A **C** B E D	B: **4** 5 2 1 3
3: B **A** C D E	C: 3 **2** 1 5 4
4: **B** C A D E	D: 4 2 3 **1** 5
5: A D C **E** B	E: 2 3 1 4 **5**

In this paper, we try to extend GS Algorithm to address problems of asymmetric information.

We formalize the concept of a "matching protocol." We define it as a centralized direct revelation protocol, in which applicants and employers report their information to the matching designer and the designer proposes who should be matched with whom. More formally, a *direct revelation matching protocol* is a function Γ from the set S of reported signals of agents to the set M of all matchings. Let μ_I be a matching generated by a direct revelation matching protocol under shared information I in which GS algorithm is applied.

3 Information Hiding Problem

Applicants who have positive information about an employer seldom disclose it on the assumption that the competition for that employer would become more severe. So the positive information is not shared, and applicants cannot get enough information to report their own preference orders. Thus, the matching results might be unstable.

Consider the case where applicants $\{a_1, a_2, a_3\}$ and employers $\{e_1, e_2, e_3\}$ exist, $p_{e_1} = \{a_2, a_3, a_1\}$, $p_{e_2} = \{a_1, a_3, a_2\}$, $p_{e_3} = \{a_3, a_2, a_1\}$ and $p_{a_1} = \{e_3, e_1, e_2\}$. Applicant a_1 has an beneficial information i_{a_1} about e_1 but a_2 and a_3 does not know the information. At the beginning, $p_{a_2} = \{e_3, e_2, e_1\}$ and $p_{a_3} = \{e_2, e_3, e_1\}$, but they might change preference if they knew this information.

– Case 1. $p_{a_2}(\{i_{a_i}\}) = \{e_1, e_3, e_2\}$
  In this case, a_2's true preference is $\{e_1, e_3, e_2\}$, but he/she reports $\{e_3, e_2, e_1\}$ to the matching designer if he/she doesn't know a_1's information. If a_1 hides

the information, a_3 reports $\{e_3, e_2, e_1\}$ as his/her preference. As a result, the matching pairs are $< (a_1, e_1), (a_2, e_3), (a_3, e_2) >$, but (a_2, e_1) is a blocking pair when the matching is evaluated based on a_2's true preference. Thus, we can not obtain stable matchings. If a_1 provides the information, the matching pairs are $< (a_1, e_3), (a_2, e_1), (a_3, e_2) >$, so we can obtain stable matchings without decreasing the utility of a_1.
- Case 2. $p_{a_3}(\{i_{a_i}\}) = \{e_1, e_2, e_3\}$
  Similarly, the matching pairs are $< (a_1, e_1), (a_2, e_3), (a_3, e_2) >$ when a_1 hides the information and $< (a_1, e_2), (a_2, e_3), (a_3, e_1) >$ when a_1 provides the information. In this situation, the utility of informer a_1 decreases by declosing his/her information.

Applicant a_1 does not know about other's preferences, so he does not disclose the information for fear of the losing (Case 2). Consequently, the stable matching like Case 1 may not be achieved.

4 Protocol Design

To address the problem shown in the previous section, we need to design new protocols in which applicants who have information have incentives to disclose it. Even if the protocol satisfies the incentive requirement, it remains possible that some applicants might declare false information. However, we cannot externally inspect the existence or nonexistence of information hiding, while it is likely that declarations of false information turn out to be false.

We propose the informers as coodinators protocol (IACP) as a two-sided matching protocol under asymmetric information in which the matching designer determines the matching based on the agents' utilities who report information as true (we call them "informers").

IACP proceeds as follows:

1. The matching designer orders applicant at random.
2. Each applicant evaluates some desired employers on the basis of information obtained over the network, and drafts his/her conditional preference list. This list consists of several pairs of conditions and a preference order for the case where the conditions are true. First, each applicant creates his/her conditional preference list based on only the private information, and reports it to the matching designer.
3. Applicants can report the matching designer whether some conditions are true. The matching designer later determines the matching based on reported true information and reporters' preferences. we call the reporting applicants "informers").
4. The matching designer transmits information about conditions, which includes all of the conditions on reported conditional preference lists and all true information. However, applicants do not know which information are truth.

