# Buckling and postbuckling of imperfect cylindrical shells: A review

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Thin-walled cylinders of various constructions are widely used in simple or complex structural configurations. The round cylinder is commonly found in tubing and piping, and in offshore platforms. Depending on their use, these cylinders are subjected (in service) to individual and combined application of external loads. In resisting these loads the system is subject to buckling, a failure mode which is closely associated with the establishment of its load-carrying capacity. Therefore, the system buckling and postbuckling behavior have been the subject of many researchers and investigators both analytical and experimental. The paper is a state-of-the-art survey of the general area of buckling and postbuckling of thin-walled, geometrically imperfect, cylinders of various constructions, when subjected to destabilizing loads. The survey includes discussion of imperfection sensitivity and of the effect of various defects on the critical conditions.

## I. INTRODUCTION

Thin-walled cylinders of various constructions find wide uses as primary structural elements in simple and complex structural configurations. The round cylinder is popular in column design, in tubing and piping, and in offshore platforms. Stiffened and unstiffened metallic and laminated composite thin (large diameter to thickness ratios) shells are used extensively in underwater, surface, air, and space vehicles as well as in the construction of pressure vessels, storage bins, and liquid storage tanks.

During their service, thin-walled cylinders are often subjected to individual and combined application of external loads. In resisting these loads, the system is subject to buckling, a physically observed failure mode, which is closely associated with the establishment of its load-carrying capacity. Therefore, the buckling strength of thin shells along with knowledge of its postbuckling behavior have been the subject of many researches and investigations both analytical and experimental. The knowledge derived from these studies is very essential in the safe design of such configurations. It is not surprising then that several hundreds of papers and reports have been written on the subject. Moreover, several review articles have found their way into the open literature, and through these reviews the interested readers and, more importantly, the potential designers are informed of the several specific questions that the reviewed articles address. In particular, attention was paid to the degree of approximation involved in the use of various kinematic relations (which led to several linear and nonlinear shell theories), to the reasons for the discrepancy between classical (linear theory), theoretical predictions for critical loads, and experimentally obtained buckling loads, to the use of stiffening for improving resistance to buckling, and to the effect of cutouts of various shapes, of foreign rigid inclusions and other defects. Moreover, as the complexity of shell-like structures increased and as the computational capability improved, efficient computer codes became necessary for the stability analysis of these configurations. Furthermore, in the more recent years, the constant demand for lightweight efficient structures led the structural engineer to the use of nonconventional materials, such as fiber-reinforced composites. The correct and effective use of these materials requires more complex analyses in order to achieve good understanding of the system response characteristics to external causes.

Finally, one should mention that designers of thin cylinders have always strived towards achieving the best possible design (lightest, most reliable, fail safe, most economical, easiest to maintain and repair, etc). An effort has been exerted in the last 15 years or so to accomplish this in a formal manner. Many refer to this effort as structural optimization.

The purpose of the present paper is twofold: (1) to review the entire field of buckling and postbuckling, which is primarily accomplished by referring to pertinent reviews, books, and recent technical articles, and (2) to provide a state-of-the-art accounting of the effect of imperfections on the response characteristics of thin cylindrical shells. The imperfections include initial geometric imperfections (out-of-roundness, load eccentricities and/or load misalignments), as well as constructional and material defects, such as small holes, cutouts, rigid inclusions, delaminations, and others.

## II. BRIEF HISTORICAL REVIEW

Because of the tremendous and continuous interest in shell buckling and because of the multitude of the reported theoretical and experimental investigations, a short, historical sketch is presented. For the sake of brevity, this sketch takes rather giant steps in moving through time.

The reader, who is interested in smaller steps and more details on every addressed aspect of the problem, is referred to

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reviews and surveys that have appeared in the open literature since the 1950s.[1-11] Moreover, he may also refer to the proceedings of several symposia[12-14] addressing shell buckling problems and to tests[15-18] that present a collection of papers on various topics of shell buckling. In addition, the reader is also referred to three recent books.[8,19,20] Yamaki's book[19] deals very thoroughly with the buckling and postbuckling behavior of elastic and isotropic cylindrical shells, while the books by Bushnell[8] and Kollar and Dulacska[20] include several shell geometries (cylindrical, conical, spherical, etc) and various constructions (stiffened, sandwich, corrugated, etc); and touch upon influences of plasticity, creep, and residual stresses. Finally, mention should be made of chapter 13, Circular tubes and shells, of Ref. 21, primarily for the benefit of a large class of practicing engineers.

