Estimating Cross-Country Differences in Product Quality*

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Abstract

We develop a methodology for decomposing countries' observed export product prices into quality versus quality-adjusted-price components. In contrast to the standard approach of equating export price with quality, our methodology accounts for cross-country variation in product prices induced by factors other than quality, e.g. comparative advantage or currency misalignment. Even though variation in quality-adjusted prices is unobserved, it can be inferred from countries' trade balances with the rest of the world. Holding observed export prices constant, for example, countries exhibiting trade surpluses must be offering higher quality (i.e., lower quality-adjusted prices) than countries running trade deficits. We implement the methodology by estimating the evolution of manufacturing product quality among the United States' top 45 trading partners. Preliminary results reveal substantial cross-sectional variation in product quality growth between 1980 and 1997 that is not apparent in export prices alone. China and Ireland, in particular, experience relatively rapid gains in manufacturing quality.

Keywords: Export Unit Values; Export Quality; Revealed Preference
JEL classification: F1; F2; F4

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1. Introduction

Theoretical and empirical research increasingly points to the importance of product quality in international trade and economic development. Cross-sectional variation in product quality is emphasized as an influential determinant of global trade patterns and international specialization[1], while quality upgrading is highlighted as a crucial dimension of the development process.[2] Unfortunately, relatively little is known about how countries increase their product quality or which policies are more likely to foster it. A major impediment to research in this area is the fact that reliable estimates of product quality do not exist for a wide range of countries, industries and years. The purpose of this paper is to develop a methodology to obtain such estimates.

Researchers typically confront the absence of quality measures by constructing *ad hoc* proxies of quality. The most common of these measures is a comparison of countries' observed export prices (unit values).[3] If goods are differentiated horizontally as well as vertically, however, export prices may vary for reasons other than their quality. Chinese shirts might be cheaper than Italian shirts because their features are less desirable, but they may also sell for less because China has lower production costs or an undervalued exchange rate.

The methodology developed here decomposes countries' observed export unit values into quality versus quality-adjusted-price components. We define quality to be any tangible or intangible attribute that increases consumer valuation of a product. We extract estimates of countries' relative product quality by combining data on their observed export prices with information about consumer demand contained in their trade balance. The intuition behind our identification is straightforward: because consumers care about price relative to quality in choosing among products, two countries with the same export prices but different trade balances must have products with different levels of quality. Among countries with identical export prices, the country with the higher trade balance is revealed to possess higher product quality.[4]

We generalize this intuition to a setting where countries also differ in terms of the number of *unobserved* horizontal varieties they export in each product category. Accounting for unobserved horizontal differentiation is difficult because it introduces an additional factor

[1] Flam and Helpman (1987) is representative of a theoretical line of research studying how product quality affects trade patterns. The role of quality as a determinant of global trade patterns is addressed empirically by Schott (2004) and Hallak (2005). Cross-country and time-series variation in product quality has also been linked to firms' export success (Brooks 2003, Verhoogen 2004), countries' skill premia (Verhoogen 2004), and quantitative import restrictions (Aw and Roberts 1986, Feenstra 1988).

[2] The contribution of quality growth to macroecomic growth is investigated theoretically by Grossman and Helpman (1991) and empirically by Hummels and Klenow (2005).

[3] Unit value differences figure prominently in surveys of countries' "quality competitiveness" (e.g., Aiginger 1998, Verma 2002 and Ianchovichina et al. 2003) and also are often used to distinguish horizontal from vertical intra-industry trade flows (e.g., Abed-el-Rahman 1991 and Aiginger 1997). More broadly, equating price with quality is often done in the computation of the U.S. Consumer Price Index (Boskin et al. 1998).

[4] The use of market shares to infer unobserved consumer valuation is well-established in the industrial organization and index number literatures (e.g. Berry 1994 and Bils 2004, respectively). Here, countries' net trade with the rest of the world (conditional on trade costs) is a natural expression of their "market share".

besides quality that can increase consumer demand for a country's products. Indeed, in the absence of horizontal differentiation, price variation is equivalent to quality variation because any differences in quality-adjusted prices would be arbitraged away. All else equal, consumer love of variety implies that countries producing a larger number of varieties in a product category export larger quantities and therefore exhibit higher trade surpluses. Unless the number of horizontal varieties that countries export is accounted for, this increase in net trade will be interpreted, erroneously, as higher product quality.[5] We pin down quality by assuming a negative relationship between quality-adjusted prices and the number of varieties a country exports. This assumption is justified by recent theoretical findings in Romalis (2004) and Bernard et al. (2005), who show that comparative advantage sectors exhibit both relatively low prices – due to relatively low factor costs – and a relatively high number of varieties – due to disproportionate use of factor inputs.

The use of countries' trade balances with the rest of the world to identify consumer demand imposes a practical constraint on the implementation of our methodology. Currently, the most reliable time-series information on countries' global net trade is recorded at the "sector" level.[6] As a result, trade balances are tracked at a coarser level of aggregation than countries' export prices, which can be observed at a finer level of aggregation (i.e., at the "product" level) in U.S. import statistics.[7] To deal with this mismatch, we develop a theoretically appropriate price index of a country's export product unit values within a sector. This index, which we denote the "impure" price index because it is "contaminated" by quality, can be decomposed into quality versus quality-adjusted-price components. We derive estimates of countries' quality relative to a numeraire country by sector and year under the assumption that each country's product quality is constant across all products in a sector. This assumption – made necessary by the different levels of aggregation at which export prices and global trade balances are observed – creates an "aggregation trade-off" in our methodology: while product quality is more likely to be constant across products the more disaggregate is the sector, data on countries' global net trade becomes more scarce, and measurement error likely increases, with disaggregation.

Our methodology has three steps. First, we show that the bilateral impure price indexes, although unobservable, are bounded by observable Paasche and Laspeyres indexes defined over the countries' common exports to a third country. Based on those bounds, we estimate an impure-price-index number for each country-sector. Second, we demonstrate that the quality-adjusted- (or "pure-") price component of countries' sectoral impure price indexes can be inferred from their sectoral trade balance with the world. In the final step, we strip away the pure price component of the impure price index to estimate changes in countries'

[5] Feenstra (1994) outlines a methodology for computing import price indexes that accounts for the introduction of new product varieties. (See also Broda and Weinstein 2004). Given its focus on changes in prices over time, that methodology requires no knowledge of cross-sectional variation in the number of varieties countries export within product categories so long as that number is constant over time for a subset of countries.

[6] The most disaggregate sector for which net trade information is available for a large set of countries is a four-digit SITC industry. In our preliminary results below, we report results for the more aggregate "all manufacturing" sector.

[7] U.S. imports are tracked according to roughly 20,000 ten-digit Harmonized System products. By comparison, there are approximately 1000 four-digit SITC sectors.

relative product quality over time. We report preliminary estimates of relative manufacturing product quality growth for the United States' top 45 trading partners relative to numeraire country Switzerland for the period 1980 to 1997. These results reveal substantial variation in product quality growth across countries that is not apparent in export prices alone. China and Ireland, in particular, experience relatively rapid gains in manufacturing quality.

This paper's focus on cross-sectional variation in product quality differentiates it from a very large index number literature devoted to constructing quality-adjusted cost-of-living indexes. Here, rather than measure quality changes in bundles of products purchased over time, we seek to identify quality variation over simultaneously purchased bundles from different sources of supply. In addition, we assume no knowledge of products' underlying attributes. As a result, we are unable to make use of standard strategies – such as hedonic pricing – that explicitly incorporate information on product characteristics that might be linked to specific dimensions of quality.[8] Our methodology complements such efforts, however, because its use of publicly available trade data permits estimation of product quality across a broad range of countries, industries and years for which surveys of product characteristics may be unavailable or prohibitively expensive to collect.[9] Our analysis is most closely related to Hummels and Klenow (2005), who use import prices and quantities to make inferences about the cross-sectional elasticity of quality with respect to per-capita income and country size. The methodology we develop here permits explicit estimation of product quality (relative to a numeraire country) by country, sector and year.

An ability to decompose export unit values into quality and quality-adjusted-price components is obviously useful for testing models of international specialization and development. It also contributes to research in other fields. In the productivity, growth and macroeconomics literatures, quality adjustment is crucial for computing the import and export price deflators used to construct real national accounts aggregates. Current estimates of "real GDP" in the Penn World Tables, for example, deflate nominal GDP using a purchasing-power-parity deflator based on final expenditure data, which may not be optimal for capturing changes in countries' *production* over time because the latter requires a terms-of-trade adjustment (Feenstra et al. 2004). An ability to net quality out of countries' import and export price indexes before performing the terms-of-trade correction would enhance their accuracy.

Development of country-sector specific quality-adjusted price indexes may also prove useful in analyzing issues of public policy. The distributional consequences of international trade implied by the Stolper-Samuelson theorem, for example, cannot be properly identified if the import and export price changes used to compute real wages do not properly account for changes in countries' product quality.

The remainder of the paper is structured as follows. Section 2 outlines our assumptions about consumer demand and introduces the Impure and Pure Price indexes that will be the

[8] Feenstra (1995), for example, demonstrates how information on product attributes can be used to establish bounds on the exact hedonic price index.

[9] The International Price Program of the U.S. Bureau of Labor Statistics constructs import and export price indexes by combining survey data on firms' prices with firms' assessments about changes in the quality of their products over time (Alterman et al. 1999).

focus of our analysis. Section 3 shows that the unobservable Impure Price Index is bounded by observable Paasche and Laspeyres indexes. Section 4 derives the relationship between the Pure Price Index and countries' sectoral net trade. Section 5 describes how information on export unit values and countries' net trade can be combined to estimate country-sector quality indexes and presents preliminary empirical estimates. Section 6 concludes.

2. Preferences and Price Indexes

2.1. Preferences

This section describes the preference structure underlying our analysis and formally introduces the price and quality indexes that are the focus of the methodology.

Goods are classified into product categories, which are in turn classified into sectors. Sectors are indexed by $s = 1, ..., S$, while product categories (within sectors) are indexed by $z = 1, ..., Z_s$.[10] There are C countries, indexed by $c = 1, ..., C$.

Preferences are common across countries, and are represented by a two-tier utility function that incorporates consumer love of variety. The upper tier is Cobb-Douglas, with expenditure shares b_s for each sector s. The lower tier has the following CES form[11]

$$u_s = \left[\sum_c \sum_z n_z^c \left(\xi_z \lambda_s^c x_z^c \right)^{\varphi_s} \right]^{1/\varphi_s} \qquad \varphi_s \epsilon (0, 1). \tag{1}$$

In the subutility function (1), n_z^c is the number of horizontally differentiated varieties of product z produced by country c, and x_z^c is the quantity consumed per variety.[12] This function includes two utility shifters, ξ_z and λ_s^c. The first shifter, ξ_z, varies across product categories but is constant across countries for a particular product category. It captures consumers' valuation of the essential characteristics common to heterogeneous varieties in a particular category (e.g. tables versus chairs). The second shifter, λ_s^c, varies across countries and sectors, but is constant across products within a particular country and sector. It represents product quality and captures the combined effect of all product characteristics, other than price and those already captured by ξ_z, on consumers' valuation of a good. Product quality encompasses both physical attributes (e.g. durability) and intangible attributes (e.g. product image due to advertising). These assumptions are formalized as follows:

Assumption 1: $\xi_z^c = \xi_z, \qquad \forall c = 1, ..., C.$

Assumption 2: $\lambda_z^c = \lambda_s^c, \qquad \forall z = 1, ..., Z_s.$

[10] In our empirical investigation below, product categories correspond to seven-digit Tariff System of the United States (TSUSA) and ten-digit Harmonized System (HS) categories, the finest possible level of aggregation.