5. All applicants update their conditional preference lists based on the reported conditions by the matching designer, and they report the updated preference lists to the matching designer again. They can ignore some conditions that do not alter their preference order.
6. All true information is denoted by I. The matching designer obtains matching results by GS Algorithm under for all the subsets of I in advance.
7. The matching designer divides all matching results into 2^n cases according to whether or not an informer provides his/her true information, and finds a subgame perfect equilibrium.
8. The matching designer notifies all applicants and employers of the matching result determined as above and also reveals all true information with the reporters' names as meta-information.

Consider the case where employers A and B both currently have offices on only the east coast. If an applicant has a preference such as if an employer had an office on the west coast, then he/she would want to get a job with the employer. His/her conditional preference list is given by Table 4. If there is no shared information, his/her preference is (C, B, A). If employer A moves office to the west coast, the applicant's preference order becomes (A, C, B).

This applicant is happy if offices are moved to the west coast, but other applicants may not be happy. In general, the directions of changes in preference differ from one applicant to another.

We also investigate how to find a desirable matching if we already have utilities of agents over matches. Each informer's strategy is whether he/she discloses the information or not. The simplest method is to select a Nash equilibrium based on the utility of informers. However, Nash equilibrium is not always available.

The matching results of all the subsets of true information $\{i_1, i_2\}$ expressed by a strategic game are given in Table 4. Each cell represents the utilities of a_2 and a_1. In this case, there are no Nash equilibria, so the matching designer cannot make a unique generation of the matching.

To find a unique solution, we incorporate the order of the provided information to be evaluated. The matching designer orders applicants at random and finds a subgame perfect equilibrium [6]. We describe ordered applicants as $a_1, a_2, \cdots, a_n$, where i_{a_k} denotes the true information reported by applicant a_k. The matching designer sequentially classifies all matching results as to whether each item of true information exists or not according to the ordering of applicants. The classification result is expressed as a n-layer binary tree, and each leaf corresponds to a matching.

Table 3. A conditional preference list of an applicant

Conditions	1	2	3
Both employers A and B move office to the west coast.	A	B	C
Only employer A moves office to the west coast.	A	C	B
Only employer B moves office to the west coast.	B	C	A
Default	C	B	A

Table 4. A strategic game

	i_1	Φ
i_2	(3, 4)	(2, 3)
Φ	(4, 1)	(1, 3)

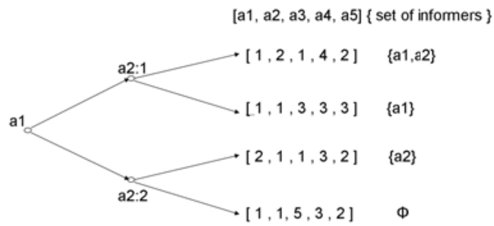

Fig. 1. An Instance in the case where two informers exist

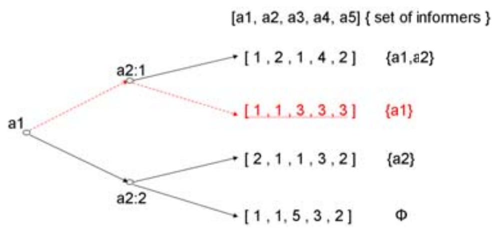

Fig. 2. Matching determination process by backward induction

A binary tree of the instance where ordered applicants $\{ a_1 ,a_2 ,a_3,a_4,a_5 \}$ exist, and $\{a_1, a_2\}$ are informers is shown in Fig.4. Each vector at a leaf represents the match ranks of applicants based on their true preferences. If both a_1 and a_2 report true information, for example, then applicants a_1 and a_5 match their first choices, a_2 and a_5 match their second choices, and a_4 matches his/her fourth choice.

The determination process is represented in Fig.4. Here, $\mu_I(a)$ denotes the matching partner of a determined by the GS algorithm over the shared information I. At node of $a_2 : 1$, the matching $\mu_{\{i_{a_1}\}}$ is left because a_2 prefers $\mu_{\{i_{a_1}\}}(a_2)$ to $\mu_{\{i_{a_1},i_{a_2}\}}(a_2)$. At node of $a_2 : 2$, $\mu_{\{i_{a_2}\}}(a_2)$ and $\mu_{\Phi}(a_2)$ are the same for applicant a_2. In this situation, the matching that includes true information reported by a_2 must be left. Thus, matching $\mu_{\{i_{a_2}\}}$ is left. In the same manner, informer a_1 prefers $\mu_{\{i_{a_1}\}}(a_1)$ to $\mu_{\{i_{a_2}\}}(a_1)$, so the matching designer determines $\mu_{\{i_{a_1}\}}$ as the final result.