The first theoretical investigations on the subject dealt with axially loaded configurations, and they were performed by Lorenz,[22,23] Timoshenko,[24,25] and Southwell.[26] The first experimental studies are those of Lilly (see Ref. 27), Robertson,[28] Flügge,[29] Wilson and Newmark,[30] Lundquist,[27] and Donnell (his experimental results are reported in Ref. 27). The initial theoretical investigations were based on many simplifying assumptions, and they reduced the mathematical model to a linear eigen-boundary-value problem (classical bifurcation approach). Comparison between theoretical predictions (critical loads) and experimental results (buckling loads) revealed discrepancy of unacceptable magnitude. A tremendous effort was made in order to explain the discrepancy both analytically and experimentally.

From the analytical point of view, the initial simplifying assumptions were later reevaluated and removed. This led to studies which attempted to attribute the discrepancy to (a) effect of prebuckling deformations,[29,31-33] (b) effect of in-plane boundary conditions,[34-37] and (c) effect of initial geometric imperfections.

Initially, the imperfection sensitivity of the system was established through strict postbuckling analyses of the perfect configuration.[38-43] In addition, some of these investigators explained that the minimum postbuckling equilibrium load is a measure of the load carrying capacity of the system. This latter thinking came to an end when Hoff, Madsen, and Mayers[44] concluded from their calculations that the minimum postbuckling load tends towards zero with increasing number of terms in the series expansion of the transverse displacement component and with diminishing thickness. In this limiting case the Yoshimura buckle pattern[45] can be achieved.

Another approach for imperfection sensitivity studies is to deal directly with the imperfect configuration and employ nonlinear kinematic relations. The first attempt is Donnell's.[46] Efforts reported in Refs. 47–52 fall into this category, but with varied success. Koiter[53] was the first to question the use of the minimal postbuckling load as a measure of the load carrying capacity. He also dealt directly with the imperfect configuration. His theory is limited to the neighborhood of the classical bifurcation load (immediate postbuckling), and therefore to small initial imperfections. Many researchers adopted this approach, and most of their investigations are reported in Ref. 4.

The single and most important conclusion of all the theoretical investigations of cylindrical shells is that the primary reason for the discrepancy between (linear) theoretical critical loads and buckling loads is that the system is extremely sensitive to initial geometric imperfections.

In parallel to the above analytical investigations, many experimental studies were performed with the same objective in mind (explain the discrepancy). While the old buckling loads fell in the range of 15–50% of the classical critical load, the new ones,[54-58] with use of carefully manufactured specimens, fall in the range of 40–90% of the classical critical load. Note that Ref. 54 obtained also postbuckling curves for both axially and pressure loaded cylinders. Moreover, Thielemann and Esslinger[59,60] extended their research to include theoretical postbuckled state calculations on the basis of observed experimental results. The theoretical predictions are based on a Galerkin procedure with the emphasis on post-limit point equilibrium positions. The theoretical study of these two papers belongs to the group of postbuckling analyses of perfect configurations.

A similar development was followed for the case of buckling under lateral loading (pressure). The first analysis is attributed to von Mises.[61] Several investigations based on linear analyses appear in the literature. The interested reader is referred to Yamaki's[19] book for a complete review. Only a few of them are mentioned here in order to discuss certain distinguishing features. The reader must also consider their cited references. Batdorf[62] used simplified Donnell-type[63] of shell equations to predict critical loads. Soong[64] employed Sanders' shell theory.[65] Simitses and Aswani[66] compared critical loads for the entire range of radius to thickness and length to radius ratios and for various load behaviors during the buckling process (true pressure, constant directional pressure, and centrally directed pressure) for a thin cylindrical shell, employing several linear shell theories: Koiter–Budiansky,[67,68] Sanders,[65] Flügge,[69] and Donnell.[63] Sobel[70] studied the effect of boundary conditions on the critical pressure.

