DOUBLE-POROSITY SOILS WITH HIGHLY CONDUCTIVE INCLUSIONS: MODELING OF WATER FLOW IN UNSATURATED CONDITIONS

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Summary The homogenization method by asymptotic expansions was used to study water flow in a double-porosity soil. The gravity effect is included. The macroscopic flow model was found to be a single diffusion-type equation with two effective parameters. The effective conductivity tensor is defined as depending on the solution of a local boundary value problem within a period. The effective specific water capacity is found to be the volumetric average. Both parameters are functions of the capillary pressure head. A numerical example of water infiltration into initially dry soil is presented. The results of homogenization show good agreement with the reference fine scale solution.

INTRODUCTION

Double-porosity soils are highly heterogeneous media composed of two porous sub-domains with contrasting hydraulic properties. In this type of media three distinct observation scales exist: (i) the microscopic scale, which corresponds to grains and pores; (ii) local (mesoscopic) scale, which is associated to the characteristic size of heterogeneities (aggregates, inclusions, shales) and (iii) the macroscopic scale, which is the scale of entire soil layers and is most important for practical purposes. Due to the particular structure the modeling of water flow and contaminant transport in double-porosity media requires specially adapted approaches. This paper concerns water flow in an unsaturated double-porosity medium which consists of highly conductive inclusions embedded in a less conductive matrix.

FORMULATION OF THE PROBLEM

We consider a heterogeneous medium of periodic structure, with two characteristic lengths: l (the characteristic size of a period) and L (the characteristic size of the macroscopic domain) (Fig 1a,b). The scale separation is written: $\varepsilon = l / L \ll 1$. The water flow in unsaturated conditions in both porous sub-domains is described by the Richards equation:

$$C_1 \frac{\partial h_1}{\partial t} - \mathrm{div}_X \left(K_1 \mathrm{grad}_X (h_1 + X_3) \right) = 0 \quad \text{in } \Omega_1 \quad \text{and} \quad C_2 \frac{\partial h_2}{\partial t} - \mathrm{div}_X \left(K_2 \mathrm{grad}_X (h_2 + X_3) \right) = 0 \quad \text{in } \Omega_2 \tag{1a,b}$$

where h_i is capillary pressure head [L], t is time [T], $\mathbf{X}$ is spatial variable [L] (X_3 oriented vertically upwards), C_i is the specific water capacity [L^{-1}] and $\mathbf{K}_i$ is the hydraulic conductivity tensor [L/T] (i denotes sub-domain 1 or 2). Both C_i and $\mathbf{K}_i$ are highly nonlinear functions of h_i. At the interface Γ the continuity of capillary pressure and flux is satisfied:

$$h_1 = h_2 \quad \text{and} \quad \left(\mathbf{K}_1 \mathrm{grad}_X (h_1 + X_3) \right) \mathbf{N} = \left(\mathbf{K}_2 \mathrm{grad}_X (h_2 + X_3) \right) \mathbf{N} \tag{2a,b}$$

where $\mathbf{N}$ is a unit vector normal to Γ. Due to the separation of scales two dimensionless spatial variables appear: $\mathbf{x} = \mathbf{X}/L$ and $\mathbf{y} = \mathbf{X}/l$. The following assumptions are introduced: (i) the characteristic time of observation corresponds to the time of flow in sub-domain 2 at the macroscopic scale; (ii) the capillarity dominates over the gravity at the scale of a period, (iii) the ratio of the characteristic diffusivities is of the order of ε: $D_{2C} / D_{1C} = (K_{2C}/C_{2C}) / (K_{1C}/C_{1C}) = O(\varepsilon)$.

HOMOGENIZATION

After normalization of the problem (1)-(2) the dimensionless quantities u^* are presented in form of an asymptotic expansion: $u^* (\mathbf{x},\mathbf{y},t^*) = u^{(0)} (\mathbf{x},\mathbf{y},t^*) + \varepsilon\, u^{(1)} (\mathbf{x},\mathbf{y},t^*) + \varepsilon^2 u^{(2)} (\mathbf{x},\mathbf{y},t^*) + \ldots$ The homogenization procedure includes three main steps: (i) introduction of the expansions into the normalized equations; (ii) identification of problems at the successive powers of ε; (iii) solution of resulting boundary value problems within the period domain. Only the final results of the homogenization are presented here. The macroscopic model is obtained as a single equation with two effective parameters K_{ij}^{eff} and C^{eff}:

$$C^{eff} \frac{\partial h^{(0)}}{\partial t^*} - \mathrm{div}_x \left(\mathbf{K}^{eff} \mathrm{grad}_x \left(h^{(0)} + X_3 \right) \right) = 0 \tag{3}$$

where $h^{(0)}$ is the macroscopic capillary pressure head. It was found that local equilibrium of capillary pressure exists during the flow. The effective water capacity is defined as: $C^{eff} = w_1 C_1 + w_2 C_2$, where w_i is the volumetric fraction of sub-domain i . The effective conductivity tensor is defined as follows:

$$K_{ij}^{eff} = \frac{1}{|S_i|} \int_{S_i} \left[K_{2ik}^{(0)} \left(I_{kj} - \frac{\partial \chi_j}{\partial y_k} \right) \right] dS \tag{4}$$

where S_i represents the surface of the period normal to i direction. The vector function χ is a solution of the following local boundary value problem:

$$\frac{\partial}{\partial y_i}\left[K_{2ik}^{(0)}\left(I_{kj}-\frac{\partial \chi_j}{\partial y_k}\right)\right]=0 \quad \text{in } \Omega_2 \quad \text{and} \quad \chi_k = y_k \quad \text{on } \Gamma \qquad \text{(5a,b)}$$

Moreover, χ should satisfy the periodicity condition. Note that $\mathbf{K}^{eff}$ is not influenced by the conductivity of inclusions.