5 Game Theoretical Analysis

At first, we define some notations for analysis. All disclosed true information are denoted by $I = \{i_k\}$ and let μ_I be a matching generated by a direct revelation matching protocol under shared information I in which GS algorithm is applied. Our protocol is a direct revelation matching protocol, so IACP is denoted by Γ^{IACP} which is a function from the set I and the set of conditional preferences of agents to $M = \{\mu_S\}$ where $S \subseteq I$. The utility of informer who discloses the information i is denoted by $U_i(\mu_S) = |E| + 1 - r_i(\mu_S(i))$ where $\Gamma^{IACP}(I) = \mu_S$.

Remark 1. Let S be the set of all subset of I.

$$\Gamma^{IACP}(I) = \mu_s, s \in S$$

It means that the number of matching candidates is limited, and is at most $2^{|I|}$ in IACP when disclosed information set I is given.

Definition 3. *A direct revelation matching protocol Γ is incentive-compatible iff:*

$$U_i(\Gamma(I - i)) \le U_i(\Gamma(I)), \forall i \in I$$

Theorem 1. *In IACP, it is incentive-compatible for an informer to disclose his/her information if no other informers exist.*

Proof. It is obvious because

$$U(\Gamma^{IACP}(I)) = \max(U(\Gamma^{IACP}(I)), U(\Gamma^{IACP}(\Phi))).$$

The protocol Γ^{IACP} enables the matching designer to determine the matching uniquely, though it is not incentive-compatible for all informers. An example is shown in Fig.5 that is not incentive-compatible for informers when a simple subgame perfect equilibrium is used.

The matching is $\mu_{\{i_{a_1}\}}$ when both a_1 and a_2 provide true information, but if only a_1 does, the matching is μ_Φ in which the utilities of both a_1 and a_2 increases.

Theorem 2. *In IACP, it might be not incentive-compatible for an informer to disclose his/her information.*

Proof. We can easily induce the following condition:

$$U_i(\Gamma^{IACP}(S - i)) \le U_i(\Gamma^{IACP}(S)), \forall S \subseteq I, S \ni i$$

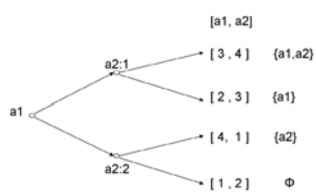

Fig. 3. An Example in the case where incentive-compatible is not satisfied

Table 5. The strategic game in the case of two informers

	1	Φ
2	$(U_1(\Gamma(I)), U_2(\Gamma(I)))$	$(U_1(\Gamma(\{2\})), U_2(\Gamma(\{2\})))$
Φ	$(U_1(\Gamma(\{1\})), U_2(\Gamma(\{1\})))$	$(U_1(\Gamma(\Phi)), U_2(\Gamma(\Phi)))$

It means that, for all subset S which does include i, the utility of i informer when S are shared must be equal or higher than the utility of i informer when $S - \{i\}$ are shared. However, the matching generation $\Gamma^{IACP}(S - \{i\})$ must be indifferent from the utility of i informer because the protocol must be applicable if the all information set was $S - \{i\}$.

The example when two informers exist is expressed by the game of strategic form is shown by Table 5. It is without loss of generalty to assume the situation that:

$$U_1(\mu_2) > U_1(\mu_S), S \in \{\Phi, \{1\}, \{1,2\}\} \tag{1}$$

$$U_2(\mu_1) > U_2(\mu_{S'}), S' \in \{\Phi, \{2\}, \{1,2\}\} \tag{2}$$

$$U_1(\mu_1) \geq U_1(\mu_\Phi) \tag{3}$$

$$U_2(\mu_2) \geq U_2(\mu_\Phi) \tag{4}$$

In this situation, $\Gamma^{IACP}(\{1\}) = \mu_{\{1\}}$ and $\Gamma^{IACP}(\{2\}) = \mu_{\{2\}}$.