Postbuckling and imperfection sensitivity analyses appear in the literature, as in the case of axial compression. These studies follow the same pattern and include strictly postbuckling analyses,[71,72] and Koiter-type of analyses.[4]

Orthotropic, stiffened, and other constructions were considered by several researchers, including Becker and Gerard,[73] Meek,[74] Hutchinson and Amazigo,[75] and Simitses, Sheinman, and Giri.[76]

Buckling analyses for torsion started with the work of Donnell.[63] Several studies followed with the emphasis on different considerations. Hayashi[77] dealt with orthotropic cylinders, and Lundquist[78] and Nash[79] are two of several that reported experimental results. Hayashi and Hirano[80] and Budiansky[81] are among those who reported on postbuckling analyses.

Before closing this section, mention should be made of at least a few studies, which consider the simultaneous application of two or more destabilizing loads. For a fairly complete list, the reader is referred to chapter 5 of Yamaki's book.[19] Some of these references deal with nonisotropic constructions. Hess,[82] Reese and Bert,[83] Sheinman and Simitses,[84] and Simitses, Shaw, and Sheinman[85] also included multiple loads in their work.

## III. BOUNDARY CONDITIONS AND PREBUCKLING DEFORMATIONS

The discrepancy between theoretical predictions based on classical buckling analyses and experimental results[9,71,72,78-80] is greater for the case of axial compression than for the case of either pressure or torsion. This was first established for cylindrical shells of isotropic construction. When the comparison is extended to stiffened configurations, the discrepancy is still present but less pronounced.

In trying to explain this discrepancy, three sets of simplifying assumptions were identified in connection with the classical approach. These are: (a) effect of prebuckling deformations, (b) effect of boundary conditions, especially of the in-plane ones, and (c) the effect of initial geometric imperfections. The first two of these are discussed with some detail in this section, and the last one in the next section.

### III.1. Prebuckling state

First, one of the assumptions in the classical theory is that the prebuckling state is one of a pure membrane. This means that in the case of compression the ends of the cylinder are free to expand, and a constant transverse displacement takes place throughout, which corresponds to zero hoop stresses. Similarly, in the case of lateral pressure the ends are free to contract or expand radially, thus resulting in a uniform radial displacement and constant in-plane stresses. The axial stress may or may not be zero, depending on whether the ends experience pressure and whether they are free to move axially.

In the real and practical case, the ends are not completely free to expand or contract. Therefore, the prebuckling state has both membrane and bending stresses.

For the case of uniform axial compression, Stein[31,33] and Fisher[32] independently showed that, by providing complete fixation against radial translation, the critical load is reduced by approximately 10% from the classical value. This effect by itself then cannot totally explain the discrepancy. Similar results are reported by Yamaki[19] for axial compression and for pressure.

### III.2. Boundary conditions

The other possible source for the discrepancy is the effect of boundary conditions, especially the in-plane type.

In the classical approach, the linearized buckling equations can be obtained by employing the perturbation technique. Small additional quantities are added to the primary membrane state in order to take us to the buckled state. Since, through this approach, one seeks the existence of a bifurcation point, the buckled state can be assumed as close to the primary state as desired. Therefore, the additional quantities can be made infinitesimal. This yields a linearized set of buckling equations (three partial differential equations) in the additional but small longitudinal, circumferential, and radial displacement components, $u^a$, $v^a$, and $w^a$. These buckling equations are subject to radial and in-plane boundary conditions. The radical boundary conditions that have received most attention are those corresponding to simply supported ends and to fixed or clamped ends. Moreover, for each case of radial or transverse boundary conditions, four sets of in-plane boundary conditions exist. By employing the standard notation of $N_{xx}$, $N_{yy}$, $N_{xy}$ for stress resultants and $M_{xx}$, $M_{yy}$, $M_{xy}$ for moment resultants, the boundary conditions can be written as SS-$i$ or CC-$i$, with $i = 1,2,3,4$, where

*Transverse*:

$$\begin{aligned} \text{SS:}\ & w^a = M^a_{xx} = 0, \\ \text{CC:}\ & w^a = w^a_{,x} = 0. \end{aligned} \tag{1}$$

*In-plane*:

$$\begin{aligned} 1:\ & N^a_{xx} = N^a_{xy} = 0, \\ 2:\ & u^a = N^a_{xy} = 0, \\ 3:\ & N^a_{xx} = v^a = 0, \\ 4:\ & u^a = v^a = 0. \end{aligned} \tag{2}$$

The in-plane boundary conditions have been arranged (1–4) according to Hoff.[2] The numbering is different in Ref. 19. According to Hoff's[2] and Ohira's[34] results, the order of Eqs. (2) corresponds to depicting a move from the weakest configuration ($i = 1$) to the strongest ($i = 4$). Please note, however, that their[2,34] results were obtained by using a linear buckling analysis.

