General Noise-Perturbed Superior Julia Sets

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The aim of this paper is to offer an integrated approach to study the additive and multiplicative noises with respect to perturbations in superior Julia sets. External and internal perturbations in superior Julia sets are analyzed under the mixed effect of additive and multiplicative noises. [DOI: 10.1115/1.4006785]

Keywords: superior Julia set, general noise, additive noise, multiplicative noise, internal perturbation, external perturbation

1 Introduction

The classical work of Gaston Julia, in 1918, has lead to new excitements in complex dynamics. Rani and Kumar [1,2] introduced superior iterates in the study of complex nonlinear dynamics and computed effective new Julia and Mandelbrot sets with respect to superior orbits (see also Refs. [3,4]).

Argyris et al. [5–8], in their pioneering work, have studied the effect of dynamic noise in complex dynamical systems. They discussed the structural characteristics of the Mandelbrot and Julia sets after importing dynamic noise (additive and multiplicative both) into the complex map $z_{n+1}^2 = z_n^2 + c$ (cf. Refs. [6–8]). Further, Wang et al. offered a nice study on the effects of additive noises on a generalized Mandelbrot map [9]. Recently, Rani and Agarwal [10] showed that under the effect of dynamical noise, distortion in Julia sets in superior orbit and Julia sets in Picard orbit is different. They also discussed the usefulness and other issues of the superior orbit in discrete dynamics.

In contrast to studying the additive and multiplicative noises separately, Negi and Rani [11] proposed a general approach and formulated a new method to combine the criterions regarding additive and multiplicative noises. Indeed, they put forward the theory of general noise, which integrates additive and multiplicative noises and allows application of both the noises simultaneously. They studied the mixed effect of additive and multiplicative noises on the superior Mandelbrot map. Using the general noise, Wang, Jia, and Zhenfeng [12] studied the mixed effect of additive and multiplicative noises on the generalized Mandelbrot map and studied the structural characteristics and the fission-evolution law of the noise-perturbed generalized Mandelbrot map. Also, Wang, Jia, and Sun [13] presented a collective study of the effect of additive noise, effect of multiplicative noise, and their mixed effected on $z^\alpha + c$ for $|\alpha| > 1$.

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The main purpose of this paper is to study perturbations in Julia sets, generated in superior orbit, under the effect of general noise. We focus on the distortions in superior Julia sets, when additive and multiplicative noises both are available simultaneously. Indeed, we apply general noise on superior Julia sets and show the pattern of perturbation. Finally, the results have been summarized and precise conclusions have been drawn.

2 Preliminaries

Consider a continuous time dynamical system in the Euclidean space R^n expressed by the equation $s = f(x, \mu, t)$, where $x, \mu \in R^n$ are vectors and $t \in R$.

Negi and Rani [11] considered the notion of general noise in the form of a disturbance, which influences the evolution of a dynamical system. Its general formulation in dynamical systems is given by $k = f(x, \mu, t, w)$, where $x, \mu \in R^n$ are vectors and $t \in R$ and w is noise.

Definition 1: General Noise. The general noise, denoted by *GN*, on the Mandelbrot map is given by

$$X_{n+1} = \lambda x_a + (1-\lambda)x_m \tag{1}$$

$$Y_{n+1} = \lambda y_a + (1-\lambda)y_m \tag{2}$$

where (x_a, y_a) are additive noises and (x_m, y_m) are multiplicative noises in a dynamical system, and $0 \leq \lambda \leq 1$.

The parameter λ controls the contribution of both the noises in *GN*. Thus, *GN* behaves as a multiplicative noise when $\lambda = 0$, and with $\lambda = 1$ it reduces to an additive noise [11].

Superior iterations, an example of a two-step feedback machine, are constructed by the formula $z_n = s_n f(z_{n-1}) + (1 - s_n) z_{n-1,}$ where z_n is a complex number and $0 < s_n \leq 1$ and $\{s_n\}$ is convergent to a nonzero number.

Definition 2: Superior Orbit. The sequence $\{z_n\}$ constructed above is called superior sequence of iterates, denoted by $SO(f, z_0, s_n)$ (see Refs. [1–4,10,11]).

The superior orbit $SO(f, z_0, s_n)$ with $s_n = 1$ reduces to the Picard orbit (also called the function iteration in discrete dynamics). In this paper, we perform experiments for $s_n = s$. Now, we recall the definition of superior Julia set, denoted by *SJ*, i.e., Julia set for a function with respect to an *SO*.

