# Incentivizing Signal and Energy Cooperation in Wireless Networks

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*Abstract*—We consider a two-hop wireless network where the source(s) in the network have the ability to wirelessly power the relay(s) who also have their own data to send to the destination. Considering the fact that each node in the network aims to maximize its own metric, we adopt a game theoretic approach that foresees offering relaying of the sources' data in exchange for energy provided to the relays, and simultaneously offering energy to the relays in exchange for their relaying services. We first study a Stackelberg competition with the single relay node as the leader, and investigate the impact of having multiple source nodes in the system. We next study the reciprocal Stackelberg game with the single source as the leader, and investigate the inter-relay competition with multiple relays. We find that in the Stackelberg games, the leader can improve its individual utility by influencing the follower's decision accordingly, even more so when there are multiple followers. We next formulate a noncooperative game between the source and the relay and show the existence of a unique Nash equilibrium by an appropriate pricing mechanism. The equilibrium maximizes the total utility of the network and allows the destination to choose how much data to receive from each node.

*Index Terms*—Energy transfer, cooperative communications, Stackelberg games, Vickrey auction, two-hop relay networks.

## I. INTRODUCTION

THE distribution of resources in a wireless network is often nonuniform, and thus requires the cooperation of the nodes in the network for the sake of network optimization. This includes signal cooperation [1] where nodes with better channel availability to destinations help forward other nodes' data to the destination, and energy cooperation [2] where nodes with better energy availability help energy deficient nodes by means of energy transfer. As we opt for a greener future for wireless communication networks, energy harvesting nodes and utilizing energy (and signal) cooperation become viable design choices for these networks [3]–[5]. Among other sources of green energy such as wind, biomass, and piezoelectric devices [6]–[8], radio frequency (RF) energy transfer, where a wireless node is powered by the energy harvested from another node's transmission, has recently gained attention, see for example [9]–[14]. Undoubtedly useful for improving network-wide performance, energy and signal cooperation may not arise naturally in practice as individual nodes would care for their individual performance. It then becomes useful to investigate how to incentivize the nodes so that they participate in signal and energy cooperation. In this work, we study the two-hop network where source(s) and relay(s) can engage in energy cooperation in return for signal cooperation and vice versa, using a game theoretic framework.

Two-hop networks have been studied extensively from a signal cooperation perspective. Reference [15], for example, has investigated two-hop networks with multiple sources or relays, and provided achievable rates. Energy cooperation has been introduced to the two-hop channel in [2] with one-way energy transfer from the source to the relay. An energy management scheme based on two dimensional waterfilling has been shown to maximize the system throughput. It has been concluded that energy transfer can improve the end-to-end throughput significantly. Reference [16] has studied energy transfer from one source to two relays in an energy harvesting diamond channel. Reference [17] has generalized throughput maximizing transmission and energy transfer policies by way of two-way energy transfer between the sources and relays of an energy harvesting two-hop network. Reference [18] has studied the multiple access and two-way channels where all transmitters can share their energy in all directions. Reference [19] has generalized the energy cooperating two-way channel to the case with finite batteries at both transmitters.

Simultaneous wireless information and power transfer (SWIPT) has been studied extensively, among others, in [11], [20], [21]. In SWIPT, signal cooperation and energy cooperation are performed using power splitting where the received signal at the relay is harvested in part for energy and in part for information decoding; time sharing which specifies disjoint time durations for energy harvesting and relaying; and relay selection for multi-relay setups where some relays harvest energy while others forward data [9]. Reference [22] has studied the trade off between energy and information transfer over a point-to-point channel. Reference [23] has considered a two-hop network with multiple relays with energy transfer from the source to the relays. Reference [24] has considered a cognitive radio setup where a secondary user can harvest energy from a nearby primary user's transmission to use for its own transmission. Cellular models where the base station wirelessly powers the nodes have also been studied, see for example [10], [25]. Reference [26] has investigated the trade off between information transmission and energy transfer. For an overview of SWIPT systems, see [12]. Other recent references that have considered wireless power transfer/harvesting in order to improve system performance include [5], [27]–[29].

Energy transfer has also been considered over more than two hops. Reference [30] has investigated wireless energy transfer over multiple hops, and observed that efficient energy transfer is possible over about twenty hops. Reference [31] has studied the free rider problem in multi-hop networks where some relays are not willing to forward the packets they receive and proposed a solution based on disconnecting them from the network. Consequently, the relays that are not willing to cooperate end up with zero utilities.

Energy and signal cooperation has proven useful in improving the end-to-end throughput in multi-terminal networks significantly. Not surprisingly, the models studied to avail these performance gains assume that nodes are altruistic and are thus willing to cooperate. In reality however, nodes may need encouragement in participating in cooperation as observed earlier for non-energy harvesting/cooperating scenarios, see for example [32] which has studied a cognitive radio setup where the secondary users are given spectrum access in exchange for signal cooperation with the primary user.

In this work, we consider an energy harvesting two-hop network with "selfish" individual nodes and identify strategies that encourage them to participate in actions that will improve the network performance, i.e., cooperation on the energy and signal levels. In this setup, all nodes wish to maximize the amount of their own data delivered to the destination. The question thus becomes how to incentivize them. An initial study in this direction has been conducted in [33] where a relay has sufficient energy to forward a source's data, but needs energy to forward its own data and harvests it from the source. The relay is assumed to be capable of amplify-and-forward only, which necessitates allocating disjoint time intervals in which the relaying of the source's messages and the transmission of the relay's data takes place. In this work, we consider a comprehensive interaction model where the relay has no energy supply other than that which can be acquired from the source. We consider a decode-and-forward relay capable of superposing cooperative transmission with its own allowing for better design. The source provides the energy required for the relaying of its data as well as the transmission of the relay's data, resulting in a model that avails an array of cooperation scenarios within a game theoretic framework including those with multiple sources and relays.

We start our investigation with a two-hop network with one source, one relay, and a destination. We formulate and solve Stackelberg games [34] for this setup where one of the two nodes is deemed the leader of the game and the other node the follower. There could be scenarios where it may be more suitable for either the source node or the relay to be the leader. For example, the source (the relay) could be a node with data that the destination has more priority on, e.g., an emergency responder. Similarly, one of the nodes can be a primary user with access rights to radio resources. As well, in a sensor network setup, both nodes may be sending reports of measurements to a collector node where one report may be more important, e.g., humidity level versus smoke. Thus, we study both cases where (i) the source, and (ii) the relay is the leader. In each case, we guarantee a unique equilibrium with positive utilities for both nodes. We demonstrate how the leader chooses a strategy that
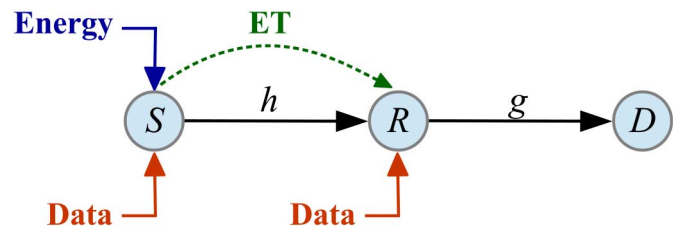


Fig. 1. The two-hop channel with energy transfer (ET).

influences the follower in a way to improve the leader's utility. We next extend each Stackelberg game to the case with multiple followers where we employ a Vickrey auction among the followers [35]. We determine the winner of the auction in each case and find that the auction improves the auctioneer's utility, even more so as the number of bidders increases.

Next, we study a noncooperative simultaneous game for the two-hop network and observe that energy or signal cooperation is not possible when the nodes are left to their own devices. We thus propose a pricing scheme that facilitates energy and signal cooperation, and maximizes the total utility of the network. We show that this scheme allows the destination to specify how much data to receive from each node. Via numerical simulations, we observe the impact of the model parameters on the nodes' optimal strategies and the resulting utilities.

The remainder of this paper is organized as follows. In Section II, we describe the two-hop channel and the proposed cooperation scheme. In Section III, we formulate and solve a Stackelberg game with the relay as the leader, and employ a Vickrey auction between multiple sources to further improve the relay's utility. In Section IV, we consider the reciprocal Stackelberg game with the source as the leader, and employ a Vickrey auction between multiple relays. In Section V, we formulate a simultaneous noncooperative game and improve the resulting utilities using a pricing scheme. In Section VI, we provide simulation results for all communication scenarios in consideration, and observe the impact of the network parameters in each case. In Section VII, we discuss our findings and conclude the paper.

## II. SYSTEM MODEL

In this paper, we study various generalizations of a fundamental communication model, namely the two-hop channel. The two-hop channel we consider requires both signal and energy cooperation by the source and relay nodes to attain a positive sum throughput. For this reason, it is suitable as a base model for a study of signal and energy cooperation in wireless networks composed of nodes with selfish interests.