[11] To simplify notation, subindexes on summations refer to all members of a set unless otherwise noted, e.g. $\sum_c$ and $\sum_{c'}$ both sum over all countries $c = 1, ..., C$ while $\sum_{c' \neq c}$ sums over all countries except c. For product categories, $\sum_z$ denotes the sum across all product categories in sector s, $z = 1, ..., Z_s$.

[12] Note that by indexing product categories instead of varieties, we implicitly assume symmetry across varieties in the same product category.

With the preference structure defined by (1), product demand depends on quality-adjusted or "pure" prices. Letting p_z^c be the export price of a typical variety of product z produced in country c, we define the "pure" price of that variety by $\widetilde{p}_z^c = p_z^c/(\xi_z \lambda_s^c)$. The pure price is a quality-adjusted price. It is also divided here by ξ_z for notational compactness, but none of the results or their interpretation is affected by this choice.

2.2. The Pure and Impure Price Indexes

Before defining price indexes of quality-adjusted and quality-unadjusted prices, we develop notation to keep track of counties unobserved numbers of varieties. Define $\overline{n}_s^c$ to be the average number of varieties across product categories produced by country c (in sector s),

$$\overline{n}_s^c = \frac{1}{Z_s} \sum_z n_z^c \qquad\qquad \forall c = 1, ...C, \tag{2}$$

and define $\overline{n}_z$ to be the (country o–normalized) world average number of varieties of product z,

$$\overline{n}_z = \frac{1}{C} \sum_c n_z^c \frac{\overline{n}_s^o}{\overline{n}_s^c} \qquad\qquad \forall z = 1, ...Z_s. \tag{3}$$

The normalization in (3) re-scales the number of varieties of each country into common, country-o units, according to the ratio of the average number of varieties between o and c. Define also $\widetilde{n}_z^c$ to be country c's "excess variety" in product z relative to the world average,

$$\widetilde{n}_z^c = n_z^c \frac{\overline{n}_s^o}{\overline{n}_s^c} - \overline{n}_z. \tag{4}$$

Note that excess variety has the convenient property that $\sum_z \widetilde{n}_z^c = 0, \forall c = 1, ..., C$.

Define an aggregator[13] of product prices produced in country c and sector s as

$$P_s^c = \left[\sum_z \overline{n}_z \xi_z^{\sigma_s - 1} \left(p_z^c\right)^{1-\sigma_s} \right]^{\frac{1}{1-\sigma_s}}. \tag{5}$$

We define the Impure Price Index between countries c and d as

$$P_s^{cd} = P_s^c / P_s^d. \tag{6}$$

The Impure Price Index is a summary measure of price variation between goods produced by countries c and d in sector s. The index is "impure" in the sense that it is defined over prices that are "contaminated" by quality. The index is transitive, so that $P_s^{cd} P_s^{do} = P_s^{co}$. Choosing country o as the numeraire country, we can associate an index number, P_s^{co},

[13] This type of price aggregator is often called a price "index" in the trade literature (e.g. Anderson and van Wincoop, 2004). We reserve the term "index" here for price comparisons between countries, in accordance with terminology employed in the index number literature.

with each country c, noting that P_s^{cd} can always be recovered from the ratio P_s^{co}/P_s^{do}. In particular, the value of this ratio is independent of which country is chosen as the numeraire.

The Impure Price Index can be decomposed into an index of quality and an index of pure prices:

$$P_s^{cd} = \widetilde{P}_s^{cd}\lambda_s^{cd}, \qquad \lambda_s^{cd} = \frac{\lambda_s^c}{\lambda_s^d}, \qquad \widetilde{P}_s^{cd} = \frac{\widetilde{P}_s^c}{\widetilde{P}_s^d} = \left[\frac{\sum_z \overline{n}_z \left(\widetilde{p}_z^c\right)^{1-\sigma_s}}{\sum_z \overline{n}_z \left(\widetilde{p}_z^d\right)^{1-\sigma_s}}\right]^{\frac{1}{1-\sigma_s}} \tag{7}$$

The Quality Index, λ_s^{cd}, between countries c and d in sector s is simply defined as the ratio of the two countries' quality levels. The Impure Price Index and the Quality Index implicitly define the Pure Price Index, $\widetilde{P}_s^{cd}$. The Pure Price Index is a summary measure of pure price variation between countries, and it is also transitive. Combining estimates of countries' sectoral Impure Price Indexes with inferences about their sectoral Pure Price Indexes derived from their global net trade, we use the decomposition in (7) to identify countries' sectoral relative product quality.

3. Bounding the "Impure" Price Index

The bilateral Impure Price Index defined in the previous section cannot be observed because it depends upon unobservables such as the number of varieties exported by the country pair and the elasticity of substitution. In this section we outline a set of assumptions which allow the Impure Price Index to be bounded by observable Paasche and Laspeyres indexes defined over the two countries' common exports to a third country. In Section 5, we demonstrate how overlapping bilateral bounds across country pairs can be used to identify Impure Price Indexes for all countries (relative to a numeraire country).

3.1. Constrained Expenditure Function

In this section, we focus on countries' exports to a single "common importer", which we refer to as the United States given the focus of our empirical examination below. The analysis would be identical were it to be applied to any other common importer.

We define a country as "active" in product z if it reports positive exports to the United States in that category. Let I_s be the set of all product categories in sector s, and let I_s^c be the subset of active categories in country c. Define vector $\mathbf{p}_s$ to include the U.S. import prices of all active categories in sector s from all countries. Define analogously vectors $\mathbf{q}_s, \mathbf{n}_s, \boldsymbol{\lambda}_s$, and $\boldsymbol{\xi}_s$. A vector of per-variety consumption $\mathbf{x}_s$ is implicitly defined by $\mathbf{q}_s$ and $\mathbf{n}_s$. Finally, stack these vectors across sectors to form vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{n}$, $\boldsymbol{\lambda}$, $\boldsymbol{\xi}$, and $\mathbf{x}$.

Since our methodology is based on comparing import prices (as measured by unit values) across pairs of U.S. trading partners, we need to use notation specific to country pairs. Index countries in a pair of U.S. trading partners by c and d. Denote by I_s^{cd} the set of active categories common to c and d in sector s. Z_s^{cd} is the number of such categories. Denote also by $I_s^{c,-d}$ the set of products in which c is active but not d, by $I_s^{d,-c}$ the set of products in which d is active but not c, and by U_s^{cd} the union of these two sets. Finally,

$\varnothing_s^{cd}$ is the set of products in which neither of the two countries is active. The set I_s can be partitioned into I_s^{cd}, U_s^{cd}, and $\varnothing_s^{cd}$. We can use I_s^{cd} to break each of vectors $\mathbf{p}$ and $\mathbf{q}$ into two components. First, alternatively for each $i = c, d$, $\mathbf{p}_{s(cd)}^i$ and $\mathbf{q}_{s(cd)}^i$ include prices and quantities, respectively, of exports by i of products in categories $z \in I_s^{cd}$. The remaining parts of $\mathbf{p}$ and $\mathbf{q}$ are denoted by $\mathbf{p}_{s(cd)}^{-i}$ and $\mathbf{q}_{s(cd)}^{-i}$. These vectors include categories $z \in I_s^{cd}$ exported by all countries other than i, and also categories $z \notin I_s^{cd}$ exported by all countries (including i).[14]

For a pair of exporting countries c and d, we now define the constrained expenditure (or import) function $m_{s(cd)}^c(\mathbf{p}_{s(cd)}^i, \mathbf{q}_{s(cd)}^{-c}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U)$. This function represents the minimum expenditure that the representative consumer in the U.S. would be required to spend on varieties exported by country c in categories $z \in I_s^{cd}$ in order to attain utility level U when import prices of those varieties are $\mathbf{p}_{s(cd)}^i$, if this consumer is constrained to consume quantities $\mathbf{q}_{s(cd)}^{-c}$ of all other products, and the number of varieties, quality, and product shifters are, respectively, $\mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}$. The constrained expenditure function solves the problem

$$\min_{q_{s(cd)}^c} \mathbf{p}_{s(cd)}^i \mathbf{q}_{s(cd)}^c \qquad s.t. \quad U(\mathbf{q}_{s(cd)}^c, \mathbf{q}_{s(cd)}^{-c}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}) = U, \quad i = c, d \tag{8}$$

where $U(.)$ is the representative consumer utility function.[15]

By revealed preference, the minimum import expenditure on products produced by country c in categories $z \in I_s^{cd}$, when import prices of those products are $\mathbf{p}_{s(cd)}^c$ while $\mathbf{q}_{s(cd)}^{-c}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}$, and U take their unconstrained equilibrium values, is the observed amount of imports:

$$m_{s(cd)}^c(\mathbf{p}_{s(cd)}^c, \mathbf{q}_{s(cd)}^{-c}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U) = \mathbf{p}_{s(cd)}^c \mathbf{q}_{s(cd)}^c. \tag{9}$$

However, when prices are $\mathbf{p}_{s(cd)}^d$ instead of $\mathbf{p}_{s(cd)}^c$, the minimum import expenditure is equal to or lower than $\mathbf{p}_{s(cd)}^d \mathbf{q}_{s(cd)}^c$, because the amount $\mathbf{p}_{s(cd)}^d \mathbf{q}_{s(cd)}^c$ is sufficient to attain utility U but $\mathbf{q}_{s(cd)}^c$ is not necessarily optimal given $\mathbf{p}_{s(cd)}^d$. Hence

$$m_{s(cd)}^c(\mathbf{p}_{s(cd)}^d, \mathbf{q}_{s(cd)}^{-c}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U) \leq \mathbf{p}_{s(cd)}^d \mathbf{q}_{s(cd)}^c. \tag{10}$$

Taking the ratio of (9) over (10), we obtain

$$M_{s(cd)}^c = \frac{m_{s(cd)}^c(\mathbf{p}_{s(cd)}^c, \mathbf{q}_{s(cd)}^{-c}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U)}{m_{s(cd)}^c(\mathbf{p}_{s(cd)}^d, \mathbf{q}_{s(cd)}^{-c}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U)} \geq \frac{\mathbf{p}_{s(cd)}^c \mathbf{q}_{s(cd)}^c}{\mathbf{p}_{s(cd)}^d \mathbf{q}_{s(cd)}^c} = H_s^{cd}. \tag{11}$$

Equation (11) displays a standard result in index number theory stating that the cost-of-utility price index $M_{s(cd)}^c$ is larger than a Paasche price index, H_s^{cd}, defined here in a cross-sectional rather than a time-series context. The left hand side of (11), $M_{s(cd)}^c$, captures the change in minimum expenditure on country c's varieties (in categories $z \in I_s^{cd}$) that would

[14] The term in parenthesis in the subindex denotes the subset of products within sector s in which countries c and d export in common to the U.S., i.e. $\{z : z \in I_s^{cd}\}$.