Definition 3: Superior Julia Set. The set of points *SK* whose orbits are bounded under superior iteration of a function $Q(z)$ is called the filled superior Julia set. Superior Julia set *SJ* of Q is the boundary of filled superior Julia set *SK* [1].

An *SJ* set for $Q(z)$ reduces to Julia set when the parameter $s_n = 1$ in *SO*. Now, we consider the polynomial $Q_c(z) = z^2 + c$. In order to compute the *SJ* sets for Q_c, a new escape criterion in *SO* is required. The superior escape criterion for the polynomial $Q_c(z) = z^2 + c$ is max $\{|c|, 2/s\}$. For the details of the superior escape criterion, one may refer to Rani and Kumar [1].

3 General Noise-Perturbed Superior Julia Sets

Argyris et al. [8] have studied the influence of dynamical noise on the quadratic Julia sets $Q_c(z) = z^2 + c$, for $c = (c_x, c_y) = (-0.3904, -0.58769)$. They showed that Julia sets suffer from external perturbation due to intrusion of additive noise and get internally perturbed due to multiplicative noise. For the sake of comparison of the results obtained by intrusion of dynamical noise and general noise, we also take the same parameters in our study and observe the effect of general noise on *SJ* for $Q_{(-0.3904,-0.58769)}$. Now, we fix up the parameter $\lambda = [1/2]$, so that both of the additive and multiplicative noises may contribute equally in our study. Also, for the symmetrical distortion in the objects, we take the parameters $x_a = y_a = m$ and $x_m = y_m = k$ (cf. Refs. [5,6]).

With all these assumptions, we made a program in C++ and applied *GN* on quadratic *SJ* sets. This section is subdivided into

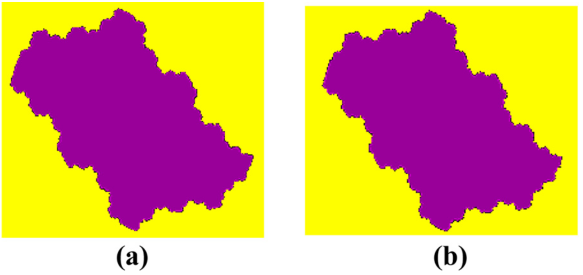

Fig. 1 (*a*) *SJ* at $s=0.9$ (*b*) *SJ* at low additive and low multiplicative noise $(s, m, k) = (0.9, 0.01, 0.01)$

parts to study *GN* perturbed *SJ* sets. Indeed, in order to consider the mixed effect of additive and multiplicative noises on *SJ* sets, we divide our study into four parts.

3.1 Low Additive and Low Multiplicative Noises. Figure 1(*a*) shows *SJ* at $s=0.9$, and when a small amount of additive and multiplicative noises are applied, there is negligible distortion in *SJ* (see Fig. 1(*b*)), which is obvious.

3.2 Low Additive and Higher Multiplicative Noises. At low additive noise, it was found that with the increasing strength of multiplicative noise, internal perturbation increases, which leads to disconnected *SJ* and further, *SJ* vanishes (see Fig. 2).

3.3 Higher Additive and Low Multiplicative Noises. At low multiplicative noise, we find that the increase in the strength of additive noise, up to $m=1$, reflects external perturbation, i.e., *SJ* tends to be somewhat circular and connectivity increases, see Fig. 3.

Further, an increase in the strength of additive noise (i.e., $m>1$) reflects internal perturbation in *SJ*. One can see distortion in the shape of *SJ* and its reducing connectivity in Fig. 4.

3.4 Higher Additive and Higher Multiplicative Noises. When both of the dynamic noises in *GN* are higher, for $m=k\leq 1$, *SJ* tends to take a disturbed elliptical shape (see Fig. 5). Further

