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INSTABILITIES OF INITIALLY STRESSED HYPERELASTIC CYLINDRICAL MEMBRANE AND SHELL UNDER INTERNAL PRESSURE

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Summary This paper investigates the static non-linear behaviour and possible instabilities of cylindrical membranes and shells. A detailed experimental analysis was carried out. The structure was analysed using appropriate membrane or shell finite elements and the resulting non-linear equilibrium equations solved using the FE software ABAQUS. A detailed parametric numerical analysis was carried out and the influence of different types of local imperfections was also studied in detail.

INTRODUCTION

The pioneering work of Green & Adkins [1] on non-linear elasticity set up the basis for the analysis of membranes under large deformations. Since then, many important papers have been published in this field, most of which studies the equilibrium and stability of thin cylindrical and spherical membranes under uniform pressure loading or loads acting along the boundaries (Alexander [2]; Ratner [3]; Chen [4]; Haughton [5]; Pamplona et al. [6]). The analysis of large deformations of shell is not so popular, nevertheless there are some important publications such as the ones by Tang et al [7] and Haussy and Ganghoffer [8], where the theory of thick hyperelastic shells was used for the modelling of carotid arteries and aneurysms, respectively. This paper investigates the large deformations of hyperelastic membranes and shells. The static non-linear behaviour and possible instabilities of these structures are both analysed. A detailed experimental analysis was carried out involving cylindrical membranes and shells and the influence of the axial force and the internal pressure were investigated. An apparatus was developed to support vertically the extended structure while it is filled with air. The membranes and shells used in the experiments are composed of an isotropic, homogeneous and hyperelastic rubber, which is numerically modelled as Neo-Hookean, Mooney-Rivlin or Ogden incompressible material. The constants of these formulations were obtained by comparing the experimental and numerical solutions for the structure under traction. The structure was analysed using appropriate membrane or shell finite elements and the resulting non-linear equilibrium equations solved using the FE software ABAQUS. A detailed parametric analysis was also carried out to study the influence of the initial traction and geometric parameters on the non-linear behaviour and load carrying capacity of the structure. The influence of different types of local imperfections was also studied in detail.

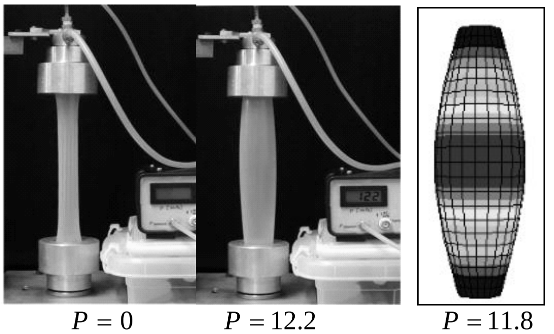
STATEMENT OF THE PROBLEM

Isotropic elastomeric materials can be conveniently represented in terms of a strain energy density function w [1]. Assuming complete recoverability after deformation, the strain energy density depends on the final state of strain and not on the loading history. Thus, given an undeformed reference state, the final stressed state is characterized by the principal stretches λ_1, λ_2 and λ_3 or equivalently by the strain invariants I_1, I_2 and I_3. A rubber-like material can be considered to behave as an incompressible material as long as stresses do not become too large. This simplifying assumption of incompressibility leads to the constraint $I_3=1$. Three material constitutive laws were chosen to analyse the problem: Mooney-Rivlin law, $w=C_1(I_1-3)+ C_2(I_2-3)$, Neo-Hookean, $w=C_1(I_1-3)$ and Ogden $w=\sum_{i=1}^{N}\frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha i}+\lambda_2^{\alpha i}+\lambda_3^{\alpha i}-3\right)$, where C_i, α_i, and μ_i, are the material constants. The analysis of the non-linear behaviour and stability for the thick-walled shell and thin membrane, both under traction and increasing air pressure, were performed using ABAQUS. Different finite elements and the three constitutive laws presented above were used to compare the numerical results with the experimental ones, obtaining the critical loads and imperfection sensitivity for different types of imperfection.