[15] Neary and Roberts (1980) and Anderson and Neary (1992) use the constrained expenditure function to analyze consumption choices under rationing.

be necessary to maintain utility U, if import prices of those varieties changed from $\mathbf{p}^d_{s(cd)}$ to $\mathbf{p}^c_{s(cd)}$, holding constant their number and characteristics (including quality), and the number, characteristics and quantity consumed of all other goods. The right hand side of (11), H^{cd}_s, is a Paasche price index defined over the observed prices of the country pair's common exports to the U.S. in sector s.

Similarly, we can focus on imports from country d to obtain

$$M^d_{s(cd)} = \frac{m^d_{s(cd)}(\mathbf{p}^c_{s(cd)}, \mathbf{q}^{-d}_{s(cd)}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U)}{m^d_{s(cd)}(\mathbf{p}^d_{s(cd)}, \mathbf{q}^{-d}_{s(cd)}, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U)} \leq \frac{\mathbf{p}^c_{s(cd)}\mathbf{q}^d_{s(cd)}}{\mathbf{p}^d_{s(cd)}\mathbf{q}^d_{s(cd)}} = L^{cd}_s, \tag{12}$$

where L^{cd}_s is a Laspeyres price index defined over the country pair's common exports to the U.S. in sector s. This is another standard result, which states that the cost-of-utility index $M^d_{s(cd)}$ is bounded from above by a Laspeyres price index.[16]

Given that the Cobb-Douglas form assumed for the upper tier of the utility function is separable into sectoral CES subutility indexes u_s, the constraint in problem (8) can be rewritten as

$$U(\mathbf{q}^c_s, \mathbf{q}^{-c}_s, \mathbf{n}, \boldsymbol{\lambda}, \boldsymbol{\xi}) = \prod_{s'} u_{s'}^{b_{s'}} = U. \tag{13}$$

The value of all subutility indexes for sectors other than s are constant, as their arguments are held constant in problem (8). Therefore, constraint (13) determines the minimum value of u_s that is required to attain utility U, conditional on the (fixed) value of the subutility indexes for the other sectors:

$$u_s = \frac{U}{\prod\limits_{s' \neq s} \overline{u}_{s'}^{b'_s}} \tag{14}$$

Since we focus on expenditure (imports) only on varieties produced by country c in categories $z \in I^{cd}_s$, it is convenient to rewrite the subutility function for sector s as

$$u_s = \left[\sum_{z \in I^{cd}_s} n^c_z \left(\xi_z \lambda^c_s x^c_z\right)^{\varphi_s} + \widehat{u}_s\right]^{1/\varphi_s}, \quad \widehat{u}_s = \sum_{z \notin I^{cd}_s} n^c_z \left(\xi_z \lambda^c_s x^c_z\right)^{\varphi_s} + \sum_{k \neq c} \sum_{z \in I_s} n^k_z \left(\xi_z \lambda^k_s x^k_z\right)^{\varphi_s}. \tag{15}$$

The first term on the right-hand side of this expression represents the utility from categories imported from country c in sector s that are not also imported from country d. The second term captures the utility from goods imported from all other countries (including d) in any category in sector s. Substituting (15) into (14), and after some algebra, we obtain

$$\left[\sum_{z \in I^{cd}_s} n^c_z \left(\xi_z \lambda^c_s x^c_z\right)^{\varphi_s}\right]^{1/\varphi_s} = \left[\left(\frac{U}{\prod\limits_{s' \neq s} u_{s'}^{b'_s}}\right)^{\varphi_s} - \widehat{u}_s\right]^{1/\varphi_s} \equiv u^*_s.$$

[16]Paasche and Laspeyres indexes are typically defined in a time series context, where there is a natural ordering of time periods. Since there is no natural ordering of countries in a multilateral context, calling one of these indexes Paasche and the other one Laspeyres rather than *vice versa* is arbitrary.

Then, we can rewrite the problem in equation (8) that defines the constrained expenditure function as

$$\min_{x_z^c} \quad \sum_{z\in I_s^{cd}} n_z^c p_z^i x_z^c \quad s.t \quad \left[\sum_{z\in I_s^{cd}} n_z^c \left(\xi_z \lambda_s^c x_z^c\right)^{\varphi_s}\right]^{1/\varphi_s} = u_s^*, \quad i = c, d.$$

The solution to this problem is the product between a CES aggregator measuring the unit cost of utility and the target level of utility, u_s^*[17]

$$m_{s(cd)}^c(\mathbf{p}_s^i, \mathbf{q}_s^{-c}, \boldsymbol{\lambda}, \boldsymbol{\xi}, U) = \left[\sum_{z\in I_s^{cd}} n_z^c \left(\widetilde{p}_z^i \frac{\lambda_s^i}{\lambda_s^c}\right)^{1-\sigma_s}\right]^{\frac{1}{1-\sigma_s}} u_s^*. \tag{16}$$

We can now obtain an explicit expression for $M_{s(cd)}^c$ in equation (11):

$$M_{s(cd)}^c = \left[\frac{\sum\limits_{z\in I_s^{cd}} n_z^c \left(\widetilde{p}_z^c\right)^{1-\sigma_s}}{\sum\limits_{z\in I_s^{cd}} n_z^c \left(\widetilde{p}_z^d \frac{\lambda_s^d}{\lambda_s^c}\right)^{1-\sigma_s}}\right]^{\frac{1}{1-\sigma_s}} = \widetilde{P}_s^{cd} \lambda_s^{cd} \left[\frac{\sum\limits_{z\in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c}\right)^{1-\sigma_s}}{\sum\limits_{z\in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^d}{\widetilde{P}_s^d}\right)^{1-\sigma_s}}\right]^{\frac{1}{1-\sigma_s}} \tag{17}$$

Taking logarithms on both sides of (17) and using the fact that $P_s^{cd} = \widetilde{P}_s^{cd} \lambda_s^{cd}$, we can combine this equation with (11) to obtain

$$\ln H_s^{cd} \leq \ln M_{s(cd)}^c = \ln P_s^{cd} + \ln \phi_s^c, \qquad \phi_s^c = \left[\frac{\sum\limits_{z\in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c}\right)^{1-\sigma_s}}{\sum\limits_{z\in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^d}{\widetilde{P}_s^d}\right)^{1-\sigma_s}}\right]^{\frac{1}{1-\sigma_s}}. \tag{18}$$

Similarly, an expression analogous to (17) can be obtained for $M_{s(cd)}^d$, which combined with (12) yields[18]

$$\ln L_s^{cd} \geq \ln M_{s(cd)}^d = \ln P_s^{cd} + \ln \phi_s^d, \qquad \phi_s^d = \left[\frac{\sum\limits_{z\in I_s^{cd}} n_z^d \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c}\right)^{1-\sigma_s}}{\sum\limits_{z\in I_s^{cd}} n_z^d \left(\frac{\widetilde{p}_z^d}{\widetilde{P}_s^d}\right)^{1-\sigma_s}}\right]^{\frac{1}{1-\sigma_s}}. \tag{19}$$

[17] It is here where Assumptions 1 and 2 are critical. In equation (16) we use these assumptions to derive $\frac{p_z^i}{\lambda_z^c \xi_z^c} = \frac{p_z^i}{\lambda_z^i \xi_z^i} \frac{\lambda_z^i \xi_z^i}{\lambda_z^c \xi_z^c} = \widetilde{p}_z^i \frac{\lambda_s^i}{\lambda_s^c}$, $i = c, d$.

[18] Note that all prices (observed and pure) considered up to now in this section are import prices. Under the assumption that trade costs constant across product categories within a sector (see Section 4), the indexes $M_{s(cd)}^c, M_{s(cd)}^d, H_s^{cd}, L_s^{cd}$ can be alternatively defined in terms of export prices if they are appropriately scaled by relative trade costs between countries c and d and the United States. As a result, the inequalities in equations (18) and (19) also hold if the indexes are defined over export prices. We use the latter definition for the indexes in the remainder of the paper.

Equations (18) and (19) relate the implications of consumer cost minimization to cross-sectional Paasche and Laspeyres price indexes, where each of the cost-of utility indexes has observable bounds on just one side. Our consideration of two cost-of-ultility indexes, as well as the one-sidedness of their bounds, differs from the standard bounding of cost-of-utility indexes from both above *and* below found in the index number literature. Here, since we allow for horizontal differentiation, we must deal with two cost-of-utility indexes because $M^c_{s(cd)}$ and $M^d_{s(cd)}$ are defined over different numbers of varieties, i.e., n^c_z and n^d_z, respectively.[19] As a result, ϕ^c_s and ϕ^d_s are also different. Under plausible assumptions described below, however, we can show that $\ln \phi^c_s < 0$ and $\ln \phi^d_s > 0$, which implies that the Paasche and Laspeyres indexes bound the Impure Price Index, i.e., $\ln H^{cd}_s \leq \ln M^c_{s(cd)} \leq \ln P^{cd}_s \leq \ln M^d_{s(cd)} \leq \ln L^{cd}_s$.

3.2. Paasche and Laspeyres Bounds on the Impure Price Index

Before describing the main result of this section, we develop additional notation specific to country pairs.

For each pair of countries c and d, define the pair's (o–normalized) average number of varieties in product category z:

$$\widehat{n}^{cd}_z = \frac{1}{2}\left(\frac{\overline{n}^o_s}{\overline{n}^c_s} n^c_z + \frac{\overline{n}^o_s}{\overline{n}^d_s} n^d_z\right), \tag{20}$$

and the country pair's (o–normalized) "multilateral excess variety" in product z relative to the world average:

$$\widetilde{\widetilde{n}}^{cd}_z = \widehat{n}^{cd}_z - \overline{n}_z. \tag{21}$$

Multilateral excess variety measures the extent to which the average number of varieties in countries c and d is above or below the world average.

Also, for each country $i = c, d$ in the country pair, define i's (o-normalized) "bilateral excess variety" in product z relative to the country-pair average,

$$\widetilde{n}^{i,cd}_z = \frac{\overline{n}^o_s}{\overline{n}^i_s} n^i_z - \widehat{n}^{cd}_z. \tag{22}$$

Bilateral excess variety measures the extent to which the number of varieties in a country is above or below the bilateral average. These measures of excess variety possess three convenient properties:

$$\sum_z \widetilde{n}^{i,cd}_z = 0, \qquad \sum_z \widetilde{\widetilde{n}}^{cd}_z = 0, \qquad \widetilde{n}^{c,cd}_z = -\widetilde{n}^{d,cd}_z \tag{23}$$

The first and second properties indicate that, across product categories within country i, both bilateral and multilateral excess variety sum to zero. The third property reveals that two countries cannot both have positive bilateral excess variety in the same category.

[19] $M^c_{s(cd)}$ and $M^d_{s(cd)}$ would be equal, for example, if the number of varieties in countries c and d were proportional to one another for every product category.

Define the bilateral difference in two countries' pure prices in product category z relative to their countries' pure price aggregator as

$$\Delta\widetilde{p}_z^{cd} = \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c}\right)^{1-\sigma_s} - \left(\frac{\widetilde{p}_z^d}{\widetilde{P}_s^d}\right)^{1-\sigma_s}. \tag{24}$$

A positive $\Delta\widetilde{p}_z^{cd}$ indicates that country c has a lower pure price of z (relative to the price aggregator) than country d. A lower pure price may arise, for example, due to comparative advantage, i.e., variation in exporters' relative production efficiency or factor costs.