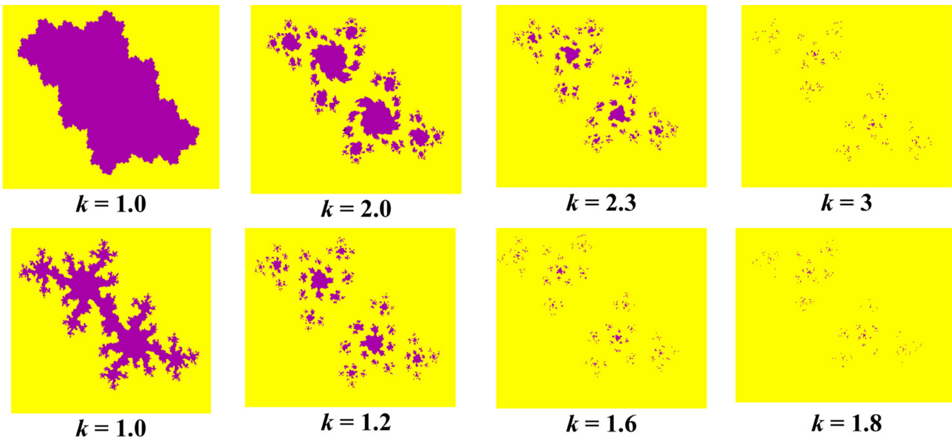

Fig. 2 Effect of increasing strength of multiplicative noise on *SJ* at low additive noise $m=0.01$ at $s=0.8$ (row 1) and $s=0.9$ (row 2)

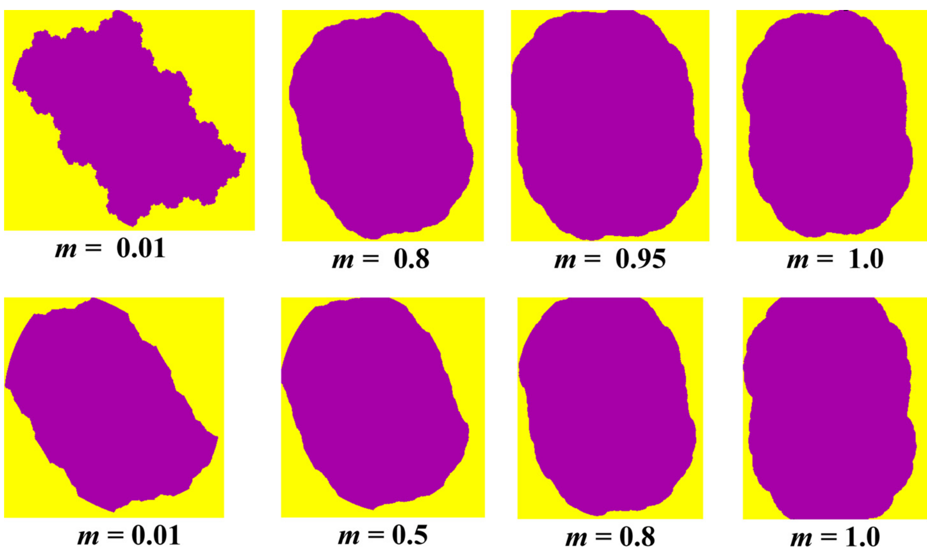

Fig. 3 Effect of increasing additive noise for $m \leq 1$ on *SJ* at low multiplicative noise $k=0.01$ at $s=0.9$ (row 1) and $s=0.8$ (row 2)

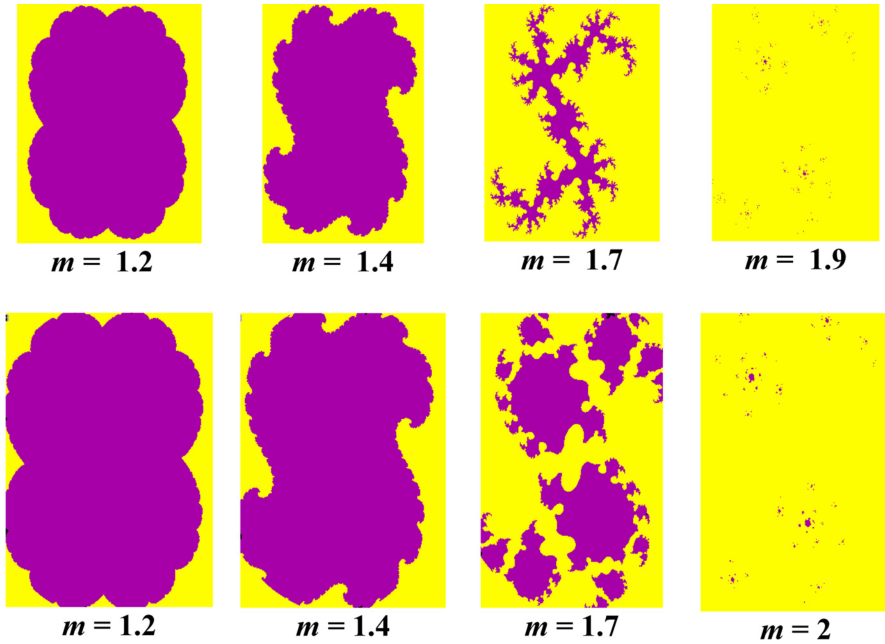

Fig. 4 Effect of increasing additive noise for $m>1$ on SJ at low multiplicative noise $k=0.01$ at $s=0.9$ (row 1) and $s=0.8$ (row 2)