Consider a two-hop network with half duplex nodes as shown in Fig. 1. Both the source, node $S$, and the relay, node $R$, have data to transmit to the destination, node $D$. The source and the relay are selfish in the sense that they will act to maximize their respective utilities, i.e., the amount of their own data delivered to the destination. The source is connected to the relay by an additive white Gaussian channel with unit variance and channel power gain $h$. We consider that the destination is far away from
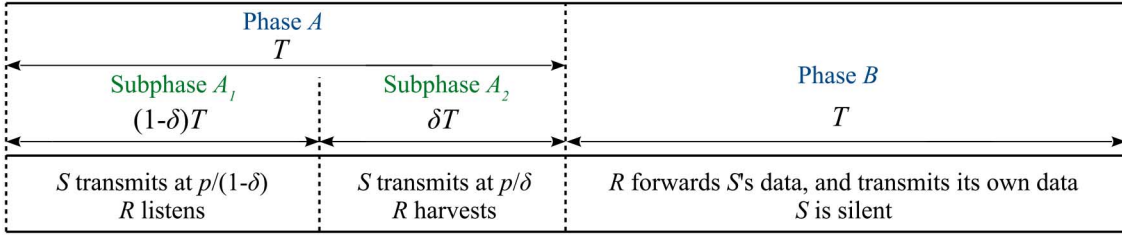
Fig. 2. The phases and subphases of the communication scenario.

the source and thus the direct channel between the source and the destination can not sustain communication. The relay on the other hand can communicate to the destination via an additive white Gaussian channel with unit variance and channel power gain $g$. Thus, the source requires the relay's data cooperation so that its data can be delivered to the destination.

We consider that the source can acquire energy from an external source at a price of $\sigma$ per unit of energy. The energy source is assumed to be capable of providing the source with any amount of energy that is required by the source's chosen strategy. The model is adopted so that we can contrast the source energy availability with that of the relay. We consider that the relay does not have access to any energy supply, and has to harvest its energy from the source's transmission at a given harvesting efficiency $\eta \in [0, 1]$. That is, the relay can use $\eta$ fraction of the energy it harvests[1] [2]. Since this is the only source of energy for the relay, it needs the source to transmit. That is, each node *needs* the other's cooperation—signal or energy—so that it can deliver its data to the destination.

We employ decode-and-forward relaying. The source can choose an average transmit power $p$ as long as it is below a maximum power value, i.e., $0 \leq p \leq P$. The relay's strategy is a fraction of time, $\delta \in [0, 1]$. The relay uses the $1 - \delta$ fraction of its time to decode the source's messages. The relay spends the remaining $\delta$ fraction harvesting energy from the source's signal to be used for the transmission of both the source's and its own messages. In other words, $1 - \delta$ fraction of the time, the relay will be operating in favor of the source's utility only, whereas the remainder of its operation improves both utilities.

The communication scenario is composed of two phases, $A$ and $B$, see Fig. 2. Without loss of generality, the two phases are considered to be of duration $T$ each, noting that the results extend to unequal phase durations in a straightforward manner. Only the source transmits in phase $A$, and only the relay transmits in phase $B$. The $\delta$ fraction chosen by the relay divides phase $A$ into two subphases: $A_1$ and $A_2$. This directly follows from the definition of $\delta$. That is, node $R$ will use $\delta$ fraction of phase $A$ to harvest energy, and the remaining fraction to decode node $S$'s data. Consequently, we have two subphases $A_1$ and $A_2$ of duration $(1 - \delta)T$ and $\delta T$, respectively.

- Subphase $A_1$: Node $S$ transmits at $p/(1 - \delta)$. Node $R$ listens to $S$'s transmission and decodes $S$'s messages.
- Subphase $A_2$: Node $S$ transmits at $p/\delta$. Node $R$ uses all of the received signal for energy harvesting. The receive power at $R$ is $hp/\delta$. Node $R$ harvests $\eta hpT$.

[1] This overall loss factor can accommodate the inefficiencies in the harvesting process as well as circuit energy costs for energy transfer [2], [36].

Note that the total energy spent by node $S$ is $2pT$, and the incurred energy cost is $2\sigma pT$. Node $R$ spends phase $B$ forwarding node $S$'s data and transmitting its own data. Node $R$ can maximize the amount of data it transmits to node $D$ by spending all of the harvested energy at a constant transmit power of $\eta hp$. Note that, node $R$ must use the $\eta hpT$ units of energy both for forwarding node $S$'s data and transmitting its own data. Node $S$ is silent in phase $B$.

Given that the strategies chosen by nodes $S$ and $R$ are $p$ and $\delta$, respectively, their utilities $u_S$ and $u_R$ defined as the average throughput over the two phases, i.e.,

$$u_S(p, \delta) = \frac{1 - \delta}{4} \log \left( 1 + h \frac{p}{1 - \delta} \right) - \sigma p, \qquad (1)$$

$$u_R(p, \delta) = \frac{1}{4} \log(1 + \eta hgp) - \frac{1 - \delta}{4} \log \left( 1 + h \frac{p}{1 - \delta} \right). \qquad (2)$$

Note that, the source utility is jointly concave in $p$ and $\delta$ [37, §3.2.6] whereas this is not necessarily the case for the relay utility.

The definition of the utilities in (1) and (2) dictates that $\delta$ be chosen large enough so that $u_R(p, \delta) \geq 0$. In other words, if node $R$ chooses a small $\delta$ and cannot harvest sufficient energy to forward all of node $S$'s data, then $u_R(p, \delta) < 0$. Therefore, node $R$ must limit the amount of data it receives from node $S$ by increasing $\delta$ accordingly so that it will have enough energy for node $S$'s data, and possibly for its own data. While the current formulation of the utilities allows node $R$ to forward more of node $S$'s data than it can with the harvested energy, this results in a negative utility for node $R$. Given that it is interested in its own utility, node $R$ will never choose such a low $\delta$ as will be demonstrated.

*Remark 1:* Our communication scheme foresees that each node is informed with the other node's chosen strategy so that their best reaction, as well as $p/(1 - \delta)$ and $p/\delta$, can be computed. This information would be collected by the destination and fed back to the nodes, causing negligible signaling overhead, which is a reasonable compromise in order to optimize the system throughput.

*Remark 2:* We consider the energy values for the source's transmission of its own data and energy transfer to be equal for ease of exposition. One can easily assume an arbitrary allocation of the total energy of $2pT$ units. This results in the same utilities up to a scaling factor for the transmit powers, and our results carry through. Moreover, this allocation can be optimized by the network operator, i.e., the destination, to maximize a weighted sum of $u_S(p, \delta)$ and $u_R(p, \delta)$. Such a

metric can further emphasize the relative importance of the source versus the relay data.

In the sequel, we consider the system model described in this section and its extensions to multi-terminal networks, i.e., with multiple sources and multiple relays, in various game theoretic scenarios. We begin with a Stackelberg competition where the relay is the leader and the source is the follower.

## III. STACKELBERG COMPETITION WITH THE RELAY AS THE LEADER

### A. Two-Hop Channel with One Source

In this subsection, we formulate and solve a Stackelberg game for the communication scenario described above with the relay node as the leader of the game. In a Stackelberg game, the follower chooses a strategy that maximizes the follower's utility given the leader's strategy. That is, the leader and the follower play a sequential game where the follower must react to the leader's strategy optimally. The leader is capable of calculating the follower's best response to any leader strategy. The leader hence chooses a strategy that maximizes its own utility knowing how the follower will react [34].

Define a Stackelberg game given by $(\{S, R\}, \{\mathcal{I}_S, \mathcal{I}_R\}, \{u_S, u_R\})$ where $S$ and $R$ are the players, the strategy spaces are given by $\mathcal{I}_S = [0, P] \ni p$ and $\mathcal{I}_R = [0, 1] \ni \delta$, and the payoffs, i.e., the utilities $u_S$ and $u_R$, are as in (1) and (2). In the sequel, we refer to nodes $R$ and $S$ as the leader and the follower, respectively.