Finally, for set of products A, define the sample covariance over that set of products as $cov_A(x,y) = (1/Z_A)\sum_{z\in A}(x_z - \overline{x})(y_z - \overline{y})$, where Z_A is the number of elements in A.

We now lay out a set of sufficient conditions for the Impure Price Index to be bounded by observable Paasche and Laspeyres price indexes.

Assumption 3 states that country c relative to country d will tend to have positive bilateral excess variety in those products in which it has a lower relative pure price.

Assumption 3: $cov_{I_s^{cd}}\left(\widetilde{n}_z^{c,cd}, \Delta\widetilde{p}_z^{cd}\right) = cov_{I_s^{cd}}\left(\widetilde{n}_z^{d,cd}, \Delta\widetilde{p}_z^{dc}\right) \geq 0$

This assumption is based on the results of theoretical models of international trade with product differentiation that do not assume factor price equalization (e.g., Romalis 2004, Bernard et al. 2005). These models find that, across goods, the relative number of varieties between two countries is a negative function of the countries' relative prices. This finding supports the intuitive notion that countries should have a relatively higher (lower) number of firms in sectors or product categories in which they are relatively more (less) competitive, i.e. those sectors with relatively lower (higher) prices. It is possible to reformulate these models in terms of quality-adjusted variables. Thus reinterpreted, these models predict that the relative number of varieties in a sector or product category is a negative function of relative pure (or quality-adjusted) prices.

Assumption 4 imposes the restriction that there is no correlation between the country-pair's multilateral excess variety and bilateral differences in pure relative prices.

Assumption 4: $cov_{I_s}\left(\widetilde{\widetilde{n}}_z^{cd}, \Delta\widetilde{p}_z^{cd}\right) = 0$

This assumption is not very strong, as there is no obvious relationship between the country pair's excess variety relative to the world average and relative comparative advantage among countries *within* the pair.

Assumption 5 requires that countries c and d be similarly active in exporting goods to the United States.

Assumption 5: $\delta_s^{cd} = \delta_s^{dc} = 0$, where

$$\delta_s^{cd} = \frac{\sum_{z\in U_s^{cd}} \widetilde{n}_z^{c,cd} \frac{1}{Z_s^{cd}} \sum_{z\in I_s^{cd}} \Delta\widetilde{p}_z^{cd} + \sum_{z\in U_s^{cd}} \widehat{n}_z^{cd} \Delta\widetilde{p}_z^{cd}}{\sum_{z\in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^d}{\widetilde{P}_s^d}\right)^{1-\sigma_s}}, \text{ and}$$

δ_s^{dc} is defined analogously.

The magnitude of the terms δ_s^{cd} and δ_s^{dc} depends on the extent to which countries c and d are "similarly active". Assumption 5 requires that these terms are zero. A sufficient condition that implies assumption 6 is that the two countries are active in the same categories. In that case, the numerator in the expression for δ_s^{cd} is zero, as it sums over elements of an empty set, U_s^{cd}. Since the sums in the numerator involve positive and negative terms, it is still possible that the numerator is zero even if U_s^{cd} is non-empty. More generally, δ_s^{cd} and δ_s^{dc} will tend to be smaller (in absolute magnitude) the smaller is the number of mismatched active categories (in the numerator) relative to the number of matched active categories (in the denominator). Also, since $\Delta\widetilde{p}_z^{cd} > 0$ and $\widetilde{n}_z^{c,cd} > 0$ for $z \in I_s^{c,-d}$, and $\Delta\widetilde{p}_z^{cd} < 0$ and $\widetilde{n}_z^{c,cd} < 0$ for $z \in I_s^{d,-c}$, these terms will tend to be smaller the more similar is the number of products in $I_s^{c,-d}$ to the number of products in $I_s^{d,-c}$.

With assumptions 3, 4 and 5 as well as our earlier assumptions about consumer utility, Proposition 1 demonstrates that a country pairs' unobservable Impure Price Index is bounded by the observable Paasche and Laspeyres indexes defined over their common exports to a third country.

Proposition 1 *Under Assumptions 1 through 5, for any two countries c and d, the (unobservable) Impure Price Index is bounded by the (observable) Paasche and Laspeyres indexes:*

$$\ln H_s^{cd} \leq \ln P_s^{cd} \leq \ln L_s^{cd}$$

Proof. See Appendix. ■

This finding provides the basis for our estimation of the Impure Price Index in the first-stage of our empirical strategy.

4. Net Trade as Indicator of Pure Price Variation

This section derives the theoretical relationship between countries net trade and their Pure Price Indexes.

4.1. Net trade as a function of pure prices

Exporting goods from country c to country c' requires paying iceberg trade costs of $\tau_s^{cc'}$. Therefore, $p_z^c \tau_s^{cc'}$ is the import price of product z in country c'. Given the CES preference structure assumed in (1), it is easy to derive country c's bilateral export and import flows (in sector s) with every other country c'. Summing export flows over $c' \neq c$, we can obtain the value of country c's exports,

$$Exports_s^c = \sum_{c' \neq c} \left[\sum_z \frac{n_z^c \left(\widetilde{p}_z^c \tau_s^{cc'} \right)^{1-\sigma_s}}{\left(G_s^{c'} \right)^{1-\sigma_s}} \right] b_s Y^{c'} \tag{25}$$

where $Y^{c'}$ is the income of country c', $\sigma_s = 1/(1-\varphi_s) > 1$ is the elasticity of substitution, and

$$G_s^{c'} = \left[\sum_{c''} \sum_z n_z^{c''} \left(\widetilde{p}_z^{c''} \tau_s^{c''c'} \right)^{1-\sigma_s} \right]^{1/(1-\sigma_s)} \tag{26}$$

is a consumption-based price aggregator capturing the impact of trade barriers on consumers in country c'. Note that the exports of country c are decreasing in $\tau^{cc'}$ and increasing (via increases in $G_s^{c'}$) in the cost of shipping goods to country c' from all other countries c''. The expression in brackets in equation (25) is country c's share in country c''s sectoral expenditure, $b_s Y^c$. This share does not depend on prices and quality levels independently of one another, but only on the ratio of the two, $\widetilde{p}_z^c$.[20]

In a similar manner, we can obtain the value of country c's imports,

$$Imports_s^c = \sum_{c' \neq c} \left[\sum_z \frac{n_z^{c'} \left(\widetilde{p}_z^{c'} \tau_s^{c'c} \right)^{1-\sigma_s}}{(G_s^c)^{1-\sigma_s}} \right] b_s Y^c = \left[1 - \sum_z \frac{n_z^c \left(\widetilde{p}_z^c \right)^{1-\sigma_s}}{(G_s^c)^{1-\sigma_s}} \right] b_s Y^c. \tag{27}$$

Subtracting equation (27) from equation (25), we obtain country c's global net trade in sector s, T_s^c, as a proportion of its expenditure in the sector,

$$\frac{1}{b_s} \frac{T_s^c}{Y^c} = -1 + \sum_{c'} \sum_z \frac{n_z^c \left(\widetilde{p}_z^c \tau_s^{cc'} \right)^{1-\sigma_s}}{(G_s^{c'})^{1-\sigma_s}} \frac{Y^{c'}}{Y^c}, \tag{28}$$

Equation (28) shows that countries' trade balance in sector s is a function of all the product-level pure prices in that sector. Our objective is to simplify this expression by relating net trade of country c in sector s to the Pure Price Index.

4.2. Net trade as a function of the Pure Price Index

To express equation (28) as a function of the Pure Price Index, we must impose structure on the relationship between pure prices and number of varieties countries produce. Note, however, that our methodology does not require that we identify the economic forces that determine pure prices in equilibrium. Variation in pure prices can be driven by traditional sources of comparative advantage, or it can be the result of macroeconomic conditions, such as over- or under-valued currencies.

Based on the same theoretical results that motivate Assumption 3, we postulate a similar negative relationship between number of varieties and pure prices, defined here across sectors rather than across categories within sectors.

Assumption 6: $\frac{\overline{n}_s^c / Y^c}{\overline{n}_s^o / Y^o} = \left(\widetilde{P}_s^{co} \right)^{-\eta_s}, \quad \forall c = 1, ..., C, \qquad \eta_s \geq 0.$

[20] We can associate and infinite price $\widetilde{p}_z^c$ with a product z that is not produced in country c. Since pure prices are elevated to a negative exponent, this product will have no effect on the volume of trade or the price aggregator.

A particular case of this assumption is when $\eta_s = 0$, in which case the average number of varieties in a sector is a constant proportion of income. More generally, the number of varieties here is allowed to decrease as pure prices increase. We also characterize the relationship between pure prices and number of varieties across product categories within sectors as the sum of a common component across countries (V_s), and a mean-zero, country-specific idiosyncratic component

$$cov\left[\widetilde{n}_z^c, \left(\widetilde{p}_z^c/\widetilde{P}_s^c\right)^{1-\sigma_s}\right] = V_s + \theta_s^c, \tag{29}$$

but we do not need to impose assumptions on this covariance.

The objective of this section is to derive an expression relating net trade at the sectoral level to the value of the Pure Price Index. Since net trade also depends on trade costs, we also want this expression to depend on summary measures of trade costs in the sector. To that end, we define some additional variables. Let $y^c = Y^c / \sum_{c'} Y^{c'}$ be the share of country c in world income, and let $r_s^c = \left(1/G_s^{1-\sigma_s}\right) \sum_z n_z^c \left(\widetilde{p}_z^c\right)^{1-\sigma_s}$ be the share of country c in the term $\left(G_s\right)^{1-\sigma_s} = \sum_{c''} \sum_z n_z^{c''} \left(\widetilde{p}_z^{c''}\right)^{1-\sigma_s}$, which is common for all countries and is thus denoted omitting the country superscript. In the free-trade equilibrium with those pure prices and number of varieties, r_s^c is also the share of country c in world expenditure (in sector s). We define a summary measure of "outbound" trade costs for country c as

$$\tau_s^{c,out} = \sum_{c'} y^{c'} \left(\tau_s^{cc'} - 1\right). \tag{30}$$

The outbound average trade cost is a weighted average, across countries, of the bilateral costs of exporting from country c to other countries (including itself).

The following Proposition describes the main result of this section.

Proposition 2 *Under Assumption 6, country c's sectoral net trade can be approximated (via a Taylor expansion) as a linear function of the Pure Price Index and this country's outbound average trade costs,*

$$T_s^c / Y^c \simeq \Psi_s + \gamma_s \ln \widetilde{P}_s^{co} + \gamma_s \mu_s \tau_s^{c,out} + b_s Z_s \theta_s^c, \tag{31}$$

$$\gamma_s = (1 - \sigma_s - \eta_s) b_s < 0, \qquad \Psi_s = b_s \left[\ln\left(Y^o\right) + \ln\left(\widetilde{P}_s^o\right)^{1-\sigma_s} + A_s + Z_s V_s\right],$$

$$\mu_s = \frac{(\sigma_s - 1)}{(\sigma_s + \eta_s - 1)} > 0, \qquad A_s = \ln \sum_{c'} \frac{Y^{c'}}{\left(G_s^{c'}\right)^{1-\sigma_s}} + (\sigma_s - 1) \sum_{c'} \sum_{c''} r_s^{c''} y^{c'} \left(\tau_s^{c''c'} - 1\right)$$

Proof. See Appendix. ■

Proposition 2 provides a simple expression for the relationship between net trade, pure prices and trade costs. It formalizes the idea that the surplus in a country's sectoral net trade should be larger the lower are its pure prices. In addition to pure prices, trade costs also influence net trade. In particular, conditional on pure prices, higher outbound trade barriers for country c imply a more negative trade balance.