NUMERICAL EXAMPLE

The numerical simulation concerns infiltration into a soil layer (Fig. 1a). The macroscopic process is 1D, while the local geometry is 2D (Fig. 1b). The scale separation parameter $\varepsilon = 0.01$. For simplicity, the function $C(h)$ is taken the same in both sub-domains, whereas the hydraulic conductivities satisfy: $\mathbf{K}_2(h)$ / $\mathbf{K}_1(h) = 0.01$. The solution of local problem is presented in Fig. 2 and the effective functions in Fig. 3. The soil is initially dry: $h_0 = -100$ cm. At the surface a constant value of h is imposed: $h_S = -2$ cm. At the bottom free drainage condition is applied: $dh/dX_3 = 0$. The macroscopic equation (3) was solved using REMOL_1D numerical code based on the method of lines (DASPK solver). A fine scale solution is also provided, where the local heterogeneous structure of the medium is explicitly represented (SWMS_2D code). The pressure profiles are compared in Fig. 4. It can be seen that the homogenization results are very close to the reference solution. The same holds for the macroscopic water flux (results not shown here).

CONCLUSIONS

The water flow in unsaturated soils with highly conductive inclusions can be described by a single macroscopic equation (local equilibrium conditions) with effective parameters derived by homogenization. The macroscopic conductivity tensor depends on the conductivity of the matrix and the local geometry of the medium and is independent of the inclusions' conductivity. There exists a continuous passage to this solution from the classical approach. Numerical results obtained using homogenization show good agreement with the reference fine scale solution.

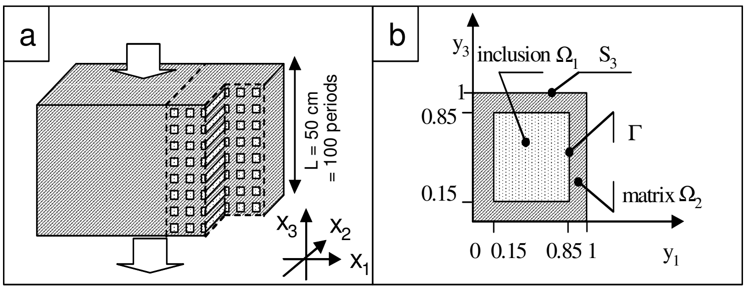

Fig. 1. Macroscopic solution domain and local geometry

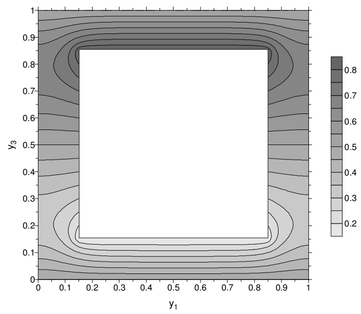

Fig. 2. Distribution of function χ in the period

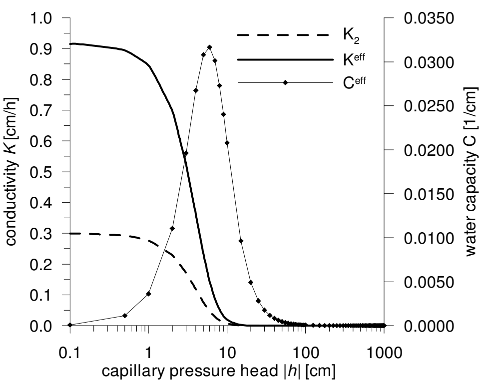

Fig. 3. Hydraulic parameters of the soil

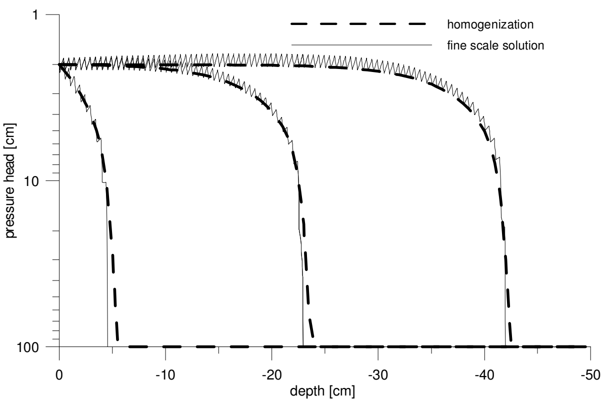

Fig. 4. Pressure profiles obtained from homogenization and reference solution

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