Given any leader strategy $\delta \in \mathcal{I}_R$, the follower solves

$$p(\delta) = \arg \max_{p' \in \mathcal{I}_S} u_S(p', \delta). \tag{3}$$

The utility $u_S(p', \delta)$ is concave in $p'$ and (3) is a one dimensional optimization problem with an interval as the feasible set. We thus solve the problem by relaxing the constraint $p' \in \mathcal{I}_S$ to $p' \in \mathbb{R}$ and projecting the solution to $\mathcal{I}_S$. The unconstrained solution is found as the $p'$ value that satisfies

$$\frac{\partial}{\partial p} u_S(p, \delta) = \left. \frac{h}{4 \ln 2 \left(1 + h \frac{p}{1-\delta}\right)} - \sigma \right|_{p=p'} = 0. \tag{4}$$

We next project this relaxed solution into the feasible set and identify the optimal solution of (3) as

$$p(\delta) = \min \left\{ \max \left\{ \left(\frac{1}{4\sigma \ln 2} - \frac{1}{h}\right)(1 - \delta), 0 \right\}, P \right\}. \tag{5}$$

It can be seen that, $p(\delta)$ is nonincreasing in $\sigma$, nondecreasing in $h$, and nonincreasing in $\delta$. Node $S$ reacts to a large $\delta$ chosen by node $R$ by lowering the average transmit power. This is because a larger $\delta$ implies less time dedicated to improving node $S$'s utility, and the throughput it can attain can no longer compensate for the incurred energy cost. The leader knows this, i.e., node $R$ can calculate $p(\delta)$ for all $\delta \in \mathcal{I}_R$. The leader takes this information into account when choosing a $\delta$, and solves

$$\delta = \arg \max_{\delta' \in \mathcal{I}_R} u_R(p(\delta'), \delta'). \tag{6}$$

Before solving (6), let us take a closer look at $p(\delta)$ in (5). If $\phi \triangleq \frac{1}{4\sigma \ln 2} - \frac{1}{h} \leq 0$, then $p(\delta) = 0$ for all $\delta \in \mathcal{I}_R$. In this case, the objective of (6) is zero, and regardless of the choice of $\delta$, the total utility is zero. This results from the energy price $\sigma$ of node $S$ being too high, or the power gain to the relay $h$ being too low, i.e., node $S$ could not attain a positive utility even if it were allocated the entire transmission session with $\delta = 0$.

Suppose now that $\phi > 0$, thus node $S$ has incentive to transmit. In this case, we can restate $p(\delta)$ as

$$p(\delta) = \begin{cases} P & \text{if } \delta \in \mathcal{I}_{R,1} \\ \phi(1-\delta) & \text{if } \delta \in \mathcal{I}_{R,2} \end{cases} \tag{7}$$

where $\mathcal{I}_{R,1} \triangleq [0, \bar{\delta})$, $\mathcal{I}_{R,2} \triangleq [\bar{\delta}, 1]$, and $\bar{\delta} \triangleq 1 - \min\{P/\phi, 1\}$. Note that $\mathcal{I}_{R,1} \cup \mathcal{I}_{R,2} = \mathcal{I}_R$ and if $\phi \leq P$, $\mathcal{I}_{R,1} = \emptyset$ and $\mathcal{I}_{R,2} = \mathcal{I}_R$. Using the piecewise description of $p(\delta)$ in (7), we separate the feasible region of (6) into two regions $\mathcal{I}_{R,1}$ and $\mathcal{I}_{R,2}$, solve the problem in each region, and finally identify the optimal $\delta$.

1) $\mathcal{I}_{R,1}$ as the feasible region of (6): In this case, $p(\delta) = P$ for all $\delta \in \mathcal{I}_{R,1}$. The objective of (6) becomes

$$u_R(P, \delta) = \frac{1}{4} \log(1 + \eta h g P) - \frac{1 - \delta}{4} \log\left(1 + h \frac{P}{1 - \delta}\right) \tag{8}$$

and is strictly increasing in $\delta$. Therefore, no $\delta \in \mathcal{I}_{R,1}$ can outperform $\bar{\delta} \in \mathcal{I}_{R,2}$. In other words, the maximizer of (6) lies in $\mathcal{I}_{R,2}$.

2) $\mathcal{I}_{R,2}$ as the feasible region of (6): In this case, $p(\delta) = \phi(1 - \delta)$ for all $\delta \in \mathcal{I}_{R,2}$. The objective of (6) becomes

$$u_R(\phi(1 - \delta), \delta) = \frac{1}{4} \log(1 + \eta h g \phi(1 - \delta)) - \frac{1 - \delta}{4} \log(1 + h \phi). \tag{9}$$

The utility $u_R(\phi(1 - \delta), \delta)$ is concave in $\delta$, and (6) is a one dimensional optimization problem where $\delta \in [\bar{\delta}, 1]$. We again remove the interval constraint first and find the unconstrained solution as the $\delta'$ value that satisfies

$$\frac{\partial}{\partial \delta} u_R(\phi(1 - \delta), \delta) = \frac{-\eta h g \phi}{4 \ln 2 \left(1 + \eta h g \phi(1 - \delta)\right)} + \left. \frac{1}{4} \log(1 + h\phi) \right|_{\delta = \delta'} = 0. \tag{10}$$

We next project this solution into the feasible set as

$$\delta' = \min \left\{ \max \left\{ 1 - \frac{1}{\ln\left(\frac{h}{4\sigma \ln 2}\right)} + \frac{1}{\eta g \left(\frac{h}{4\sigma \ln 2} - 1\right)}, \bar{\delta} \right\}, 1 \right\} \tag{11}$$

which maximizes $u_R(\phi(1 - \delta), \delta)$.

We can observe from (11) that as $\sigma$ increases or $h$ decreases, node $R$ tends to choose a lower $\delta$. This follows from the fact

that node $R$ knows that such changes in $\sigma$ and $h$ will cause node $S$ to lower $p$. Therefore, node $R$ proactively lowers $\delta$ so as to counteract the influence of $\sigma$ and $h$ on node $S$'s decision. This demonstrates how the leader uses its knowledge of how the follower reacts to the leader's strategy.

### B. Two-Hop Channel with Multiple Sources

In this subsection, we study the two-hop channel with multiple source nodes. We introduce an additional layer of competition by utilizing an auction scheme between the multiple source nodes where the auctioned item is the relay's signal cooperation. That is, only the winning source node can deliver its data to the destination using the relay. We foresee that this will improve the relay's utility since the sources need to outbid each other in order to obtain positive utilities, and consequently, the winner will be a source that is most willing to compromise its own utility. In this work, we employ a Vickrey auction to model the inter-source competition.

*Definition 1:* A Vickrey auction is a sealed bid second price auction mechanism where an item is to be assigned to a bidder in exchange for a payment [34], [35]. The bidders submit their bids simultaneously with no knowledge of other bids, hence the sealed bid property. The winner of the auction is the bidder who has placed the highest bid. The winner is required to pay the second highest bid, hence the second price property.

Due to the second price property of Vickrey auctions, it is a weakly dominant action for each bidder to bid their true valuation of the item [34]. That is, for each bidder, bidding the true valuation results in a payoff that is no less than the payoff that is obtained by submitting any other bid. This encourages the bidders to bid the maximum price they are willing to pay. The second price property is thus a desirable property of Vickrey auctions, and it is particularly useful in this work since it results in an improvement in the relay's utility while attaining a positive utility for the source as well. In our formulation of the auction, the source nodes are the bidders and their bids are the relay utilities that can be obtained with the transmit powers chosen by the sources.

Consider a half-duplex Gaussian two-hop network with $J$ source nodes, $S_j$, $j \in \mathcal{J} \triangleq \{1, 2, \ldots, J\}$, a decode-and-forward relay node, $R$, and a destination node, $D$, as shown in Fig. 3. The power gain is $h_j$ from node $S_j$ to $R$, $j \in \mathcal{J}$, and $g$ from $R$ to $D$. As before the Gaussian noise variance of all links is assumed to be unity without loss of generality. The source nodes are not directly connected to the destination. All source and relay nodes are assumed selfish in the sense that each node wishes to maximize the amount of its own data delivered to the destination. Node $S_j$ has energy available from an external source of energy at a price of $\sigma_j$ per unit of energy, $j \in \mathcal{J}$. The relay node can harvest a fraction of the energy in the source's transmission at efficiency $\eta \in [0, 1]$, and use this energy to transmit its data to $D$.

The communication scenario studied in this section is based on an auction between the source nodes. Node $S_j$ chooses average transmit power $p_j \in [0, P_j]$ and bids the resulting relay utility. Here, $P_j$ models a maximum average power constraint at node $S_j$, $j \in \mathcal{J}$. Only the winning source will be given the
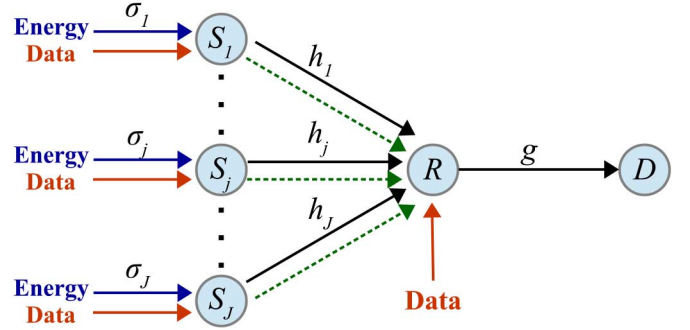


Fig. 3. The two-hop channel with multiple source nodes. Data links are shown in solid lines and energy transfers are shown in dashed lines.

chance to adjust its average transmit power, and transmit its data to the relay and subsequently to the destination. The auction scheme will be explained in the sequel, but suppose, for the moment, node $S_{j*}$ wins the auction, and settles on average transmit power $p^* \in [0, P_{j*}]$. The relay chooses $\delta \in [0, 1]$ denoting the fraction of the received signal that will be used for energy harvesting.