Equation (31) does not include an expression for inbound average trade costs due to our use of a Taylor approximation around a free-trade equilibrium. For intuition regarding the absence of inbound trade costs, suppose that under free trade country c imposes a bilateral tariff on the imports from one country. That tariff has an obvious negative impact on the net trade of the targeted country, which is captured by an increase in the targeted country's outbound average tariff. On the other hand, the imposition of this tariff raises the market share of all other countries selling in the domestic market (including c) – via an increase in the price index G_s^c. Note that country c benefits from its tariff increase as much as it would benefit from a tariff increase in any other country with the same income. This symmetry is due to the fact that under free trade country c has the same market share in each country. For the same reason, foreign countries benefit from country c's tariff as much as they would benefit from a tariff increase in any other country with the same income (including themselves). The improvement in the non-targeted countries' net trade is captured by an increase in the term A_S, which is constant across countries. A_s increases for the targeted country as well, but in this case the increase in net trade is more than offset by the negative impact of the tariff (via an increase in the outbound average trade cost).[21]

The effect of trade costs on net trade characterized in Proposition 2 are "conditional on pure prices". This implies that, while they appropriately adjust the relationship between net trade and pure prices, they do not provide a comparative statics assessment of the impact of trade costs on net trade. Changes in those costs will typically affect pure prices in general equilibrium, implying an indirect effect on net trade not captured in equation (31).

Equation (31) can be interpreted as a relative demand function, where net trade is the "quantity" variable, the Pure Price Index is the "price" variable, and the trade costs are demand shifters. The first term captures movements along the demand curve: higher pure prices of country c in sector s are associated with a worsening of this country's net trade position in that sector. The second term captures movements of the demand curve. Conditional on pure prices, higher outbound trade costs shift this curve to the left.

Before concluding this section, we note that our assumptions of constant quality and elasticities of substitution across product categories within sectors highlight an aggregation trade-off in our methodology. While these assumptions are more likely to be satisfied for more disaggregate sectors, data on countries' global net trade becomes more scarce, and measurement error likely increases, with disaggregation.

5. Empirical Implementation and Results

In this section we use the results of Propositions 1 and 2 to estimate product quality for the United States' top trading partners. Our estimation strategy proceeds in two stages. In the first stage, based on the results of Section 3, we use data on export unit values and quantities to derive an estimate of each country's Impure Price Index. In the second stage, using the results of Section 4, we use information on countries' global net trade and trade

[21] Away from the free-trade equilibrium, inward tariffs should have a higher impact on the net trade of the country imposing it, as this country commands a higher relative market share in the domestic market. In our empirical analysis, we control for inbound tariffs to capture this effect.

costs to infer movement in countries' pure prices, and strip these movements away to extract estimates of product quality from the first-stage results. We begin by describing our data sources and outlining our estimation strategy. We then present Quality Index estimates for the "All Manufacturing" sector.

5.1. Data

The first stage of our estimation requires product-level export prices for every country. These prices are derived from product-level U.S. import data available from the U.S. Census Bureau and compiled by Feenstra et al. (2002). The database records the customs value of all U.S. imports by source country from 1972 to 2001. Imports are recorded according to thousands of finely detailed seven-digit Tariff System of the United States (TSUSA) categories (1974 to 1988) and ten-digit Harmonized System (HS) categories (1989 to 2001). We focus here on products in All Manufacturing, i.e., products in SITC aggregates 5 through 8.

The U.S. trade data include information on both quantity and value for many goods. We compute the unit value, or "price", of product z from country c, p_z^c, by dividing import value (v_z^c) by import quantity (q_z^c), $p_z^c = v_z^c/q_z^c$.[22] Examples of the units employed to classify products include dozens of shirts in apparel, square meters of carpet in textiles and pounds of folic acid in chemicals.

Product-level trade data are noisy due to both aggregation bias and measurement error.[23] Aggregation bias is minimized by using detailed data, but is likely to remain. We therefore trim the data along two dimensions before using them to compute Paasche and Laspeyres indexes. The first trim involves dropping country-year-product observations with value less than \$10,000 or quantity equal to 1. The second trim eliminates country-pair-year-product observations when the relative quantity or the relative price of the country-pair-product is either below the 2nd percentile or above the 98th percentile of all country-pair-product observations in that year. The first trim gets rid of unusual and unrealistic imports while the second trim discards unreliable country comparisons.

The second stage of our estimation requires measures of trade balance and trade costs at the sectoral level. We measure countries' sectoral trade balance relative to GDP by dividing nominal dollar-denominated trade flow data from the World Trade Flows database compiled by Feenstra et al. (2004) with GDP data from the World Bank's World Development Indicators database. For the real exchange rate we rely on version 6.1 of the Penn World Tables (i.e., PPP/XRAT).

Ideally, our estimates of trade costs between countries would include measures of transportation costs, tariffs and non-tariff barriers as well as other costs due to language barriers, etc. Here, due to data constraints, we focus on the former.[24] We measure bilateral transport costs using the U.S. import data, which records both the customs-insurance-freight (cif) and

[22] Availability of unit values averages about 80 percent over the years in our sample.

[23] See, for example, GAO (1995) and Schott (2004).

[24] Our technique will benefit from the ongoing development of datasets such as TRAINS that record estimates of countries' tariff and non-tarriff barriers. Though we are exploring the use of TRAINS in our estimation, the sparseness of its coverage prior to the late 1990s severely restricts the sample size of the second stage of our estimation.

free-on-board (fob) value for most import flows. We estimate *ad valorem* transport costs per mile for industry s in year t by regressing the relative value spent on customs, insurance and freight on imports from country c on the distance the exports have travelled,

$$\frac{cif_{st}^{c} - fob_{st}^{c}}{fob_{st}^{c}} = \delta_{st} D^{c,US} + \in_{st}^{c} \tag{32}$$

where $D^{c,US}$ represents the great circle distance in miles between the United States and country c. In our estimations below, we set τ_{st}^{cd} equal to $\widehat{\delta}_{st} D^{cd}$. For each country, we compute average outbound trade costs by weighting destination countries according to their share of world GDP. We also calculate average inbound trade costs as $\tau_s^{c,in} = \sum_{c'} w_s^{c'} \left(\tau_s^{c'c} - 1\right)$, where we weight source countries according to their share $w_s^{c'}$ of world exports in industry s.

We report quality estimates for the top 45 non-OPEC U.S. trading partners for the period 1980 to 1997. This sample was chosen to yield a relatively long and balanced panel. We exclude years prior to 1980 because trade is dominated by a relatively small group of high-income countries. We exclude years after 1997 because of significant outliers in the trade balance data between 1998 and 2001.[25]

5.2. *Estimation Strategy*

5.2.1. *First Stage: Estimation of the Impure Price Index*

In the first stage of the estimation strategy, we use the results of Proposition 1 to estimate each country's Impure Price Index, $\widehat{P}_s^{co}$, where country o is the numeraire country.[26] The idea of the identification strategy is as follows. For generic country pair c and d, the estimated indexes $\widehat{P}_s^{co}$ and $\widehat{P}_s^{do}$ implicitly determine a bilateral index $\widehat{P}_s^{cd} = \widehat{P}_s^{co}/\widehat{P}_s^{do}$. This index should satisfy the Paasche and Laspeyres bounds for that country pair, as outlined in Proposition 1. Similarly, for C trading partners, the estimation of $C-1$ Impure Price Indexes, $\widehat{P}_s^{co}$ $\forall c \neq o$, implicitly determine $C(C-1)$ bilateral indexes, $\widehat{P}_s^{cd}$ $\forall c, d$, which should satisfy the bilateral Paasche and Laspeyres price index bounds for all country pairs. If the Paasche and Laspeyres bounds were observed without error, estimation would entail searching for an interior solution to the set of restrictions imposed by the bounds across country pairs. Here, in light of evidence that import data (mainly quantities) are mis-recorded on customs documents (GAO 1995), we instead allow for the possibility that the true Paasche and Laspeyres indexes are observed with error.

Denote the "true" Paasche and Laspeyres indexes by H_s^{*cd} and L_s^{*cd}, respectively. We assume that the observed indexes, H_s^{cd} and L_s^{cd}, vary from the true indexes by a multiplicative error, $\ln H_s^{cd} = \ln H_s^{*cd} + \zeta_{h,s}^{cd}$ and $\ln L_s^{cd} = \ln L_s^{*cd} + \zeta_{l,s}^{cd}$. We also assume that each error is distributed normally, $\zeta_{h,s}^{cd} \sim N(0, \psi/w_s^{cd})$ and $\zeta_{l,s}^{cd} \sim N(0, \psi/w_s^{cd})$, and that the errors

[25] We are currently investigating these outliers and plan to extend the analysis to 2001 once they are verified.

[26] The choice of numeraire is made without loss of generality. In the results presented below, Switzerland (CHE) is the numeraire.

for each bound are independent both of each other and of error terms for other bilateral pairs.[27] Note that we weight the standard deviation of the error distribution by w_s^{cd}. In the results below, this weight is set equal to the square root of the number of categories that countries c and d export in common to the United States. This weight is meant to increase the contribution of country pairs with a relatively large number of exports in common.

Satisfying the inequality constraints of Proposition 2 for a given pair of countries implies:

$$\ln P_s^{cd} \geq \ln H_s^{*cd} \Longrightarrow \zeta_{h,s}^{cd} \geq \ln H_s^{cd} - \ln P_s^{cd} \tag{33}$$

$$\ln P_s^{cd} \leq \ln L_s^{*cd} \Longrightarrow \zeta_{l,s}^{cd} \leq \ln L_s^{cd} - \ln P_s^{cd}. \tag{34}$$

We estimate the set of index numbers $\widehat{P}_s^{co}$, $\forall c \neq o$, and the variance parameter $\widehat{\psi}$, for a given year t, by maximizing the probability that the "true" Paasche and Laspeyres bounds contain the estimates.