1. Suppose $\Gamma^{IACP}(I) = \mu_{\{1,2\}}$. It is not incentive-compatible for informer 1 and informer 2 according to the conditions (1)(2). It is similary when $\Gamma^{IACP}(\{1,2\}) = \mu_\Phi$.
2. Suppose $\Gamma^{IACP}(I) = \mu_{\{1\}}$. It is not incentive-compatible for informer 1 because $U_1(\Gamma^{IACP}(I)) < U_1(\Gamma^{IACP}(\{2\})) = U_1(\mu_{\{2\}})$.
3. Similarily, It is not incentive-compatible for informer 2 when $\Gamma^{IACP}(\{1,2\}) = \mu_{\{1\}}$.

There are no other matching candidates in $\Gamma^{IACP}(I)$ because of the property shown by Remark 1. Therefore, it is not incentive-compatible for both informer 1 and informer 2 in IACP.

6 Evaluations

6.1 Rationality of Agents

Theoretically, IACP cannot satisfy incentive compatibility of disclosing information. However, we investigated how many such cases actually occur through simulations. Table. 6 shows how the utility of the informer who disclose information varies when another informer disclose information in one-to-one matching situations. The number of applicants is 8, and two of them are informers. As a result, we figured out that the informer seldom move down the match rank, even if the information is positive about an employer. Therefore, in most cases, it is Nash equilibrium for informers to disclose information.

Table 6. The relation between informers and match rank

information types	rank up	even	rank down
only negative	22240	77721	39
at random	12500	87425	75
only positive	1544	98400	56

6.2 Quality of Matching

The more applicants are affected by the sharing of true information, the worse the matching result is likely to be if no information is shared. However, the directions of preference changes are not homogeneous, so not sharing information may bring a better matching result. Therefore, we should check the influence that the ratio of affected applicants has on the matching result. Through simple simulations, we evaluated the utilities of applicants and the matching stability when the ratio of affected applicants was changed.

First, we explain the simulation settings. We set the number of applicants as 32, the number of employers as 8, the number of job openings as 8 for all employers, and the number of informers as 2. We suppose that informers have information about their first choice's employers.

In general, the larger the number of applicants submitting true information, the more the ratio of affected applicants increases. However, we suppose that the preference changes do not depend on the number of informers, but depends on the shared information itself. And in this simulation, we presumed that each condition depends on one employer, so the order of preference except for employer e does not vary according to the true information about employer e.

We simulated more than 10000 incidents and compared the results for IACP with those obtained by the Gale-Shapley protocol with shared information ("All Shared") and without shared information ("Simple GS").

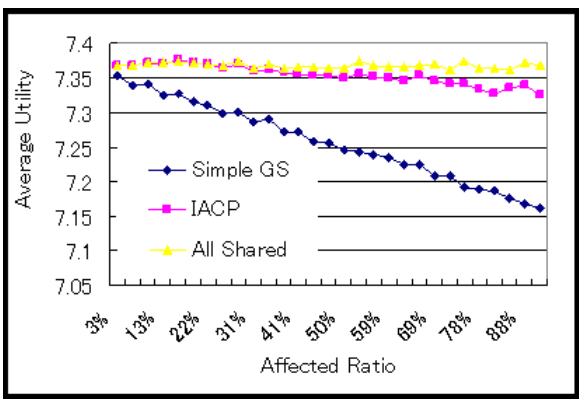

Fig. 4. Average utilities of applicants

The graph in Fig. 4 shows the average utilities of applicants who are not informers organized by the ratio of affected applicants.

It indicates that the larger the number of affected applicants, the worse their average utilities tended to be. The introduction of IACP prevented the utilities of non-informers from decreasing and kept them as high as in the Gale-Shapley protocol with shared true information. Fig. 5 shows the case that disclosed information is positive. In that case, applicants who accept the information about an employer may raise its rank, so we can easily predict that informer's match rank is likely to decline. However, the graph indicates that the proposing protocol is effective even if informers have positive information.