Without loss of generality, suppose a two-phase communication scheme with phases $A$ and $B$ of equal duration $T$ is employed for the transmission of $S_{j*}$ and $R$, respectively. In phase $A$, $R$ listens to $S_{j*}$'s transmission for $(1 - \delta)T$ seconds while $S_{j*}$ transmits at $p^*/(1 - \delta)$, resulting in an average transmit power of $p^*$. During the remaining $\delta T$ seconds of phase $A$, $S_{j*}$ transmits at transmit power $p^*/\delta$ while $R$ harvests $\eta h_{j*} p^* T$. In phase $B$, the relay uses the harvested energy to forward $S_{j*}$'s data and send its own data to node $D$. For equal duration of phases and the winning bidder $S_{j*}$, the utilities $u_{S_j}$ for node $S_j$, and $u_{R|j*}$ for node $R$ are

$$u_{S_{j*}}(p^*, \delta) = \frac{1 - \delta}{4} \log\left(1 + h_{j*}\frac{p^*}{1 - \delta}\right) - \sigma_{j*}p^*, \quad (12)$$

$$u_{S_j}(p_j, \delta) = 0, \qquad j \in \mathcal{J} \setminus \{j^*\}, \quad (13)$$

$$u_{R|j*}(p^*, \delta) = \frac{1}{4}\log(1 + \eta h_{j*}gp^*)$$
$$- \frac{1 - \delta}{4}\log\left(1 + h_{j*}\frac{p^*}{1 - \delta}\right). \quad (14)$$

Instead of serving all of the source nodes at the same time, the relay puts its relaying services up for auction. Each source node chooses an average transmit power $p_j$ given leader strategy $\delta$, and bids the leader utility $u_{R|j}(p_j, \delta)$ that results from these strategies. Due to the truthful bidding property of the Vickrey auction, the sources are willing to increase their bids as long as their own utilities are nonnegative. That is, source $S_j$ wishes to ensure that

$$\frac{1 - \delta}{4}\log\left(1 + h_j\frac{p_j}{1 - \delta}\right) \geq \sigma_j p_j, \qquad \forall j \in \mathcal{J}. \quad (15)$$

This condition results in an upperbound on the relay utility given by

$$u_{R|j}(p_j, \delta) \leq \frac{1}{4}\log(1 + \eta h_j gp_j) - \sigma_j p_j, \qquad \forall j \in \mathcal{J}. \quad (16)$$

Since the sources wish to bid the highest relay utility while maintaining a nonnegative utility for themselves, they calculate their transmit powers in such a way that results in an equality for (16), and thus for (15). That is, node $S_j$ computes $p_j$ by solving

$$\frac{1-\delta}{4} \log\left(1 + h_j \frac{p_j}{1-\delta}\right) - \sigma_j p_j = 0, \qquad \forall j \in \mathcal{J}. \quad (17)$$

This equation has a solution, other than $p_j = 0$, for all sources which can be stated as

$$p_j = -\frac{1-\delta}{4\sigma_j \ln 2} W\left(-\frac{4\sigma_j \ln 2}{h_j} e^{-\frac{4\sigma_j \ln 2}{h_j}}\right) - \frac{1-\delta}{h_j} \quad (18)$$

for all $j \in \mathcal{J}$. Here, we use the Lambert W function [38] to identify $p_j$. Note that $W(\cdot)$ denotes the lower branch of the Lambert W function. We can rewrite (17) as

$$f(h_j p_j) \triangleq \left(\frac{h_j p_j}{1-\delta}\right)^{-1} \log\left(1 + \frac{h_j p_j}{1-\delta}\right) = 4\frac{\sigma_j}{h_j}, \quad (19)$$

for all $j \in \mathcal{J}$. Since $x \log(1 + 1/x)$ is strictly increasing in $x \geq 0$, $f(h_j p_j)$ is strictly decreasing in $h_j p_j$. Thus, we can infer that $h_j p_j$ increases as $\sigma_j / h_j$ decreases. That is, the source node with the lowest $\sigma_j / h_j$ value can provide the highest receive power at the relay. Hence, the sources that can buy energy at a lower price, or have a better link to the relay are willing to bid higher average receive powers at the relay. The winner of the auction is the source node that can provide the highest utility to the relay with its bid, i.e.,

$$j^* = \arg\max_{j \in \mathcal{J}} u_{R|j}(p_j, \delta) \quad (20)$$

$$= \arg\max_{j \in \mathcal{J}}(\log(1 + \eta g h_j p_j) - f(h_j p_j) h_j p_j) \quad (21)$$

where (21) follows from (17). Recall that $-f(h_j p_j)$ is strictly increasing in $h_j p_j$. Thus, we have

$$j^* = \arg\min_{j \in \mathcal{J}} h_j p_j = \arg\min_{j \in \mathcal{J}} \frac{\sigma_j}{h_j}. \quad (22)$$

In other words, the winner of the auction is the source node with the highest received power at the relay, or equivalently, the one with the lowest $\sigma_j / h_j$ ratio[2]. Note that, the source nodes need not calculate their bids in order to determine who will win the auction. Recall that in the single source case, $\delta$ turned out to be decreasing in $\sigma / h$ as observed in (11). It is thus shown that with multiple sources, the source node with the lowest $\sigma_j / h_j$ will agree with the largest $\delta$ chosen by the relay, and therefore provide the largest utility to the relay. Consequently, the auction improves the auctioneer's payoff, in this case, the relay's utility.

Let $S_{j\dagger}$ be the runner up. Node $S_{j*}$ must provide at least $u_{R|j\dagger}(p_{j\dagger}, \delta)$. We know that $h_{j*} p_{j*} \geq h_{j\dagger} p_{j\dagger}$, and thus $S_{j*}$ can lower its transmit power to $h_{j\dagger} p_{j\dagger} / h_{j*}$ and provide the required relay utility. However, if $p_{j*}(\delta)$ computed as

$$p_{j*}(\delta) = \arg\max_{p' \in [0, P_{j*}]} u_{S_{j*}}(p', \delta) \quad (23)$$

is larger than $h_{j\dagger} p_{j\dagger} / h_{j*}$, then both $S_{j*}$ and $R$ can have higher utilities if $S_{j*}$ lowers its power only to $p_{j*}(\delta)$. Thus,

$$p^*(\delta) = \max\{p_{j*}(\delta), h_{j\dagger} p_{j\dagger} / h_{j*}\}. \quad (24)$$

That is, the Vickrey auction results in a minimum average power requirement at node $S_{j*}$. Since the auctioneer, i.e., node $R$, would have to sacrifice its utility by lowering $\delta$ in order to increase the average transmit power of the source node if there were only one source node, the auction results in an increase in the auctioneer's utility.

## IV. STACKELBERG COMPETITION WITH THE SOURCE AS THE LEADER

In this section, we consider the reciprocal Stackelberg games to those in Section III with the source node as leader.

### A. Two-Hop Channel with One Relay

Consider first when there is only one relay[3]. Given leader strategy $p$, the follower, node $R$, solves

$$\delta(p) = \arg\max_{\delta' \in \mathcal{J}_R} u_R(p, \delta'). \quad (25)$$

Since $u_R(p, \delta)$ is increasing in $\delta$, it is immediate that the solution of (25) is $\delta(p) = 1$ for all $p > 0$, and $u_R(p, \delta) = 0$ for any $\delta$ if $p = 0$. The leader knows $\delta(p)$, and solves

$$p = \arg\max_{p' \in \mathcal{J}_S} u_S(p', \delta(p')). \quad (26)$$

If the leader picks a positive $p$, then the first term in its utility will be zero, but the second term will be negative. Thus, the optimal strategy for the leader is to stop transmission, resulting in vanishing utilities for both nodes. Therefore, with a single relay in a Stackelberg setup with the source as the leader, the two players cannot agree on a strategy pair that yields positive utilities. However, positive utilities can be facilitated by introducing multiple relay nodes, and another layer of competition to the system. As will be shown next, this approach improves the utilities under the source's leadership.

### B. Two-Hop Channel with Multiple Relays

The communication system we study in this section is similar to the one in Section III-B, except here, there are multiple relay nodes instead of the multiple source nodes. We thus have a half-duplex Gaussian two-hop network with a source node, $S$, $K$ decode-and-forward relay nodes, $R_k$, $k \in \mathcal{K} \triangleq \{1, 2, \ldots, K\}$, and a destination node, $D$, as shown in Fig. 4. All nodes have data to transmit to node $D$, and they wish to maximize their individual throughputs. The power gain is $h_k$ from $S$ to $R_k$, and $g_k$ from $R_k$ to $D$ with unit noise variance. Node $S$ can purchase energy at price $\sigma$. Relay $R_k$ can harvest energy from $S$'s transmission at efficiency $\eta_k \in [0, 1]$ to transmit its data to $D$.