According to Hoff,[2] the effect of in-plane boundary conditions for infinitely (very) long cylinders is such that for SS-1 and SS-2 the critical load is approximately half of the classical load, while for all other cases it is approximately equal to the classical load.

Simitses et al,[86] in investigating the imperfection sensitivity of laminated thin cylindrical shells, studied numerous effects including the effect of in-plane boundary conditions and the effect of eccentricity in axial load. The nonlinear solution scheme used accounts for prebuckling effects and initial geometric imperfections of specified amplitude and shape. Results were generated for an isotropic configuration with the following structural geometry:

$$\begin{aligned} &E = 72.4\ \text{GPa}\ (10.5 \times 10^6\ \text{psi}), \quad \nu = 0.30, \\ &R = 10.16\ \text{cm}\ (4\ \text{in.}), \quad L/R = 1, \quad R/t = 1000, \end{aligned} \tag{3}$$

where $L$, $R$, and $t$ denote the length, radius, and skin thickness, respectively.

The imperfection shape was taken to be (almost) axisymmetric and the imperfection amplitude parameter $\xi$ ($= W^0_{max}/t$, maximum imperfection amplitude/shell thickness) was varied from 0 to 2. The results are shown on Fig. 1. Note that for small $\xi$ values the results support the trend suggested by Hoff[2] and Ohira,[34] ie, the weakest configuration corresponded to SS-1, the next one to SS-2, and then SS-3 and SS-4. Moreover, the results at $\xi = 0$ (obtained through extrapolation as $\xi \to 0$) agree well with those of Hoff.[2] For SS-1 the ratio of critical load to classical load is 0.55, for SS-2, 0.68, and for SS-3 and SS-4, 0.98.

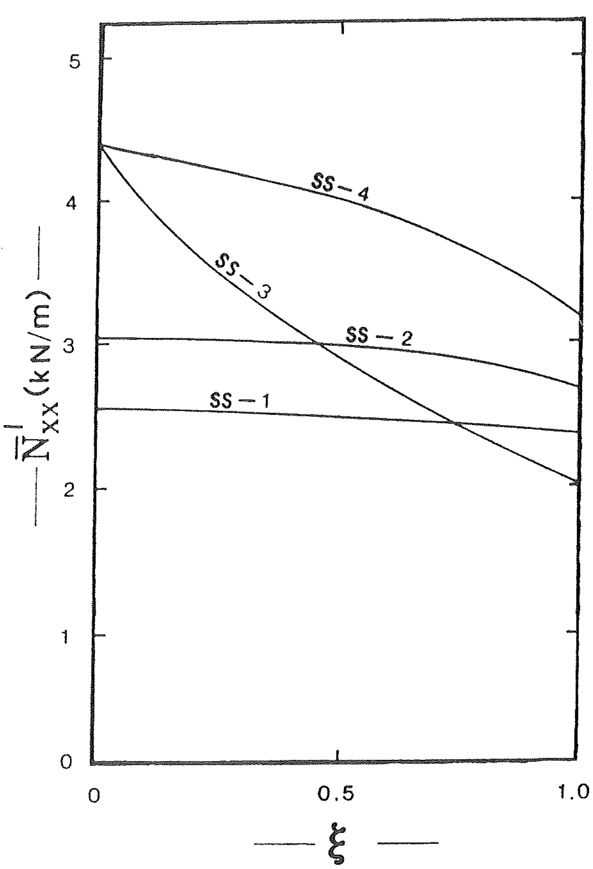


FIG. 1. Effect of in-plane boundary conditions on the imperfection sensitivity of isotropic geometry (SS-$i$).