5.2.2. Second Stage: Estimation of Product Quality

Variation in estimates of countries' Impure Price Indexes contains information about pure prices and product quality. Proposition 2 demonstrates that countries' pure prices, as summarized by the Pure Price Index, determine their sectoral trade balance. In the second stage, we use the results of that proposition to strip away the pure-price component of the Impure Price Index. Incorporating $\ln \widetilde{P}_s^{cd} = \ln P_s^{cd} - \ln \lambda_s^{cd}$ from equation (7), and neglecting the error arising from the linear approximation described in the proof of Proposition 2, we can rewrite equation (31) as

$$T_{st}^c/Y_t^c = \Psi_{st} + \gamma_s \ln \widehat{P}_{st}^{co} + \gamma_s \mu_s \tau_{st}^{c,out} - \gamma_s \ln \lambda_{st}^{co} + b_s Z_s \theta_{st}^c + \gamma_s \kappa_{st}^{co} \tag{35}$$

where $\kappa_s^{co} = \ln P_s^{co} - \ln \widehat{P}_s^{co}$ is the estimation error in the first-stage estimates, and subscript t indexes time periods. Equation (35) highlights the fact that countries' unobserved product quality relative to the numeraire country (λ_{st}^{co}) is part of a compound error term that also includes the estimation error in the first stage (κ_{st}^{co}) and the idiosyncratic component of the covariance between excess variety and pure prices (θ_{st}^c) from equation (29). We assume that both κ_{st}^{co} and θ_{st}^c are uncorrelated with $\widehat{P}_s^{co}$. However, assuming that the quality component of the error term ($\ln \lambda_{st}^{co}$) is uncorrelated with the regressor $\ln \widehat{P}_{st}^{co}$ is untenable. Developed countries, which tend to have higher export prices, are also likely to produce higher quality. (This presumption is confirmed later by our results.)

To deal with this endogeneity problem, we first specify a time path for the evolution of product quality relative to the base country:

$$\ln \lambda_{st}^{co} = \alpha_{0s}^{co} + \alpha_{1s}^{co} t + \varepsilon_{st}^{co} \tag{36}$$

[27] Our assumptions about the normality and independence of the errors represent a potentially strong simplification. Errors across country pairs with one country in common are likely to be correlated as they are constructed using similar information. The within-country-pair Paasche and Laspeyres errors are also likely to be correlated: a high negative Paasche error will coincide with a high positive Laspeyres error. We are currently working on relaxing these assumptions.

where α_{0s}^{co} and α_{1s}^{co} are a country fixed effect and the slope of a country-specific time trend, respectively, and ε_{st}^{co} represents deviations of quality from this trend.[28] Incorporating this (country-specific) linear trend for quality into equation (35), we obtain our second-stage estimating equation

$$T_{st}^{c}/Y_{t}^{c} = \Psi_{st} + \gamma_s \ln \widehat{P}_{st}^{co} - \gamma_s \left(\alpha_{0s}^{co} + \alpha_{1s}^{co} t\right) + \gamma_s \mu_s^{out} \tau_{st}^{c,out} + \upsilon_{st}^{co} \tag{37}$$

where $\upsilon_{st}^{co} = \gamma_s(\kappa_{st}^{co} - \varepsilon_{st}^{co}) + b_s Z_s \theta_{st}^{c}$.

The inclusion of country fixed effects in (37) eliminates the most obvious source of endogeneity, i.e. the cross-sectional correlation between the time-invariant components of countries' prices and quality levels. The inclusion of country-specific time trends further reduces the remaining correlation between regressor and error term, as the latter term now only includes deviations of quality from country-specific trends. However, correlation between ε_{st}^{co} and $\widehat{P}_{st}^{co}$ may still persist, as shocks to quality are likely to be accompanied by increases in (impure) prices. To address this potential endogeneity problem, we use the real exchange rate as an instrument for $\widehat{P}_{st}^{co}$. As usual, the instrument needs to satisfy two conditions. First, since the estimating equation includes country-specific fixed effects and time trends, the instrument has to be (partially) correlated with $\widehat{P}_{st}^{co}$, *after controlling for the fixed effects and time trends*. In other words, deviations of the real exchange from its own time trend have to be correlated with similar deviations of $\widehat{P}_{st}^{co}$. Macroeconomic conditions typically determine periods of over- and under-valuation of countries' real exchange rate around long-run trends. These periods also determine changes in the international competitiveness of a countries' exports, captured in our model by $\widetilde{P}_{st}^{co}$. Since $\widetilde{P}_{st}^{co}$ is a component of $\widehat{P}_{st}^{co}$, periods of over- or under-valuation will also be associated with movements of $\widehat{P}_{st}^{co}$. Second, the instrument has to be uncorrelated with the error term ε_{st}^{co}, which requires that shocks to quality around the trend in sector s are not correlated with the real exchange rate. While we cannot rule out that such a correlation exists, we think that it is unlikely to be important. Shocks to quality in sector s might be accompanied by exactly offsetting changes in prices, leaving pure prices – and hence net trade in that sector – unchanged. Even if these shocks affect pure prices, they might have a negligible effect on the real exchange rate. This is more likely to be true if the shocks are temporary deviations around a trend, and if they are specific to sector s, i.e. not correlated with shocks to quality in other sectors.

We estimate equation (37) in first differences using two-stage least squares.[29] As discussed in footnote 21 of Section 4.2., we also include the average inbound trade cost ($\tau_{st}^{c,in}$) as an additional control. Our estimation of countries' trend in export quality over the

[28] The choice of numeraire country is made without loss of generality, as the empirical speficication and the estimated Impure Price Indexes satisfy transitivity. It is only the standard errors on the quality trends that are specific to the difference between the relevant country and the numeraire. However, standard errors for the difference between any country pair can be recovered using the variance-covariance matrix of the estimated coefficients.

[29] We report results for first differences because residuals in levels are autocorrelated while there is no evidence of autocorrelation of residuals in first differences. In any case, estimation in levels or in second differences yields similar results.

sample period is

$$\ln \widehat{\lambda}_{st}^{co} = \widehat{\alpha}_{0s}^{co} + \widehat{\alpha}_{1s}^{co} t \tag{38}$$

where t indexes years starting in 1980.[30] Note that we can only identify the linear trend in quality. Deviations of quality from the trend are confounded with the other two components of the error term and are therefore not included in equation (38).

Equation (37), the definition of $\ln \widehat{\lambda}_{st}^{co}$ in equation (38) and our inclusion of the inbound trade cost ($\tau_{st}^{c,in}$) define our second-stage estimate of the Pure Price Index to be

$$\ln \widehat{\widehat{P}}_{st}^{co} = \ln \widehat{P}_{st}^{co} - \ln \widehat{\lambda}_{st}^{co} = -\frac{\widehat{\Psi}_{st}}{\widehat{\gamma}_s} + \widehat{\mu}_s^{out} \tau_{st}^{c,out} + \widehat{\mu}_s^{in} \tau_{st}^{c,in} + \frac{1}{\widehat{\gamma}_s} T_{st}^c / Y_t^c - \frac{1}{\widehat{\gamma}_s} \widehat{\upsilon}_{st}^{co}. \tag{39}$$

This definition includes the compound residual υ_{st}^{co}. As a result, first-stage measurement error (κ_{st}^{co}) as well as shocks to quality (ε_{st}^{co}) are attributed to the Pure Price Index. On the other hand, idiosyncratic deviations in the relationship between pure prices and the number of varieties (θ_{st}^c) are included – with opposite signs – both in the residual and in the trade balance (T_{st}^c / Y_t^c) and so are cancelled out.

5.3. Estimation Results

In this section we report preliminary estimates of export quality for All Manufacturing. While we intend for our methodology to be applied to disaggregate sectors within manufacturing once it is sufficiently refined, we start with a relatively aggregate sector to focus on the fundamental aspects of the methodology while abstracting from sector-specific nuances. Examination of aggregate manufacturing is also useful for assessing how our priors about countries' manufacturing prowess compare to the methodology's estimates.

Optimization results for the first-state estimates are summarized in Table 1. First-stage Impure Price Indexes relative to numeraire Switzerland for all countries in the sample in 1980 and 1997 are reported in Figure 1. In each panel of the figure, countries are sorted from low to high according to the natural log of their Impure Price Index relative to numeraire Switzerland (CHE), whose log index equals zero. The ordering of countries accords with their level of development, with higher-income developed economies like France (FRA) and Great Britain (GBR) exhibiting higher Impure Price Indexes than lower-income developing countries like Bangladesh (BGD) and Pakistan (PAK).

Table 2 reports the second-stage 2SLS estimates of γ_s from equation (37) where the real exchange rate is used to instrument for our estimates of countries' Impure Price Indexes. Four sets of coefficients are reported, accompanied by standard errors that are robust to heteroskedasticity and are clustered at the country level. The first column reports results for OLS, while the second through fourth columns report results for 2SLS excluding and including outbound and inbound transport costs, respectively. The OLS estimate for γ_s, while negative, is close to zero and statistically insignificant. The 2SLS estimates of γ_s are

[30] The recovered country fixed effect $\widehat{\alpha}_{0s}^{co}$ from our first-differenced estimation is equal to $(1/\widehat{\gamma}_s)\,\overline{T_s^c/Y^c} - \overline{\widehat{P}_s^{co}} - \widehat{\alpha}_{1s}^{co}\overline{t} - \widehat{\mu}_s^{out}\overline{\tau_s^{c,out}} - \widehat{\mu}_s^{in}\overline{\tau_s^{c,in}} - (1/\widehat{\gamma}_s)\,\overline{\widehat{\Psi}}_s$, where a bar over a variable denotes the average for each country over the sample period.

substantially more negative and statistically significant. The coefficient on outbound trade costs when it is the only trade cost variable included in the regression is positive and statistically insignificant. Coefficient estimates on outbound and inbound transportation costs when they are included together have the predicted sign and are statistically significant: net trade falls with higher outbound trade costs and increases with higher inbound trade costs. The difference between OLS and 2SLS coefficients, as well as the first-stage F- statistics reported in the final row of the table, supports our use of instrumental variables.

Based on the coefficient estimates in the last column of Table 2, Figure 2 plots the decomposition of the Impure Price Indexes into Quality Indexes and Pure Price Indexes relative to numeraire Switzerland using equations (38) and (39) for four countries: Argentina (ARG), China (CHN), Germany (DEU) and Ireland (IRL). As indicated in the figure, relative quality trends vary substantially across countries, increasing significantly for China and Ireland while declining moderately for Argentina and Germany. The crossing of the Impure Price and Quality Indexes for China and Ireland is due to the transition of those countries' manufacturing trade balances' from deficit to surplus over the sample period.

Pure price indexes decline over time for both Ireland and China: in the early part of the sample period, both countries' goods were more expensive in quality-adjusted terms than goods originating in Switzerland, but the opposite is true in later years. Pure prices in Argentina do not show a particular trend in the '80s but they increase in the 90's, while they are relatively stable in Germany after an increase in the first third of the sample period.

Figure 2 highlights the inappropriateness of the *ad hoc* assumption that export unit values are equivalent to quality. Indeed, the relatively flat Impure Price Indexes for Ireland and China contrast starkly with their upward sloping Quality Indexes (and downward sloping Pure Price Indexes). Close examination of the results for China illustrate how our identification of quality works. During the sample period, China's manufacturing trade balance (not shown) moved from deficit to surplus. For this to happen, its unobserved relative pure prices must have fallen, as shown by its declining Pure Price Index. If relative pure prices fall while the Impure Price Index remains relatively constant, quality must rise. Note that the validity of this inference does not depend on why pure prices decreased, i.e., whether they declined due to increasing comparative advantage or an increasingly undervalued exchange rate. Indeed, consider the latter. If pure prices had decreased because of currency misalignment, but quality *had not increased*, the Impure Price Index would have fallen. Instead, it remained constant.