[2]In case there are multiple maximizers to (22), the winner is picked randomly among the source nodes that provide the highest received powers.

[3]We consider this scenario as a pedagogical step in order to build upon this model.
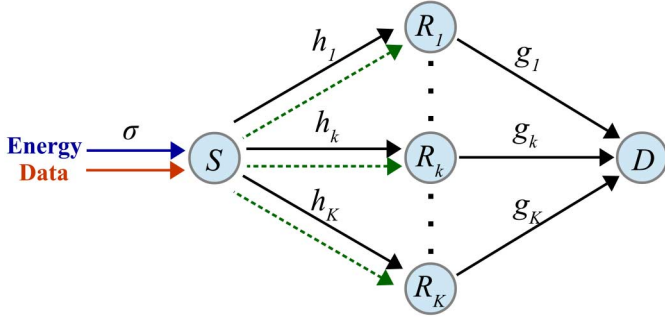
Fig. 4. The two-hop channel with multiple relay nodes. Data links are shown in solid lines and energy transfers are shown in dashed lines. Data arrivals at the relays are omitted.

We consider an auction between the relay nodes. The underlying communication scheme is again carried out in two phases. Node $S$'s strategy is the average transmit power $p \in [0, P]$ where $P$ is the maximum average power. Node $R_k$ bids harvesting fraction $\delta_k \in [0, 1]$, $k \in \mathcal{K}$. The winning relay will be able to adjust its fraction, and take part in the two-phase communication scheme. Suppose the winning relay is node $R_{k*}$, and the adjusted harvesting fraction is $\delta^* \in [0, 1]$. Given that $k*$ is the winning bidder index, the resulting utilities $u_{S|k*}$ for node $S$ and $u_{R_k}$ for node $R_k$ can be stated as

$$u_{S|k*}(p, \delta^*) = \frac{1 - \delta^*}{4} \log\left(1 + h_{k*}\frac{p}{1 - \delta^*}\right) - \sigma p, \quad (27)$$

$$u_{R_{k*}}(p, \delta^*) = \frac{1}{4}\log(1 + \eta_{k*}h_{k*}g_{k*}p)$$
$$- \frac{1 - \delta^*}{4}\log\left(1 + h_{k*}\frac{p}{1 - \delta^*}\right), \quad (28)$$

$$u_{R_k}(p, \delta_k) = 0, \qquad k \in \mathcal{K} \setminus \{k*\}. \quad (29)$$

Let us now consider a Vickrey auction where the auction item is the energy that can be harvested from the source's transmission, and the bids are the utilities for the source resulting from the harvesting fractions chosen by the relays. Similar to Section III-B, the relays are motivated to pay the maximum price they can pay. Since the relay utilities are increasing in $\delta$, each relay is willing to lower their $\delta$ until the relay utility vanishes. That is, node $R_k$ solves

$$\log(1 + \eta_k h_k g_k p) - (1 - \delta_k)\log\left(1 + h_k\frac{p}{1 - \delta_k}\right) = 0, \quad (30)$$

for all $k \in \mathcal{K}$, which yields

$$\delta_k = \max\left\{\left[\frac{1}{\psi_k}W\left(-\frac{\psi_k}{h_k p}e^{-\frac{\psi_k}{h_k p}}\right) + \frac{1}{h_k p}\right]^{-1} + 1, 0\right\} \quad (31)$$

for all $k \in \mathcal{K}$ where $\psi_k = \ln(1 + \eta_k h_k g_k p)$. To determine the winner of the auction, one need not calculate $\delta_k$ however. Instead, we use (30) to express the source utility in a simpler form as follows.

The winner of the auction is the relay node that can provide the source with the highest utility. That is,

$$k* = \arg\max_{k \in \mathcal{K}} u_{S|k}(p, \delta_k) \quad (32)$$
$$= \arg\max_{k \in \mathcal{K}}(\log(1 + \eta_k h_k g_k p) - 4\sigma p) \quad (33)$$

where we use (30) to arrive at (33). Since the source utility is strictly increasing in $\eta_k h_k g_k$, we have

$$k* = \arg\max_{k \in \mathcal{K}} \eta_k h_k g_k. \quad (34)$$

In other words, the winner of the auction is the relay that can utilize its harvested energy most efficiently, and thus deliver the most data to the destination. Let $R_{k^\dagger}$ be the runner up. Relay $R_{k*}$ must provide at least $u_{S|k^\dagger}(p, \delta_{k^\dagger})$. Since the winning relay's own utility is increasing in the harvesting fraction, it is optimal for $R_{k*}$ to provide exactly $u_{S|k^\dagger}(p, \delta_{k^\dagger})$. Thus, the adjusted fraction can be found by solving

$$\log(1 + \eta_{k^\dagger}h_{k^\dagger}g_{k^\dagger}p) = (1 - \delta^*)\log\left(1 + h_{k*}\frac{p}{1 - \delta^*}\right) \quad (35)$$

The unique solution to (35) can be computed as

$$\delta^* = \left[\frac{1}{\psi_{k^\dagger}}W\left(-\frac{\psi_{k^\dagger}}{h_{k*}p}e^{-\frac{\psi_{k^\dagger}}{h_{k*}p}}\right) + \frac{1}{h_{k*}p}\right]^{-1} + 1 \quad (36)$$

by using the lower branch of the Lambert W function.

Since $\eta_{k*}h_{k*}g_{k*} \geq \eta_{k^\dagger}h_{k^\dagger}g_{k^\dagger}$ due to (34), the right-hand side of (35) is at least $\log(1 + \eta_{k^\dagger}h_{k^\dagger}g_{k^\dagger}p)$ when $\delta^* = \delta_{k*}$, and it decreases to 0 as $\delta^*$ increases to 1. Thus, a unique solution exists, and unless $\eta_{k*}h_{k*}g_{k*} = \eta_{k^\dagger}h_{k^\dagger}g_{k^\dagger}$, the follower can also have a positive utility while delivering the required leader utility. The leader can now calculate $p$ using (26). As a final remark, we note that no Stackelberg game with the source as the leader and any one of the relays as the follower results in a positive utility for any node. By introducing competition between the relays by means of an auction, we obtain positive utilities.

## V. SIMULTANEOUS GAMES FOR THE TWO-HOP CHANNEL

We have so far studied sequential games for the two-hop channel where the leader node plays first knowing how the follower will react, and then the follower reacts to the leader's strategy in the optimal way. In this section, we consider a distributed setting where all nodes are equal, and study simultaneous games where both nodes will react to each other's strategies optimally.

### A. Noncooperative Game

Consider the noncooperative game given by $(\{S, R\}, \{\mathcal{I}_S, \mathcal{I}_R\}, \{u_S, u_R\})$. A strategy pair $(p^*, \delta^*)$ is a Nash equilibrium if and only if neither player has incentive to unilaterally deviate from these two strategies, i.e., each player plays its best response to the other player's strategy at the equilibrium [34]. The best response is defined as the strategy that maximizes

a player's utility given the other player's strategy. We aim to identify the Nash equilibria of this game.

The observation that $u_R(p, \delta)$ is strictly increasing in $\delta$ for $p > 0$ yields that the best response of node $R$ to any $p > 0$ is $\delta = 1$. But, with $\delta = 1$, node $S$'s strategy is nonpositive and is maximized by $p = 0$. In other words, we have

- $(p^*, \delta^*) \notin (0, P] \times [0, 1)$ since node $R$ has incentive to increase $\delta$ to 1,
- $(p^*, \delta^*) \notin (0, P] \times \{1\}$ since node $S$ has incentive to decrease $p$ to 0, and
- $(p^*, \delta^*) \notin \{0\} \times [0, 1)$ since node $S$ has incentive to increase $p$ to a positive value.

Therefore, the unique equilibrium is $(p^*, \delta^*) = (0, 1)$ which results in $u_S(0, 1) = u_R(0, 1) = 0$, and thus zero total utility.

This result can be interpreted as follows. Although there exists $(p, \delta) \in \mathcal{I}_S \times \mathcal{I}_R \setminus \{(0, 1)\}$ with a positive total utility, the players cannot agree upon this strategy pair due to the noncooperative nature of the setting at hand. We can, however, not only ensure a positive total utility, but also maximize it at a noncooperative equilibrium by modifying the utilities with pricing as explained next.