There are two important observations in connection with the results of Fig. 1. First, for low values of $\xi$ (imperfection amplitude), the $v^a = 0$ boundary conditions (SS-3 and SS-4) yield stronger configurations than the $N^a_{xy} = 0$ boundary conditions (SS-1 and SS-2). For this same range $u^a = 0$ yields a stronger configuration in comparison to $N^a_{xx} = 0$ (SS-2 stronger than SS-1 and SS-4 stronger than SS-3). For higher values of the imperfection amplitude ($\xi$), the $u^a = 0$ boundary conditions yield stronger configurations (SS-4 and SS-2) than the $N^a_{xx} = 0$ conditions (SS-3 and SS-1). The only difference is that the $v^a = 0$ effect is not the same for the two sets. The order in going from the weakest to the strongest configuration for $\xi > 0.7$ appears to be SS-3, SS-1, SS-2, and SS-4. On the other hand, for the clamped case (not shown herein) the order of going from the weakest to the strongest configuration is the same for the entire range of $\xi$ values considered ($0 \le \xi \le 2$) and it is CC-1, 3 to CC-2 to CC-4. The CC-1 and CC-3 results are indistinguishably close, while the CC-2 results are slightly smaller than the CC-4 ones. Moreover, the CC-4 results are slightly (2–5%) higher than the SS-4 results, and the CC-1, 3 results are consistently smaller than the CC-4 results by approximately 12%. Thus, for the clamped case, the $u^a = 0$ boundary conditions (CC-1 and CC-2) yield stronger configurations than the $N^a_{xx} = 0$ conditions, for all $\xi$ values considered.

The second important observation is that the initially (low $\xi$ values) weak configurations (SS-1 and SS-3) are not as sensitive to initial geometric imperfections by comparison to the stronger one (SS-4 and SS-2) (see Fig. 1). For the clamped boundary conditions, it appears that all cases (CC-$i$, $i = 1, 2, 3, 4$) are as sensitive as the SS-4 (and SS-2) configuration.

Similar conclusions are drawn from the results on an asymmetric laminated geometry.[86]

Although these observations are based on results from only a few geometries, they suggest that a substantial part of the discrepancy may be attributed to various ways of experimentally supporting a thin cylinder in a laboratory experiment. These results cannot fully account for the tremendous scatter in experimentally obtained critical loads, nor can they explain the large discrepancy, in some cases by a factor of 4, between classical theoretical predictions and experimental results.

The above statements suggest that the primary reason for the discrepancy must lie elsewhere. Indeed, this is the case, as discussed in the next section.

## IV. IMPERFECTION SENSITIVITY

As already mentioned, it is the generally accepted conclusion that most of the discrepancy between classical theoretical predictions and experimental results can be attributed to the presence of small initial geometric imperfections.

There exist several types of imperfections and defects that affect the response of the configuration. They are generally grouped into two broad categories: (a) initial geometric imperfections and (b) material or constructional defects. The first group is addressed in this section, and the second group in the next.

The phrase "imperfection sensitivity" means to sensitivity of response of the configuration, because of initial geometric imperfections. There exist two types of initial geometric imperfections: initial geometric shape imperfections and load eccentricities.

### IV.1. Initial geometric shape imperfections

These imperfections refer to deviations in shape of the structural configuration. Examples of these imperfections include a local bubble in a thin circular cylindrical shell, which makes the geometry locally nonsymmetric; a global out-of-roundness, which may or may not be dependent on axial position, that makes the geometry globally noncircularly cylindrical; or a small initial curvature in a flat plate or rod.

As already mentioned in the introduction, several approaches have been used to establish the imperfection sensitivity of circular cylindrical shells and to predict critical conditions for these configurations. The approach that seems to have a better chance for accomplishing both is the one that deals directly with the imperfect configuration.[47–52] In this approach nonlinear shell theory is employed. The shell is subject to limit point instability. The limit point is a measure of the critical condition. Several analytical investigations exist. They are based on different kinematic approximations, different solution schemes, and various computer codes. It is reasonable to expect that this approach could and should yield critical loads which compare well with the experimentally obtained buckling loads, provided that one has complete knowledge of the initial geometric shape imperfections and that this knowledge is accommodated in the mathematical model. Unfortunately, this is a virtually impossible task, especially for commercially manufactured shells.