Fig. 5 Effect of higher additive noise and higher multiplicative noise at $(m, k, s)=(0.9, 0.9, 0.9)$

increase in both of the dynamic noises, i.e. ($m=k\geq 1$), causes internal perturbation in the shape of *SJ* as shown in Fig. 6.

4 Discussion and Conclusion

The results obtained in the paper are summarized as follows:

- When both the noises in *GN* are low, there is negligible distortion in the shape of *SJ*, which is obvious.
- At low additive and higher multiplicative noises, there is internal perturbation, which tends to destroy the connectivity of *SJ*.
- The distortion in *SJ* is due to dominance of multiplicative noise. The result is the same as given by Argyris et al. [8].

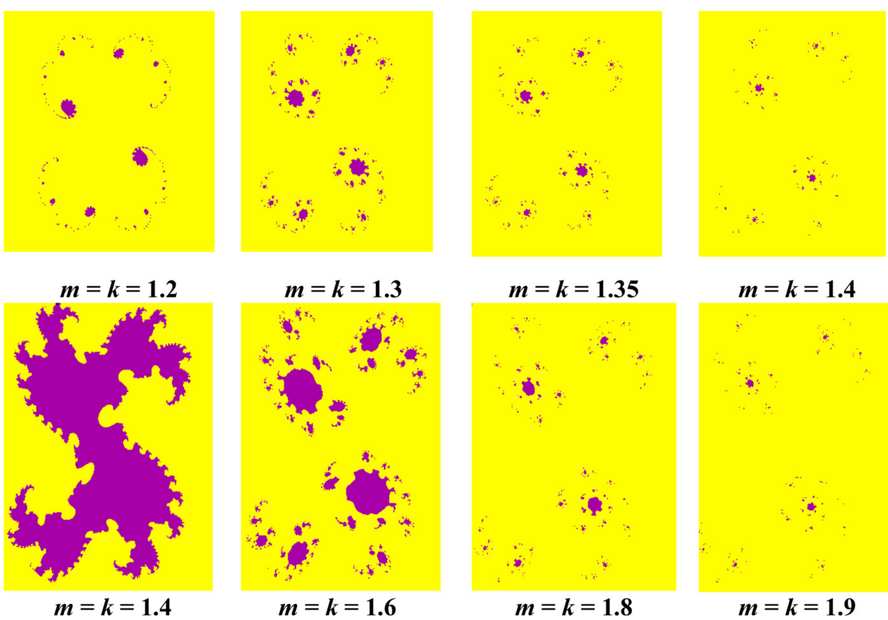

Fig. 6 Effect of higher additive noise and higher multiplicative noise at $m>1$ and $k>1$ at $s=0.9$ (row 1) and $s=0.6$ (row 2)

- ◆ At low multiplicative noise, for higher additive noise, a mixed effect is observed. Initial increase in the additive noise reflects external perturbation. However, its further higher strengths lead to internal perturbation in *SJ* (cf. the *SJ* for additive noise > 1).
- ◆ Here, distortion in *SJ* is due to dominance of additive noise. As per Argyris et al. [8] under the dominance of additive noise, Julia sets get externally perturbed.
- ◆ When both the noises in *GN* are higher, again a mixed effect of them is observed. Initial increase in the strength of *GN* reflects external perturbation. However, further higher strengths lead to internal perturbation in *SJ* (cf. the *SJ* for $GN > 1$).
- ◆ Distortion in all the above four cases is controlled by the parameter *s*. Distortion can be increased or decreased by increasing or decreasing the value of the parameter in any case.

We conclude by asserting that:

In a general-noise perturbed *SJ*, if we keep the additive noise on the higher side, then by decreasing and increasing the strength of additive noise, we can achieve external and internal perturbations respectively, irrespective of the strength of multiplicative noise. Distortion in *SJ* can be decreased or increased by decreasing or increasing the value of the controlling parameter *s*, respectively.

We close our study by observing that there is a good scope to continue the present study for various choices of the parameter *s* and possibly considering a sequence of parameters $\{s_n\}$ instead of *s*.

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