### B. Social Optimality of the Two-Hop Channel

We employ a pricing scheme on the utilities for both players to facilitate an equilibrium that results in social optimality for the entire network. The pricing scheme is similar to the interference compensation scheme in [39], except here, the prices are not determined by the players, but are announced by node $D$. Consider the noncooperative game $(\{S, R\}, \{\mathcal{I}_S, \mathcal{I}_R\}, \{\tilde{u}_S, \tilde{u}_R\})$ where the modified utilities are given by

$$\tilde{u}_S(p, \delta; \pi_R) = u_S(p, \delta) - p\pi_R, \qquad (37)$$

$$\tilde{u}_R(p, \delta; \pi_S) = u_R(p, \delta) - \delta\pi_S, \qquad (38)$$

where the prices $\pi_R$ and $\pi_S$ model a penalty charged to the two nodes as a result of the influence of their individual strategies on the other node's utility. For instance, $p\pi_R$ is a penalty charged to node $S$ as a result of the impact of its strategy $p$ on node $R$'s utility. The prices are announced by node $D$ and can be used by node $D$ to maximize the amount of data it receives. Node $D$ wishes an optimal solution of the social problem which can be expressed as

$$\max_{(p, \delta) \in \mathcal{I}_S \times \mathcal{I}_R} (u_S(p, \delta) + u_R(p, \delta)) \qquad (39)$$

or equivalently as

$$\max_{(p, \delta) \in \mathcal{I}_S \times \mathcal{I}_R} (\log(1 + \eta h g p) - 4\sigma p) \qquad (40)$$

Problem (40) is a convex program whose objective is strictly concave in $p$ and constant in $\delta$. The optimal solution can be computed as

$$p^{\ddagger} = \min \left\{ \max \left\{ \frac{1}{4\sigma \ln 2} - \frac{1}{\eta h g}, 0 \right\}, P \right\}. \qquad (41)$$

Recall that $\delta$ determines what portion of the data received at the destination is node $R$'s data and what portion is node $S$'s

data. Since the social problem as expressed in (40) maximizes the total data delivered to node $D$, regardless of nodes $S$ and $R$'s contribution, any feasible $\delta$ is optimal. However, node $D$ may wish that the data it receives be composed of node $S$'s and node $R$'s data in a particular ratio. Let $\delta^{\ddagger}$ be the corresponding harvesting fraction. We rewrite (40) as

$$\max_{(p, \delta) \in \mathcal{I}_S \times \mathcal{I}_R} (u_S(p, \delta) + u_R(p, \delta) - (\delta - \delta^{\ddagger})^2). \qquad (42)$$

Problem (42) is a convex program with a strictly concave objective, thus it admits a unique optimizer, say $(p^{\ddagger}, \delta^{\ddagger})$. This strategy pair can be made the unique Nash equilibrium if node $D$ calculates the prices using

$$\pi_S(p, \delta) = -\frac{\partial u_S(p, \delta)}{\partial \delta} + 2(\delta - \delta^{\ddagger}), \qquad (43)$$

$$\pi_R(p, \delta) = -\frac{\partial u_R(p, \delta)}{\partial p}. \qquad (44)$$

We employ a modified version of the asynchronous distributed pricing (ADP) algorithm in [39]. The strategies are initially set as $p = 0$ and $\delta = 0$, and are then asynchronously updated by nodes $S$ and $R$ using their best response updates. Consequently, any limit point of the modified ADP algorithm is a Nash equilibrium. After each update of $p$ or $\delta$, node $D$ calculates $\pi_S$ and $\pi_R$ using (43) and (44), and announces them. This selection of the prices guarantees that any limit point of the algorithm will satisfy the Karush-Kuhn-Tucker (KKT) conditions of (42), and will thus be uniquely optimal [39, Theorem 1].

In order to prove the convergence of the modified ADP algorithm, we take an approach that is based on transforming the strategy space as was done in [39, Theorem 1]. Suppose that the prices are announced by the source and the relay nodes themselves, not the destination, and that the source and the relay will compute the prices using (43) and (44). We have

$$\frac{\partial^2 \tilde{u}_S(p, \delta; \pi_R)}{\partial p \partial \pi_R} = -1 < 0, \qquad (45)$$

$$\frac{\partial^2 \tilde{u}_R(p, \delta; \pi_S)}{\partial \delta \partial \pi_S} = -1 < 0. \qquad (46)$$

That is, the utilities have decreasing differences. Letting $\pi'_R \triangleq -\pi_R$ and $\pi'_S \triangleq -\pi_S$, we consider the same game with strategies $(p, \delta, \pi'_S, \pi'_R)$. For this game, the strategy space for each strategy is nonempty and compact, and the utilities are continuous in all strategies. Moreover, each node's utility has increasing differences in its own strategies since we have

$$\frac{\partial^2 \tilde{u}_S(p, \delta; \pi_R)}{\partial p \partial \pi_S} = 0, \qquad (47)$$

$$\frac{\partial^2 \tilde{u}_R(p, \delta; \pi_S)}{\partial \delta \partial \pi_R} = 0. \qquad (48)$$

Lastly, the derivatives in (45) and (46) are positive if we replace $\pi_S$ and $\pi_R$ by $\pi'_S$ and $\pi'_R$, respectively. Hence, the utilities have increasing differences, and we have a supermodular game for which the best response updates converge to a Nash equilibrium [40]. The original game is equivalent to the supermodular game
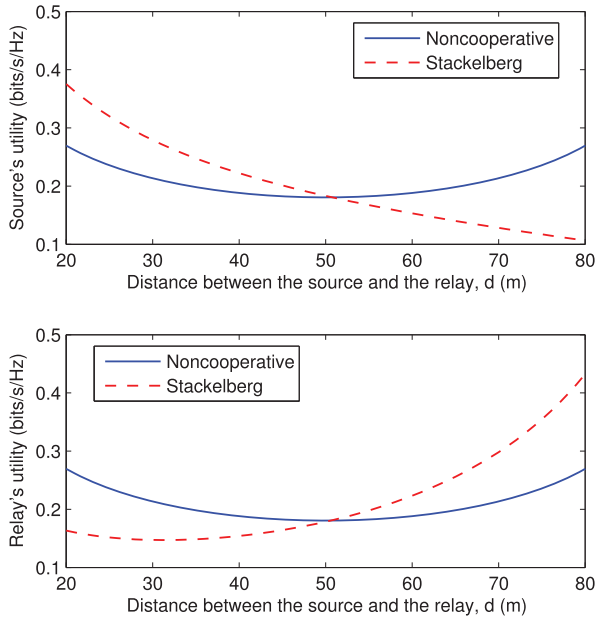
Fig. 5. The two players' utilities versus the location of node $R$ with $d' = 100$ m, $\sigma = 0.05$ bps/W, $\eta = 0.4$, and $P = 500$ mW for the Stackelberg game in Section III-A and the noncooperative game in Section V-B.
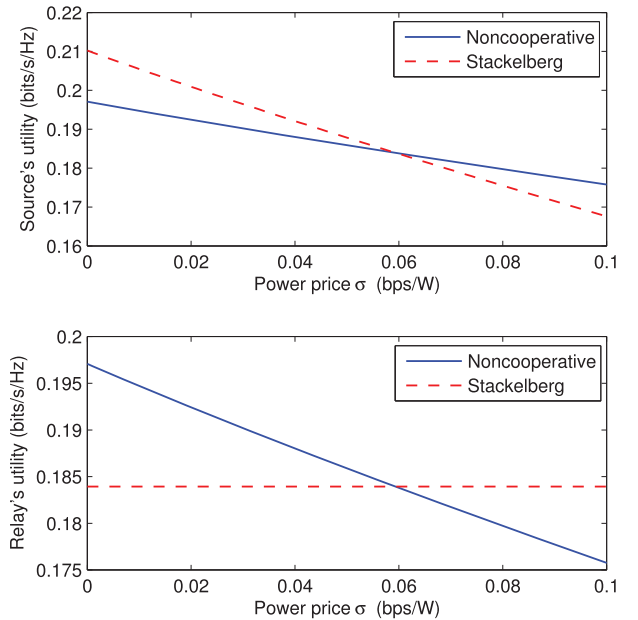


Fig. 6. The two players' utilities versus the power price with $d = 50$ m, $d' = 100$ m, $\eta = 0.4$, and $P = 500$ mW for the Stackelberg game in Section III-A and the noncooperative game in Section V-B.

since the prices are computed and announced in the same way. Note that, although the two players can choose any $p$ or $\delta$ value they want in the supermodular game, they are constrained to use (45) and (46) to compute the prices by definition of the game. Therefore, $(p^{\ddagger}, \delta^{\ddagger})$ is the unique Nash equilibrium of the supermodular game as well, and hence is the limit point of the modified ADP algorithm [39].

This completes the description of the socially optimal pricing scheme for the two-hop channel since we already know that $(p^{\ddagger}, \delta^{\ddagger})$ solves (42) optimally. Next, we implement all equilibria found in Sections III–V and compare the resulting utilities for varying channel parameters.