It has been suggested and efforts have been made towards the establishment of an International Initial Imperfection Data Bank[87,88] with a dual purpose[89]: first, to present all available imperfection data in identical form, and, second, to make these data available to future potential users, who may devise improved computer codes based on nonlinear shell analyses, free of many currently used simplifying assumptions. Along these lines Elishakoff[90] proposes a method for incorporating experimentally obtained imperfection distributions into a statistical imperfection-sensitivity analysis. The feasibility of this approach has been demonstrated[91] for axially loaded imperfect cylindrical shells.

The effort to establish an Imperfection Data Bank can be traced to the experimental buckling program reported by Singer, Arbocz, and Babcock.[92] In particular, the no. AS-2 stringer-stiffened specimen, for which carefully measured imperfection data are reported, has been used extensively as a benchmark in correlating theoretical results with experimentally obtained buckling loads.[92–96,89] In these efforts several solution methodologies were used (Koiter-type $b$-factor analyses, STAGS-code, etc), and a comparison with the experimentally obtained value (226.3 N/cm[92]) was presented.[96] The best numerical prediction listed in these references is 243.8 N/cm,[93] and it was obtained by using the STAGS code (30-modes model). The difference between this and the experimental value is only 7%, a margin that is well within the accuracy that can be expected for imperfection-sensitive buckling load calculations. However, as was pointed out in Ref. 93, the initial geometric imperfection shape, recalculated by using 30 Fourier coefficients (30-modes analysis) does not physically resemble the measured initial imperfection shape. In order to remove this restriction, Arbocz[96] tried various fitting methods to recompute the imperfection shape, and he also employed the STAGS code to calculate the corresponding critical loads. His new computations[96] did not improve the best numerical prediction.[93] It is indeed surprising to see that computations using 74 modes, 132 modes, and cubic spline fit are not as close to the experimental results as the 30 modes analysis.[93] The imperfection shapes in Ref. 96 are much closer to the true imperfection than the shape used in the 30-modes model. All of these calculations employed CC-4 boundary conditions. In view of the results two questions can be raised: (1) Do we need to apply so complicated a shape for the imperfection in order to accurately predict critical loads? (2) Do CC-4 boundary conditions accurately describe the physical boundary conditions for shell AS-2 of Ref. 92?

These questions were addressed by Simitses and Shaw.[97] In so doing, they employed a solution procedure developed earlier,[52] in which the imperfection can be expressed as a Fourier series in the circumferential direction with "axial coordinate" dependent coefficients. They employed several imperfection shapes from the measured data for the AS-2 shell (converted into double Fourier series). In each one of these, though, only one circumferential wave shape of measured imperfection is used. Moreover, they used two simplified, one axisymmetric and one symmetric, imperfection shapes, in order to study the possibility of simplification of imperfection shape.

In the first group, the imperfection shape is taken as:

$$W^0 = t\sum_{k=0}^{N}\left[A_{k0}\cos\frac{k\pi x}{L} + \cos\frac{k\pi x}{L}\left(A_{kl}\cos\frac{ny}{R} + B_{kl}\sin\frac{ny}{R}\right)\right], \tag{4}$$

where $A_{kl}$ and $B_{kl}$ are Fourier series coefficients of measured imperfection shape, corresponding to wave number $l$. The values of $A_{kl}$ and $B_{kl}$ are part of Table 3 of Ref. 95. Moreover, Eq. (4) implies that, only one $l$ value is used. In Table 3 of Ref. 95, the $A_{il}$ and $B_{il}$ terms are shown for several $l$ values, but only one column is used in the imperfection shape of Eq. (4). The solution procedure[52] employs a single $l$ representation, and $l$ is computed by requiring both minimum potential energy response and lowest limit point load. This implies that the response has a single $l$ representation and the imperfection shape is similar to the response shape (transverse displacement; $l = n$).