The average number of agents who have blocking pairs organized by the ratio of affected applicants is shown in Fig. 6. The figure indicates that the stability became worse as the ratio of affected applicants increased in "Simple GS". It is natural that the number of blocking pairs is always 0 in the case of "All Shared" because the matchings were obtained by the GS algorithm. Fig. 6 shows that

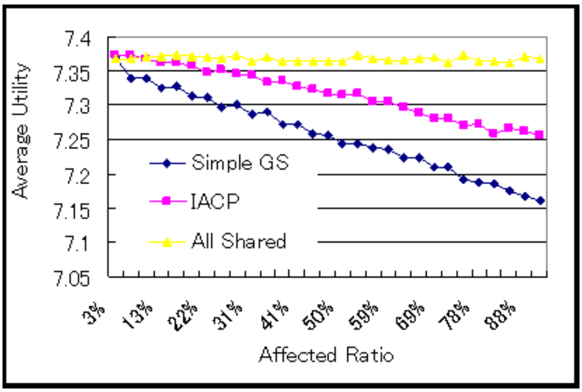

Fig. 5. Average utilities of applicants when disclosed information is positive

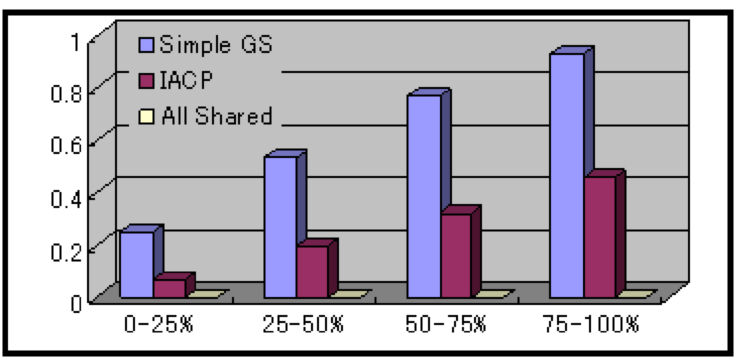

Fig. 6. The stability of the matching

the average stabilities were improved in IACP. We checked the rate of incidents in which the number of agents who have blocking pairs was 0 and it turned out that the matching was stable in more than 90% of incidents.

7 Related Work

Roth [4] has analyzed residence matching in the USA and pointed out the problem if some couples of applicants exist. A couple of applicants may prefer each first choice of hospitals to hospitals that are close geographically. He has designed protocols that enable couples to submit the preference of the pair. The preference of a pair consists of the ordering of pairs of hospitals. Their research is similar to ours in dealing with a situation that an applicant behavior affect another applicant preference.. However, the preferences of couples are not affected by the preferences of other single applicants. Thus, the problem of information revelation does not occur. Therefore, Roth's technique cannot solve the problems treated here.

Golle discusses the private stable matching algorithm [7], His motivation is to keep the preference secret to other people. So, the problem setting is completely different from ours.

Caldarelli and Capocci investigate the case of the preference is correlated to others [9]. However, they did not consider the problem of information revelation.

Teo, *et.al.* discuss a strategic issue in a stable matching problem [11]. However, they did not deal with cases that asymmetric information among applicants exists.

The design of protocols under asymmetric information has been studied in the area of auctions. Ito *et.al.* have proposed auction protocols under asymmetric information of natural choices [5]. They deal with a situation that an antique pot is put up for auction and there exist experts who know whether it is genuine or not and amateurs who do not know and pointed out the problem of information revelation by the experts. The introduction of conditional bids enables these problems to be avoided. However, the matching protocol is different from the auction protocols in that no monetary transfer occurs in our problem setting.

8 Conclusions and Future Work

We addressed the following issues in two-sided matching on a network.

Matching under Asymmetric Information. In the two-sided matching protocol using the GS algorithm, all applicants must submit true preference orders. However, it may be difficult for applicants to reveal their true preferences over the network because of asymmetric information. Therefore, new two-sided matching models are required to treat this problem.

Strategic Actions Like Information Hiding. Applicants who have positive information about an employer seldom disclose it on the assumption that the competition for that employer would become more severe. As a result of strategic actions like this, other applicants cannot get useful information.

To solve the issues above, we studied IACP on the basis of conditional preferences and information sharing as a matching model under asymmetric information. We analyzed the disclosure strategies of applicants who have information by means of game theories and designed a protocol in which information holders generally have an incentive to provide information over the network via the matching designer. We considered asymmetric information only on the applicant side in this paper, so the study of asymmetric information on both sides remains for future work. In addition, our approach in this paper needs a lot of calculation. We will study protocols that have low computational costs, considering the relationship between information and preference changes.

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