## VI. NUMERICAL RESULTS

We first present the simulation results for the two-hop channel with one source and one relay. We evaluate the utilities of the two players at the equilibria found in Sections III-A and V-B. We omit the equilibria found in Sections IV-A and V-A since they yield zero utilities for both players. The available bandwidth is 1 MHz, and the additive white Gaussian noise density is $10^{-19}$ W/Hz. We assume Rayleigh fading and average over 1000 realizations of the fading coefficients. We denote by $d$ the distance between the source and the relay, i.e., between nodes $S$ and $R$, and by $d'$ the distance between the source and the destination, i.e., between nodes $S$ and $D$. The channel power gains are computed using a model where the power gains corresponding to the mean fading level are $h = K/d^2$ and $g = K/(d' - d)^2$ respectively for the channel gain between $S$ and $R$, and $R$ and $D$. The carrier frequency is 900 MHz, and the reference distance for the path loss model is 1 m resulting in $K = -40$ dB [41], [42]. We vary the remaining parameters $d$, the power price $\sigma$, the harvesting efficiency $\eta$, and the maximum power $P$ in order to assess their impact on the utilities.

For the utilities resulting from the noncooperative equilibrium in Section V-B, we consider that node $D$ wishes to receive equal amounts of data from node $S$ and node $R$.

Fig. 5 shows the utilities resulting from the Stackelberg equilibrium in Section III-A, and the noncooperative Nash equilibrium with pricing in Section V-B. Recall that the latter is socially optimal, thus it maximizes the total utility in all settings considered here. For this set of simulations, node $S$ and node $D$ are stationary and node $R$ moves from node $S$ to node $D$ on a line. We observe for the noncooperative equilibrium that both nodes have higher utilities when node $R$ is close to node $S$ or node $D$. This follows from the fact that $R$ can harvest more energy when near node $S$, and can send more data when near node $D$. At the Stackelberg equilibrium, however, as node $R$ moves away from node $S$, its utility is increased while node $S$'s utility is decreased. Recall that $\delta$ is decreasing in $g$, and thus decreases as node $R$ moves away. This encourages node $S$ not to lower its transmit power. As a result, node $R$ has a higher utility as its channel to node $D$ is improved while node $S$'s utility suffers from a smaller channel gain.

Fig. 6 shows the utilities for varying power price $\sigma$, in order to observe the impact of the energy cost at node $S$. The power price causes node $S$ to be more conservative with its power usage. Thus, at the social optimum, both nodes' utilities are decreasing in $\sigma$. At the Stackelberg equilibrium, the power price does not impact node $R$'s operations. Node $R$ chooses a lower $\delta$ as $\phi$ decreases in order to give node $S$ incentive to transmit even though $\sigma$ is increasing. This demonstrates the advantage of having the additional knowledge of the follower's best reaction to any leader strategy.

Fig. 7 shows the utilities versus the harvesting efficiency $\eta$. The harvesting efficiency is varied from 0 with no harvested energy to 1, i.e., the ideal case. Since the energy harvested by
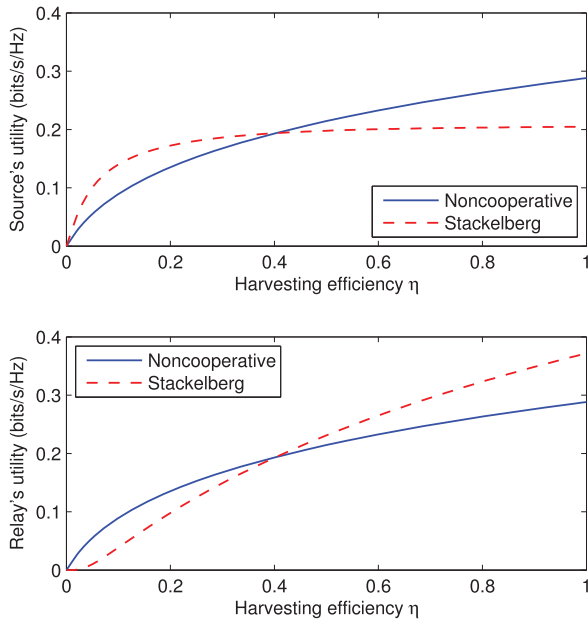
Fig. 7. The two players' utilities versus the harvesting efficiency with $d = 50$ m, $d' = 100$ m, $\sigma = 0.05$ bps/W, and $P = 500$ mW for the Stackelberg game in Section III-A and the noncooperative game in Section V-B.



Fig. 8. The two players' utilities versus the maximum average transmit power for node $S$ with $d = 50$ m, $d' = 100$ m, $\sigma = 0.05$ bps/W, and $\eta = 0.4$ for the Stackelberg game in Section III-A and the noncooperative game in Section V-B.

node $R$ is increasing in the harvesting efficiency, both utilities are increasing in $\eta$ at the social optimum. For low values of $\eta$ at the Stackelberg equilibrium, node $R$ cannot harvest enough energy for both node $S$'s data and its own data. Thus, in this regime, more of node $S$'s data can be delivered as $\eta$ increases. However, once $\eta$ is high enough, node $R$ limits the energy usage for forwarding node $S$'s data, and attains a larger utility as $\eta$ increases.

Fig. 8 shows the utilities for varying maximum power constraints. The maximum average transmit power that node $S$ can choose is varied from 0 to 1 W. We observe that both utilities are increasing in $P$ at the social optimum since the feasible space for node $S$'s strategy is larger with a higher $P$. However, the utilities for the socially optimal case are concave, which means the gain from a high $P$ diminishes as $P$ is further increased. This is due to the energy cost of node $S$ being a part of the total utility. At the Stackelberg equilibrium, node $R$ uses its knowledge of $p(\delta)$ in (5) to entice node $S$ to increase its transmit power. This eventually results in a decrease in node $S$'s utility due to the energy cost. However, at $P = 0.88$ W, the maximum power constraint is active and limits the transmit power chosen by node $S$. Node $R$ needs to sacrifice more of its own utility beyond this point.

Fig. 9 provides a comparison of the source utility if it were to simply use a direct link and did not participate in signal or energy cooperation. In this example, the direct link is 20 dB weaker than its path loss based attenuation, for example due to shadowing. Again, $S$ and $D$ are 100 m apart. As we can observe from the figure, the source obtains a larger utility with the help of the relay node as compared to its utility over the direct link to the destination without any relaying in this example. The two-hop scheme is even more advantageous when the relay is close to the source or the destination. This example is
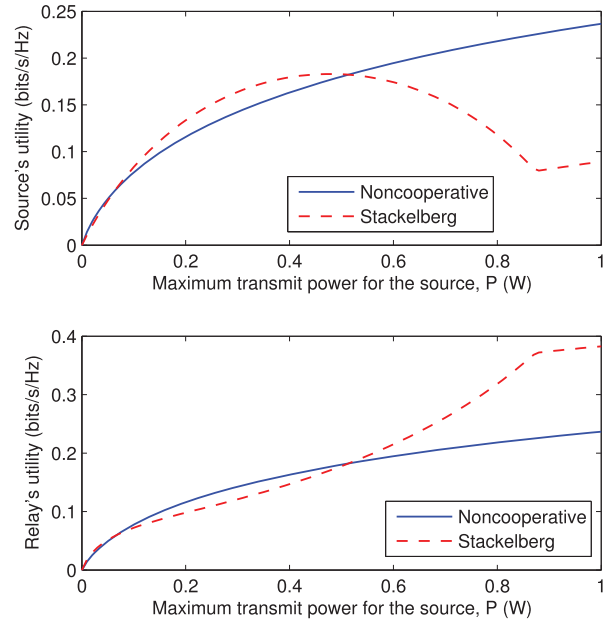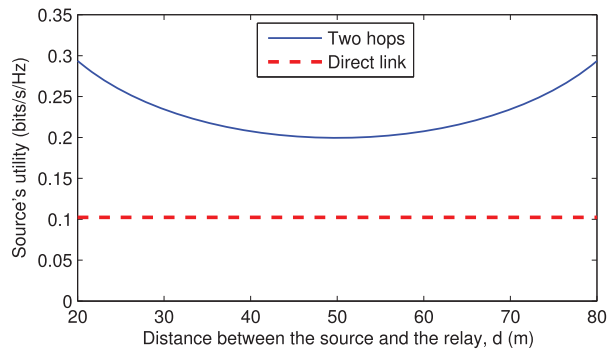


Fig. 9. Node $S$'s utility versus the location of node $R$ with $d' = 100$ m, $\sigma = 0.05$ bps/W, $\eta = 0.4$, and $P = 500$ mW for the noncooperative game in Section V-B and direct transmission from node $S$ to node $D$.

meant to demonstrate that the source would prefer employing the relay's services rather than sending its data directly to the destination, in conditions similar to those that render classical two-hop communications preferable, e.g., when the direct link is under unfavorable fading conditions; even though the relay demands additional energy for its own data in exchange, and the RF energy transfer leads to a significant loss due to channel gains.