The second group employs a virtually axisymmetric imperfection which has the form

$$W^0 = \xi t\left(\cos\frac{2\pi x}{L} + 0.1\sin\frac{\pi x}{L}\cos\frac{ny}{R}\right). \tag{5}$$

The third group employs a symmetric imperfection of the form

$$W^0 = \xi t\sin\frac{\pi x}{L}\cos\frac{ny}{R}. \tag{6}$$

In Eqs. (5) and (6), $\xi$ is a measure of the imperfection amplitude. Note that for the symmetric imperfection, $\xi = W^0_{max}/t$. While for the virtually axisymmetric imperfection, $\xi = W^0_{max}/1.1t$. Two kinds of boundary conditions were used in calculating buckling loads, CC-3 and CC-4. The conclusions is that a simple axisymmetric or symmetric imperfection shape can be employed, and the actual imperfection data are only used to establish the maximum amplitude of the $l$-dependent (circumferential, $l = n$) imperfection data. Moreover, from the description of the experiments CC-3 is a better boundary condition than CC-4. Two figures (Figs. 2 and 3) are shown herein to support their findings of Ref. 97. From the imperfection data, when converted to Fourier series coefficients[95] the maximum amplitudes corresponding to $l = n = 9$, 10, and 11 are $W^0_{max}/t = 0.052$, 0.062, and 0.079, respectively. The critical loads corresponding to calculations based on simplified shapes, Eqs. (5) and (6), are 228 and 223 N/cm. Both of these are extremely close to the experimental value of 226.3 N/cm.

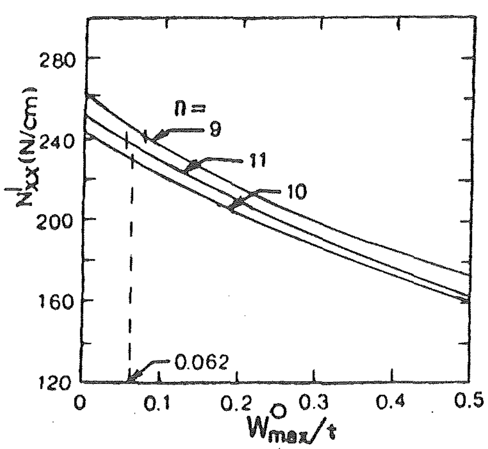


FIG. 2. Critical load for axisymmetric imperfection (CC-3).

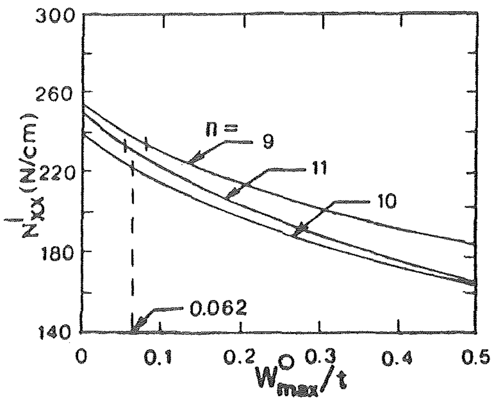


FIG. 3. Critical load for symmetric imperfection (CC-3).

These findings support the contention of several researchers[21,98] that the critical load degradation is most significant when the imperfection shape is similar to the buckling mode shape.

In spite of these findings, agreements, and observations, more concrete proof is needed to fully assess the effect of initial geometric shape imperfections.

## IV.2. Load eccentricities

Another important class of initial geometric imperfections consist of load eccentricities. Such imperfections have been used to establish the imperfection sensitivity[99] or insensitivity[100] of structural elements.

In the case of cylindrical shells, it is possible to apply the in-plane destabilizing loads (axial compression and shear) with an eccentricity relative to the neutral surface for metallic configurations. Since, for axial compression, buckling is followed by a predominantly inward lateral deflection, it has been suggested by some to stabilize the shell by applying the load eccentrically, so that an outward prebuckling deformation is present. This effect was studied and reported in Ref. 86 for shells of various construction, isotropic, orthotropic, and laminated. The findings reported in Ref. 86 do not support the above suggestion. According to Ref. 86 for small eccentricities (minus half to plus half the isotropic shell thickness), the response seems to be insensitive to the eccentric application of the load. For very large eccentricities (±10, or more times the skin thickness), positive eccentricity, which induces outward prebuckling motion, has a stabilizing effect, and this observation supports the contention. Similarly negative eccentricities have a relatively small destabilizing effect. In the intermediate range of load eccentricities (±0.5–10 times the skin thickness) an irregularity is observed. It is suspected that one possible reason for this behavior (stabilizing effect for large positive eccentricities) may be associated with the Poisson effect. As the load is applied quasistatically, the cylindrical shell moves outward, because of the Poisson effect; it reaches a maximum expansion at the critical load, and then an inward motion takes place; finally, at and after collapse, this inward motion continues. This sequence of events and the corresponding stabilization or destabilization is heavily affected by the value of the Poisson ratio or the $A_{12}$ term in the extensional stiffness matrix $[A_{ij}]$. The smaller the $A_{12}$ term (low value of $\nu$ for isotropic construction or placing the strong axis along the cylinder axis for an orthotropic construction), the greater the stabilization effect of the positive eccentricity.[86]