Figure 3 compares the Impure Price and Quality Indexes for all countries in 1997, the final year of the sample. Countries with the largest trade surpluses in manufacturing – i.e., Ireland, Taiwan and China – exhibit the largest positive gaps between relative quality and impure prices. Guatemala (GTM) and El Salvador (SLV), with relatively high manufacturing trade deficits in 1997, exhibit the largest negative gaps. Countries with relatively balanced manufacturing trade, such as Belgium (BEL) and Italy (ITA) have second-stage Quality Indexes roughly equal to their first-stage Impure Price Indexes.

6. Conclusion

This paper attempts to fill an important gap in the international trade and development literature by providing the first reliable estimates of the evolution of countries' sectoral product quality. We develop a methodology for decomposing countries' observed export unit values into quality versus quality-adjusted-price components. This methodology exploits information on consumers' valuation of countries products contained in countries' net trade with the world. In contrast to a vast literature that associates cross-country variation in export unit-values with variation in product quality – implicitly assuming away cross-country variation in quality-adjusted prices – our methodology allows for price variation induced by factors other than quality, e.g. comparative advantage or currency misalignment. Our estimates reveal trends in product quality not apparent in export prices alone. For example, while China's export unit values in manufacturing are stable over the 1980-1997 period, our methodology reveals a substantial increase in product quality.

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A Proof of Proposition 1

We have already shown that $\ln H_s^{cd} \leq \ln P_s^{cd} + \ln \phi_s^c$. Here, we need to show that $\ln \phi_s^c \leq 0$, which implies that $\ln H_s^{cd} \leq \ln P_s^{cd}$. A similar proof shows that $\ln P_s^{cd} \leq \ln L_s^{cd}$.

The central part of the proof is to show that

$$\sum_{z \in I_s^{cd}} n_z^c \Delta \widetilde{p}_z^{cd} \geq -\sum_{z \in U_s^{cd}} \widetilde{n}_z^c \frac{1}{Z_s^{cd}} \sum_{z \in I_s^{cd}} \Delta \widetilde{p}_z^{cd} - \sum_{z \in U_s^{cd}} \widehat{n}_z^c \Delta \widetilde{p}_z^{cd}$$

This is done first:

$$\begin{aligned}
\sum_{z \in I_s^{cd}} n_z^c \Delta \widetilde{p}_z^{cd} &= \frac{\overline{n}^c}{\overline{n}^o} \left[\sum_{z \in I_s^{cd}} \widetilde{n}_z^{c,cd} \Delta \widetilde{p}_z^{cd} + \sum_{z \in I_s} \widehat{n}_z^{cd} \Delta \widetilde{p}_z^{cd} - \sum_{z \in U_s^{cd}} \widehat{n}_z^{cd} \Delta \widetilde{p}_z^{cd} \right] = \\
&= \frac{\overline{n}^c}{\overline{n}^o} \left[\begin{array}{c} Z_s^{cd} cov_{I_s^{cd}} \left(\widetilde{n}_z^{c,cd}, \Delta \widetilde{p}_z^{cd} \right) + \sum_{z \in U_s^{cd}} \widetilde{n}_z^{c,cd} \frac{1}{Z_s^{cd}} \sum_{z \in I_s^{cd}} \Delta \widetilde{p}_z^{cd} \\ + Z_s cov_{I_s} \left(\widetilde{\widetilde{n}}_z^{cd}, \Delta \widetilde{p}_z^{cd} \right) + \sum_{z \in I_s} \overline{n}_z \Delta \widetilde{p}_z^{cd} - \sum_{z \in U_s^{cd}} \widehat{n}_z^{cd} \Delta \widetilde{p}_z^{cd} \end{array} \right] \\
&\geq \frac{\overline{n}^c}{\overline{n}^o} \left[-\sum_{z \in U_s^{cd}} \widetilde{n}_z^{c,cd} \frac{1}{Z_s^{cd}} \sum_{z \in I_s^{cd}} \Delta \widetilde{p}_z^{cd} - \sum_{z \in U_s^{cd}} \widehat{n}_z^{cd} \Delta \widetilde{p}_z^{cd} \right]
\end{aligned}$$

The first equality uses $n_z^c = \widetilde{n}_z^{c,cd} + \widehat{n}_z^{cd}$ and the fact that $\sum_{z \in I_s^{cd}} \widehat{n}_z^{cd} \Delta \widetilde{p}_z^{cd} = \sum_{z \in I_s} \widehat{n}_z^{cd} \Delta \widetilde{p}_z^{cd} - \sum_{z \in U_s^{cd}} \widehat{n}_z^{cd} \Delta \widetilde{p}_z^{cd}$. The second equality uses $\widehat{n}_z^{cd} = \widetilde{\widetilde{n}}_z^{cd} + \overline{n}_z$ to decompose the second term, and also uses the fact that $\sum_{z \in I^j} x_z y_z = Z_j cov_{I^j} (x_z, y_z) + \frac{1}{Z^j} \sum_{z \in I^j} x_z \sum_{z \in I^j} y_z$. The inequality uses assumptions 4 and 5, and also the definition of $\widetilde{P}_s^c$ in (7), which implies that $\sum_{z \in I_s} \overline{n}_z \Delta \widetilde{p}_z^{cd} = 0$.

Decomposing $\Delta \widetilde{p}_z^{cd}$ according to its definition in (24) and using assumption 6, after some simple algebra manipulation we obtain

$$\frac{\sum_{z \in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c} \right)^{1-\sigma_s}}{\sum_{z \in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^d}{\widetilde{P}_s^d} \right)^{1-\sigma_s}} \geq 1 \tag{40}$$

which implies that

$$\ln \phi_s^c = \ln \left(\frac{\sum_{z \in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c} \right)^{1-\sigma_s}}{\sum_{z \in I_s^{cd}} n_z^c \left(\frac{\widetilde{p}_z^d}{\widetilde{P}_s^d} \right)^{1-\sigma_s}} \right)^{\frac{1}{1-\sigma_s}} \leq 0 \tag{41}$$

Substituting this result into $\ln H_s^{cd} \leq \ln P_s^{cd} + \ln \phi_s^c$ in equation (18), we obtain

$$\ln H_s^{cd} \leq \ln P_s^{cd} \tag{42}$$

An analogous proof shows that $\ln P_s^{cd} \leq \ln L_s^{cd}$. Hence, the Paasche and Laspeyres indexes bound the Impure Price Index,

$$\ln H_s^{cd} \leq \ln P_s^{cd} \leq \ln L_s^{cd}.$$

B Proof of Propostition 2

We start by reproducing equation (28):

$$\frac{1}{b_s}\frac{T_s^c}{Y^c} = -1 + \sum_{c'}\sum_{z} \frac{n_z^c \left(\widetilde{p}_z^c \tau_s^{cc'}\right)^{1-\sigma_s}}{(G_s^{c'})^{1-\sigma_s}} \frac{Y^{c'}}{Y^c} \tag{43}$$

Solving for n_z^c using equation (4), and substituting into equation (43), we can rewrite the right hand side of this equation as

$$-1 + \frac{\overline{n}^c}{\overline{n}^o}\frac{1}{Y^c}\left(\sum_{c'} Y^{c'}\left(\frac{\tau_s^{cc'}}{G_s^{c'}}\right)^{1-\sigma_s}\right)\left(\sum_z \overline{n}_z (\widetilde{p}_z^c)^{1-\sigma_s} + \left(\widetilde{P}_s^c\right)^{1-\sigma_s}\sum_z \widetilde{n}_z^c \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c}\right)^{1-\sigma_s}\right)$$

Using the definition of $\widetilde{P}_s^c$ in equation (7) and the fact that, since $\sum_z \widetilde{n}_z^c = 0$, $\sum_z \widetilde{n}_z^c (\widetilde{p}_z^c)^{1-\sigma_s} = Z_s cov\left[\widetilde{n}_z^c, (\widetilde{p}_z^c)^{1-\sigma_s}\right]$, this expression can be rewritten as

$$-1 + \frac{\overline{n}^c}{\overline{n}^o}\frac{1}{Y^c}\left(\widetilde{P}_s^c\right)^{1-\sigma_s}\left(\sum_{c'} Y^{c'}\left(\frac{\tau_s^{cc'}}{G_s^{c'}}\right)^{1-\sigma_s}\right)\left(1 + Z_s cov\left[\widetilde{n}_z^c, \left(\frac{\widetilde{p}_z^c}{\widetilde{P}_s^c}\right)^{1-\sigma_s}\right]\right)$$

Using Assumption 3 and equation (29), we can substitute the latter expression for the right hand side of (43). Rearranging terms and taking natural logarithms, we obtain

$$\ln\left(1 + \frac{1}{b_s}\frac{T_s^c}{Y^c}\right) = \ln\left[Y^o\left(\widetilde{P}_s^o\right)^{1-\sigma_s}\left(\widetilde{P}_s^{co}\right)^{1-\sigma_s-\eta_s}\left(\sum_{c'} Y^{c'}\left(\frac{\tau_s^{cc'}}{G_s^{c'}}\right)^{1-\sigma_s}\right)\left[1 + Z_s\left(V_s + \theta_s^c\right)\right]\right] \tag{44}$$

Using $\ln(1+x) \simeq x$, and abstracting from the approximation error, we can express equation (44) as

$$\frac{1}{b_s}\frac{T_s^c}{Y^c} = \ln\left(Y^o\left(\widetilde{P}_s^o\right)^{1-\sigma_s}\right) + (1-\sigma_s-\eta_s)\ln\widetilde{P}_s^{co} + Z_s\left(V_s+\theta_s^c\right) + \ln\left(\sum_{c'} Y^{c'}\left(\frac{\tau_s^{cc'}}{G_s^{c'}}\right)^{1-\sigma_s}\right) \tag{45}$$

We will perform a first-order Taylor expansion of the last term in equation (45). Using the definition of $G_s^{c'}$ in (26), we can rewrite this term as

$$\ln\sum_{c'}\left(\frac{Y^{c'}(\tau_s^{cc'})^{1-\sigma_s}}{\sum_{c''}\sum_z n_z^{c''}\left(\widetilde{p}_z^{c''}\tau_s^{c''c'}\right)^{1-\sigma_s}}\right) \tag{46}$$