For the multi-source/relay auction schemes, we consider a two-hop line network with one source and one relay, and add more randomly placed source or relay nodes to observe the inter-source or inter-relay competition. Similar to the previous simulations, we assume Rayleigh fading and compute the mean power gain between two nodes that are $d$ meters apart as $-40$ dB$/d^2$. The harvesting efficiencies, maximum average powers, and energy prices are drawn uniformly from $\beta_\eta[0, 1]$, $\beta_p[0, 100]$ mW, and $\beta_\sigma[0, 0.1]$ bps/W, respectively where $\beta_\eta, \beta_p, \beta_\sigma \in [0, 1]$ vary for each simulation to demonstrate the impact of these model parameters.
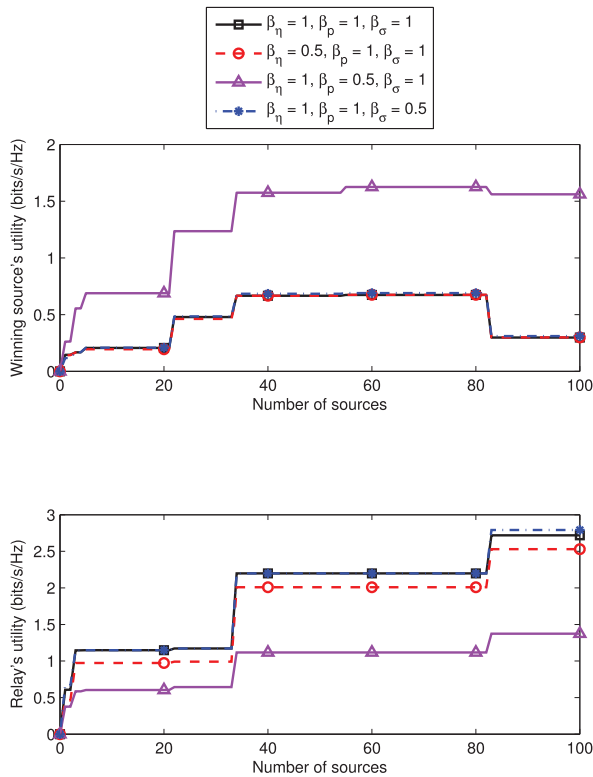
Fig. 10. The winning source's utility and the relay's utility in a two-hop network with multiple sources versus the number of sources in the network for the setup in Section III-B.
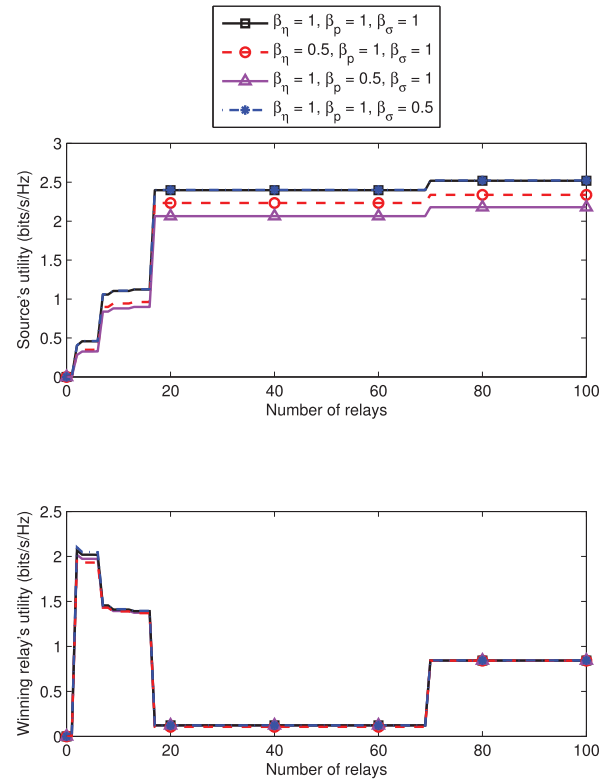


Fig. 11. The source's utility and the winning relay's utility in a two-hop network with multiple relays versus the number of relays in the network for the setup in Section IV-B.

Fig. 10 shows the leader and follower utilities for the setup in Section III-B with one relay and $J$ sources versus the number of sources in the network. The distance from $R$ to $D$ is set to be 50 m, and the distance between each $S_j$ and $R$ is uniform on [0, 100] m. As more randomly placed sources are introduced to the system, the sources face more intense competition whereas the relay has more options to choose from. Therefore, the relay's utility is nondecreasing in the number of sources in the system. However, the winning source's utility is not monotone since the addition of new sources with random distances and random energy prices can impact the source's utility in any direction. Note that the new sources added to the system during the constant portions of both utilities cannot win the auction or have the second best offer, thus their involvement does not impact the utilities. The utility curves are also drawn for lower harvesting efficiencies by setting $\beta_\eta = 0.5$, lower maximum powers by setting $\beta_p = 0.5$, and lower energy prices by setting $\beta_\sigma = 0.5$. We observe that a lower harvesting efficiency at the relay results in a lower utility for the relay, but it does not impact the winning source's utility. A lower maximum power constraint helps the source and causes a lower utility at the relay. In addition, a lower energy price results in higher bids from all sources, and thus a higher relay utility. It also results in a lower energy cost at the winning source.

Fig. 11 shows the leader and follower utilities for the setup in Section IV-B with one source and $K$ relays versus the number of relays in the network. The distance from $S$ to $D$ is set to be 100 m, and the relays are placed uniformly on the line between $S$ and $D$. Similarly, the auctioneer's utility is nondecreasing in
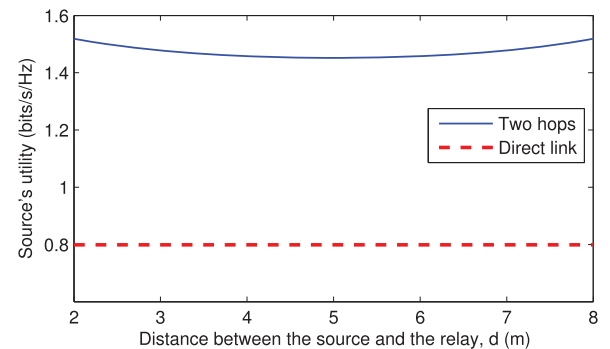


Fig. 12. Node $S$'s utility versus the location of node $R$ with $d' = 10$ m, $\sigma = 0.05$ bps/W, $\eta = 0.4$, and $P = 1$ mW for the noncooperative game in Section V-B and direct transmission from node $S$ to node $D$.

the number of bidders whereas the auction winner's utility is not monotone. The source's utility is increasing in the harvesting efficiency when the source is the leader of the game. With a higher $\eta$, the source can encourage node $R$ to use the additional harvested energy for forwarding node $S$'s data. While the source can adjust the average transmit power and in turn the harvesting fraction at the relay in accordance with the changes in the energy price, a lower maximum power constraint at the source results in a lower source utility. This is because with a lower maximum power, the source has a smaller feasible set, and hence the resulting utility potentially decreases.

Lastly, we consider a low frequency short range RFID application example such as an asset tagging system in which an intermediate node can relay another node's data in exchange

for transferred energy [43]. We consider a carrier frequency of 13.56 MHz and reference distance of 1 m. Fig. 12 shows the source's utility over two hops as well as over the direct link. Here, $S$ and $D$ are 10 m apart, and $R$ moves from $S$ to $D$. The figure demonstrates that our two-hop cooperation scheme can outperform the direct link in this setup, without any additional fading on the direct link. We observe a similar phenomenon to that of Fig. 9 in that the proximity of the relay to the source or the destination leads to improved source utility since the relay can harvest more energy or has a better channel to the destination, respectively.

## VII. DISCUSSION AND CONCLUSION

In this paper, we have studied signal and energy cooperating two-hop wireless networks in a game theoretic setup, with the goal of properly incentivizing the nodes in the network to participate in both forms of cooperation. We have considered the two-hop network where the source has energy to offer to the relay in exchange for relaying its data, and the relay can use part of this energy to transmit its own data. We have focused on the setup that, despite wishing to maximize their individual utilities, the nodes find it beneficial to cooperate. We have formulated Stackelberg games, where the relay and the source is the leader respectively, and observed how the leader can influence the follower's decision in order to obtain a higher individual utility. By introducing an additional layer of competition by means of a Vickrey auction, we have shown that the leader's utility can be improved. We have observed that auctions incentivize the source nodes to transfer more energy to the relay, or the relay nodes to forward more of the source's data, as compared to the single-source single-relay case. Moreover, as there are more bidders participating the auction, the inter-source or the inter-relay competition becomes more intense, and consequently the leader has an even higher utility. Additionally, we have considered a noncooperative game with nodes of equal stature, and shown that a pricing scheme can be employed to improve all utilities to social optimality of the two-hop channel. In this case, the destination can employ the pricing scheme to choose how much data to receive from the source and the relay.

We note that the two-hop setup considered in this work is a stepping stone in the direction of fully characterizing the cooperation performance in networks with energy harvesting and selfish nodes. In arbitrary network topologies, there would be interference between the terminals which needs to be carefully managed, but this would also provide further opportunities for energy harvesting. In this case, our cooperation schemes need to be extended to those that allow some source nodes to transmit data while other source nodes transmit energy to their relays which harvest energy from the interference they receive as well. Considering such a general network interaction is an interesting future direction.

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