In stringer-stiffened configurations, the load eccentricity effect is more complex because it ties boundary effects (moments induced at the boundaries) with the effect of stiffener eccentricity. This effect was addressed by Stuhlman, DeLuzio, and Almroth[101] in the 1960s. Moreover, in the early 1970s[102] parametric studies and many tests were carried out at the Technion, for evaluating this important interaction. Their[102] studies showed that differences in buckling load of up to 50% can be attributed to this effect in some practical configurations.

Before closing this section, I should mention another loading effect, such as nonuniform axial compression. Many have viewed this as a combined bending-axial compression loading. The interested reader is referred to the work of Libai and Durban[103] and their cited references.

## V. MATERIAL OR CONSTRUCTIONAL DEFECTS

This group consists of small holes or cutouts, small rigid inclusions, cracks, nonmonolithic skin-stiffener connections, delaminations in laminated configurations, and others. Most of the emphasis in the past has been on establishing stress concentrations and local stress distributions in order to predict material failures.

For metallic materials, one can find several studies which deal with the effect of material imperfections on the fatigue life of the structural component. Moreover, there exists a number of investigations that deals with the effect of *small cutouts* on the stress and deformation (local) response of thin, circular, cylindrical shells.[104-108] Savin[109] has provided an extensive bibliography on the subject, and it covers various shapes of holes (circular, square, elliptic, and triangular) for both cylindrical and spherical thin shells. Furthermore, there exists a small number of publications,[110-115] which deals with the effect of small and large cutouts on the buckling characteristics of cylindrical shells. The problem is extremely difficult and use of linear buckling theory is highly questionable. Finally, as far as the effect of small holes on the response of shells and plates of nonmetallic construction is concerned, the reader is referred to the work of Hoff[116] and Shnerenko.[117]

Another material imperfection is the small *rigid inclusion*. The effect of rigid inclusions on the stress field of the medium in the neighborhood of the inclusion has received limited attention in the past 25 years.[118-122] As far as this author knows, the effect of rigid inclusion on buckling characteristics has not been studied.

The effect of delamination is receiving more attention especially for fiber-reinforced composite construction. Very few efforts are reported in the literature that deal with delamination buckling of cylindrical configurations. All of them[123-125] use special construction and linear classical buckling analysis. Because of the importance of the effect of this material type of imperfections, more work will soon appear.

## VI. CLOSURE

Very little has been mentioned, so far, for buckling of laminated configurations. With the advent of fiber-reinforced composites, metallic configurations are continuously being replaced by laminated configurations with or without stiffening. As expected then, all of the problems and questions related to buckling of metallic structures are subjected to reexamination in view of the new construction. Questions of shell theory approximation[126] and imperfection sensitivity[86,127,128] are as important a consideration for composites as for metallic construction. The interested reader is also referred to a recent review by Tennyson.[129]

Because of the complexity of the problem, several finite element algorithms have been developed for the nonlinear buckling and postbuckling analysis of shells, including cylindrical configurations. For a fairly extensive review, the interested reader is referred to Bushnell's[8] book.

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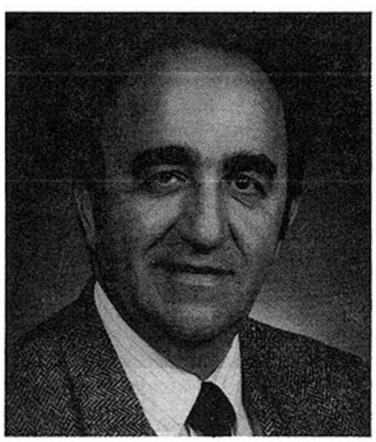

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