Since this expression is a function of all consumption price indexes $G_s^{c'}$, it is in turn a function of the bilateral trade costs between all pairs of countries, $\tau_s^{c''c'}$. We will perform the Taylor expansion around a free-trade equilibrium, i.e. a point at which $\tau_s^{c''c'} = 1, \forall c'', c'$. Under free trade, the price index in the denominator is the same for every country, $G_s^{c'} = G_s, \forall c'$. A first-order Taylor expansion of (46) around the free-trade point results in

$$\begin{aligned} & \ln \sum_{c'} \frac{Y^{c'}}{G_s^{1-\sigma_s}} + \sum_{c'} \sum_{c''} \left(\frac{\partial \ln [.]}{\partial \tau_s^{c''c'}} \mid \tau_s^{c''c'} = 1 \right) \left(\tau_s^{c''c'} - 1 \right) \qquad (47) \\ = \; & \ln \sum_{c'} \frac{Y^{c'}}{G_s^{1-\sigma_s}} + \sum_{c'} \left[\left(\frac{\partial \ln [.]}{\partial \tau_s^{cc'}} \mid \tau_s^{cc'} = 1 \right) \left(\tau_s^{cc'} - 1 \right) + \sum_{c'' \neq c} \left(\frac{\partial \ln [.]}{\partial \tau_s^{c''c'}} \mid \tau_s^{c''c'} = 1 \right) \left(\tau_s^{c''c'} - 1 \right) \right] \end{aligned}$$

where

$$\begin{aligned} \left(\frac{\partial \ln [.]}{\partial \tau_s^{cc'}} \mid \tau_s^{cc'} = 1 \right) &= \frac{G_s^{1-\sigma_s}}{\sum_{c'} Y^{c'}} Y^{c'} \left[\frac{(1-\sigma_s) G_s^{1-\sigma_s} - (1-\sigma_s) \sum_z n_z^c \left(\widetilde{p}_z^c \right)^{1-\sigma_s}}{\left(G_s^{1-\sigma_s} \right)^2} \right] = \\ &= (1-\sigma_s) \frac{Y^{c'}}{\sum_{c'} Y^{c'}} \left[1 - \frac{\sum_z n_z^c \left(\widetilde{p}_z^c \right)^{1-\sigma_s}}{G_s^{1-\sigma_s}} \right] = -(\sigma_s - 1) \left(1 - r_s^c \right) y^{c'} \end{aligned}$$

and

$$\left(\frac{\partial \ln [.]}{\partial \tau_s^{c''c'}} \mid \tau_s^{c''c'} = 1 \right) = \frac{G_s^{1-\sigma_s}}{\sum_{c'} Y^{c'}} Y^c (\sigma_s - 1) \left(\frac{\sum_z n_z^{c''} \left(\widetilde{p}_z^{c''} \right)^{1-\sigma_s}}{\left(G_s^{1-\sigma_s} \right)^2} \right) = (\sigma_s - 1) y^{c'} r_s^{c''}.$$

Substituting these results into equation (47) and using the definition of average outbound trade cost in equation (30), we obtain

$$\begin{aligned} \ln \sum_{c'} Y^{c'} \left(\frac{\tau_s^{cc'}}{G_s^{c'}} \right)^{1-\sigma_s} &= \ln \sum_{c'} \frac{Y^{c'}}{G_s^{1-\sigma_s}} + (\sigma_s - 1) \qquad (48) \\ & \sum_{c'} \left[- \left(1 - r_s^c \right) y^{c'} \left(\tau_s^{cc'} - 1 \right) + \sum_{c'' \neq c} r_s^{c''} y^{c'} \left(\tau_s^{c''c'} - 1 \right) \right] \\ &= \ln \sum_{c'} \frac{Y^{c'}}{G_s^{1-\sigma_s}} + (\sigma_s - 1) \sum_{c'} \sum_{c''} r_s^{c''} y^{c'} \left(\tau_s^{c''c'} - 1 \right) \\ & -(\sigma_s - 1) \sum_{c'} y^{c'} \left(\tau_s^{cc'} - 1 \right) \\ &= \ln \sum_{c'} \frac{Y^{c'}}{G_s^{1-\sigma_s}} + (\sigma_s - 1) \sum_{c'} \sum_{c''} r_s^{c''} y^{c'} \left(\tau_s^{c''c'} - 1 \right) - (\sigma_s - 1) \tau_s^{c,out} \end{aligned}$$

Finally, substituting (48) into (45), we obtain

$$\frac{T_s^c}{Y^c} = \Psi_s + \gamma_s \ln \widetilde{P}_s^{co} + \gamma_s \mu_s \tau_s^{c,out} + \iota_s^c \tag{49}$$

where $\gamma_s = b_s(1 - \sigma_s - \eta_s) < 0$, $\mu_s = \frac{(\sigma_s - 1)}{(\sigma_s + \eta_s - 1)} > 0$, $\iota_s^c = b_s Z_s \theta_s^c$,

$$\Psi_s = b_s \left[\ln (Y^o) + \ln \left(\widetilde{P}_s^o \right)^{1-\sigma_s} + A_s + Z_s V_s \right],$$

$$A_s = \ln \sum_{c'} \frac{Y^{c'}}{\left(G_s^{c'} \right)^{1-\sigma_s}} + (\sigma_s - 1) \sum_{c'} \sum_{c''} r_s^{c''} y^{c'} \left(\tau_s^{c''c'} - 1 \right).$$

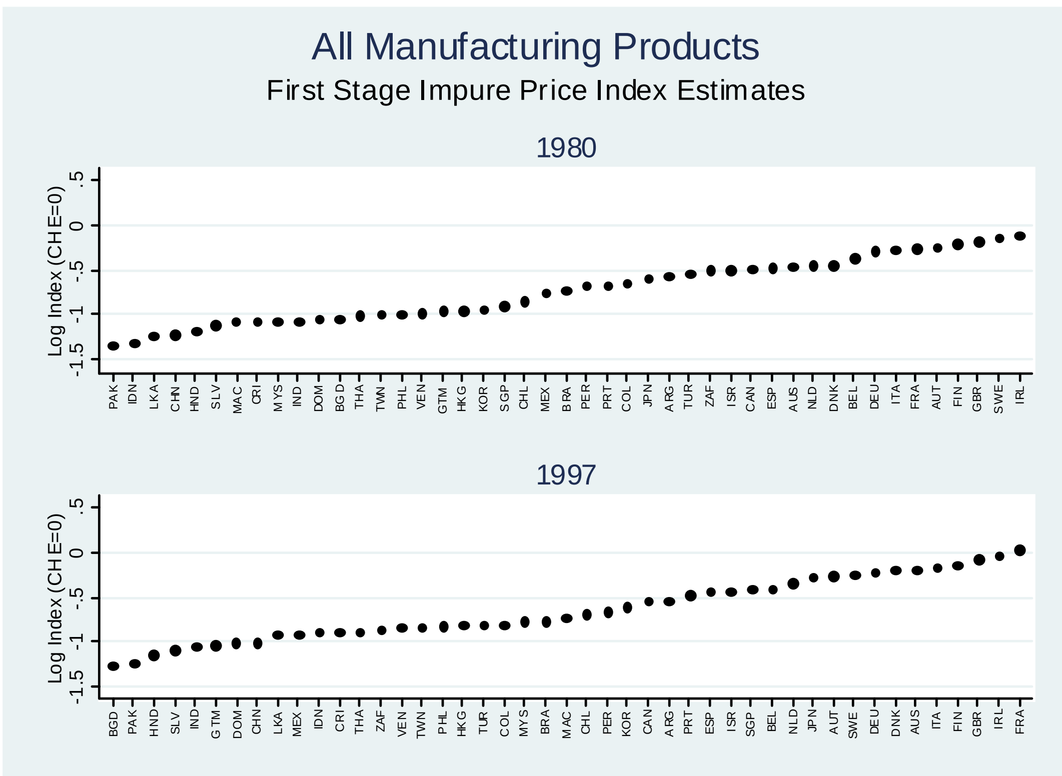

Figure 1: First Stage Impure Price Indexes $\left(\ln \widehat{P}_{st}^{co}\right)$, 1980 and 1997

Figure 2: Second-Stage Relative Quality and Relative Pure Price Indexes, 1980 to 1997

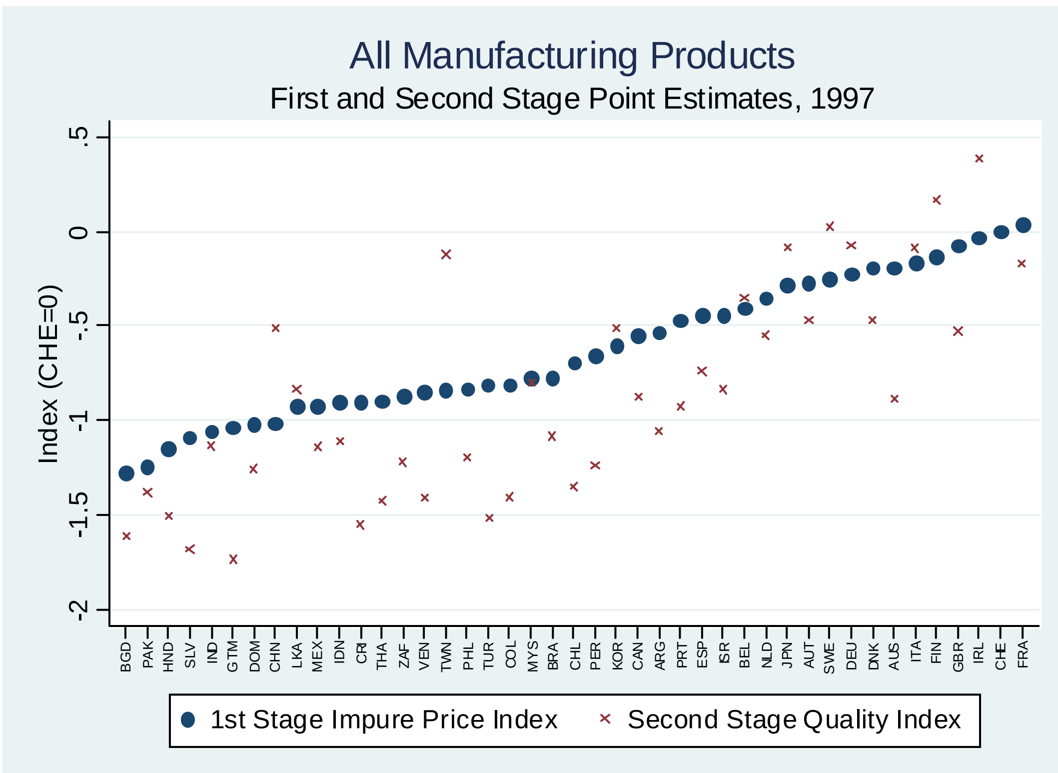

Figure 3: Comparison of First- and Second-Stage Estimates Relative Quality Indexes, 1997

Year	Log Probability	Mean Commonly Exported Products	Psi
1980	-801	332	0.30
1981	-738	363	0.30
1982	-758	387	0.30
1983	-821	435	0.37
1984	-725	570	0.40
1985	-645	697	0.49
1986	-666	717	0.46
1987	-733	717	0.43
1988	-615	722	0.37
1989	-509	896	0.38
1990	-456	882	0.34
1991	-460	856	0.34
1992	-507	876	0.34
1993	-513	923	0.33
1994	-451	1009	0.34
1995	-383	1113	0.31
1996	-362	1167	0.35
1997	-346	1260	0.36

Notes: Table summarizes results from first-stage estimation on sample of 45 U.S. trading partners relative to base country Switzerland (CHE). First column reports sum of the log probability that the "true" Paasche and Laspeyres bounds contain the estimates. Second column reports mean number of common products across country pairs. Final column reports estimate of variance.

Table 1: First-Stage Estimates, All Manufacturing

	OLS	2SLS	2SLS	2SLS
Impure Price Index	-0.007	-0.160 **	-0.161 **	-0.179 ***
	0.018	0.070	0.071	0.068
Outbound Transport Cost			0.288	-1.683 *
			0.555	0.983
Inbound Transport Cost				1.940 **
				0.796
Observations	730	730	730	730
R^2	0.11	.	.	.
First-Stage Fstat	.	28	29	28

Notes: Results of 2SLS estimation of equation (38) for the years 1980 to 1997. The instrument for the Impure Price Index is the real exchange rate. ***, ** and * denote statistical significance at the 1, 5 and 10 percent levels, respectively.

Table 2: Second Stage IV Estimation, All Manufacturing