NUMERICAL AND EXPERIMENTAL RESULTS

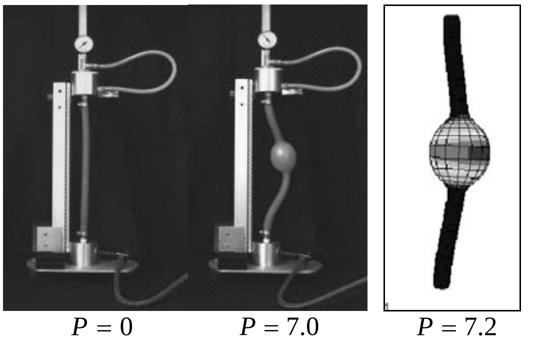
To establish the material constitutive law, each membrane and shell were subjected to increasing traction forces and the material constants were obtained using ABAQUS. Figure 1 illustrates a typical experiment of membrane under traction and increasing air pressure. For the stressed membrane the internal pressure increases with injected air. However, for a specific critical pressure although the volume still increases, there was a gradual decrease of the internal pressure, giving two or more stable equilibrium configurations for different volumes. For an unstressed shell, as the pressure increases the shell deflects laterally as a long column until a critical pressure is reached and a bubble is formed near the middle of the specimen, Figure 2. At this critical point, there was a sudden drop in pressure indicating the existence of limit point instability. When the shell is initially stressed, the global buckling mode is no longer observed and the shell remains in a vertical position until the critical pressure is reached. Typical non-linear equilibrium paths are shown in Figures 3 and 4 for cylindrical membranes and shells respectively where the internal pressure is plotted as a function of the maximum radial displacement for increasing values of the structure thickness. Initially the load-displacement relation is almost linear. As the pressure increases the shell stiffness decreases and a limit point is reached after which the load decreases as the maximum displacement increases. This behaviour is typical of a softening, imperfection sensitive system. As the thickness increases, the stiffness of the shell increases, increasing the critical load. Similar parametric analysis was carried out both experimentally and numerically. The influence of the structure's geometry, several types of imperfections and load history on the non-linear behaviour and stability of the tube was investigated. Results show that the localization, number of bubbles and critical load may vary depending on the system parameters and imperfections.

Cylindrical membrane and shell



$P = 0$ $P = 12.2$ $P = 11.8$

Figure 1 – Sequence of equilibrium configurations. Membrane under constant traction (ΔL/L=1) and increasing air pressure and FE results. *P(10^{-4} MPa).*



$P = 0$ $P = 7.0$ $P = 7.2$

Figure 2 – Sequence of equilibrium configurations. Unstressed shell under increasing air pressure and FE results. *P(10^{-2} MPa).*

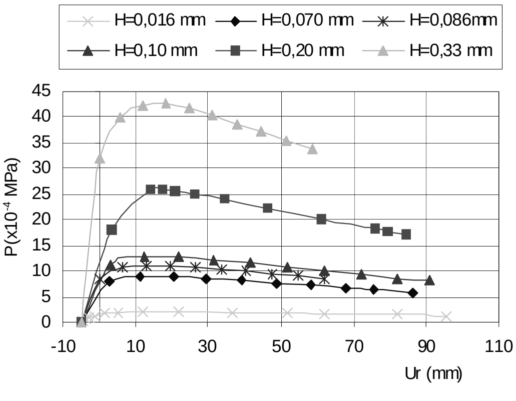


Figure 3 –Influence of the membrane thickness on the load-displacement response. Limit point instability

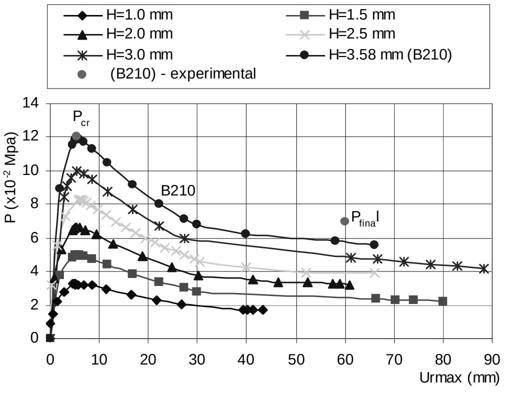


Figure 4 –Influence of the shell thickness on the load-displacement response. Limit point instability

CONCLUSIONS

Finite deformations of isotropic circular cylindrical shell and membrane subjected to a finite extension and gradually filled with air were investigated theoretically and experimentally. When the extended structure was filled with air, it was observed that the pressure increased initially as the volume increased until a critical value was reached, after which a bubble was suddenly formed along the structure (local buckling) and the internal pressure decreased markedly with increasing volume. The experimental results are, as shown in this paper, in satisfactory agreement with the theory.

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