Hydraulic Actuator Tuning in the Control of a Rotating Flexible Beam Mechanism

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The end point position and vibration control of a rotating flexible beam mechanism driven by a hydraulic cylinder actuator are considered. An integrated nonlinear system model comprised of beam dynamics, hydraulic actuator, control valves, and control scheme is presented. Control based on simple position feedback, along with a hydraulic actuation system tuned to suppress beam vibration over a wide range of angular motion, is investigated. For positioning to small to moderate mechanism angles, a linear system model with the actuator tuned for good open-loop performance is developed. Actuator tuning is accomplished by varying the system hydraulic resistance according to a dimensionless parameter defining the interaction between actuator dynamics and the fundamental mode of the flexible beam. Simulation results for a closed-loop system indicate that this simple tuned control provides comparable performance and requires less control effort than an untuned system with a more complex state feedback optimal controller. To compensate for geometric nonlinearities that cause instability when positioning to large mechanism angles, an active actuator tuning scheme based on continuous variation of hydraulic resistance is proposed. The active variable resistance controller is combined with simple position feedback and designed to provide a constant dimensionless actuator-flexible beam interaction parameter throughout the motion. Simulation results are presented to show the stabilizing effect of this control strategy.

1 Introduction

The problem of controlling the position of a rotating flexible beam or a single link flexible manipulator arm while minimizing link flexural vibrations has received considerable attention in recent years. For example, Fukuda (1985) and Sakawa et al. (1985) used a beam base encoder, tachometer, and strain gauge measurements in state feedback control schemes. Dancose et al. (1989) applied optimal state feedback control methods. Combinations of beam base and tip measurements with control schemes consisting of PID plus modal control (Singh and Schy, 1986) and pole placement plus integral control (Chalhoub and Ulsoy, 1987) have also been investigated. Siciliano and Book (1988) separated the beam dynamics into slow and fast subsystems and applied state feedback pole placement control to the fast part representative of flexural vibrations. Castelazo and Lee (1990) applied a nonlinear damping scheme. The efforts discussed above have shown favorable results regarding reduced beam vibrations during position control. The control schemes have used state feedback which generally requires measurements of both beam dynamic motion and flexure vibration variables with the appropriate observers. All of these studies use linearized models and consider the control force or torque to be applied directly to the beam or hub without including the actuator dynamics.

The complex dynamics for the rotating flexible beam control problem consist of the interaction of rigid body motion, beam flexural vibration, actuator dynamics, and control input. Possibilities of damping structural vibrations by understanding and adjusting certain parameters which describe the overall system were outlined in Panza et al. (1988). It was shown in Sah and Mayne (1990) that gear ratio played an important role in damping vibrations in a complete open-loop system model for an electromagnetically actuated rotating flexible beam. The case of adjusting hydraulic resistance and capacitance to reduce vibrations in a complete open loop system for a hydraulically actuated rotating flexible beam model was presented in Panza and Mayne (1989). An integrated overall system approach describing actuator-flexible beam coupling given in Panza and Mayne (1994) showed that actuator tuning based on the selection of key dimensionless parameters resulted in fast open-loop dynamic response with well damped beam vibrations. Sah et al. (1993) showed that a well-tuned electromagnetic actuator-slewing beam system with simple control could give better overall performance than optimal control of a poorly tuned system. These studies indicate that the inclusion of actuator dynamics modeling and parameter selection may be quite valuable in the design of slewing systems with flexible beams. In closed loop control, of course, beam flexibility causes vibrations that degrade accurate tip positioning.

Figure 1 gives a schematic of a hydraulic cylinder actuation system driving a rotating flexible beam. The system investigated in this paper consists of the interactive dynamics between a flexible beam, a hydraulic actuator, and feedback control. The focus is on tuning the actuator to the flexible beam by varying hydraulic resistance. Control valves in the flow lines from cylinder to a servovalve are used to vary the resistance. The control input u_v to the servovalve may then reflect a simple output feedback scheme. This study includes actuator parameter consideration as an integral part of system design for closed-loop slewing control. Hydraulic resistance (a useful adjustable parameter) is tuned from open-loop system studies and coordinated with a feedback control scheme and gain selection to realize a simple closed-loop system with good behavior and low control effort. The actuator tuning is designed to provide significant damping via actuator-flexible beam interaction to minimize structural vibrations and provide a smooth dynamic response. For positioning to small to moderate mechanism angles, a linear system model is applied and the hydraulic resistance is fixed throughout the response. This situation was the

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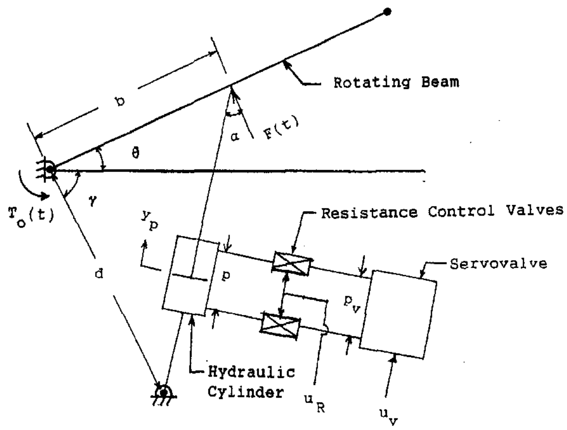

Fig. 1 Schematic of hydraulic cylinder actuation system

focus for the open-loop study of Panza and Mayne (1994). For positioning to large angles, a nonlinear system model is needed and the hydraulic resistance is continuously varied by adjusting the resistance control valves with an active control u_R applied to these valves.

In Section 2, Lagrange's equations are applied to obtain general nonlinear differential equations for the mechanism including beam dynamics. Additionally, hydraulic actuator dynamics are coupled to the beam to provide an integrated model for the flexible beam-actuator system. In Section 3, actuator tuning to the beam dynamics is defined in terms of a dimensionless flexible beam-actuator interaction parameter. Also developed in Section 3 are the application of three basic feedback control schemes appropriate with hydraulic actuator tuning. These include an error driven beam angular position control, a minor loop beam angular velocity feedback scheme, and a variable hydraulic resistance adjustment scheme. In Section 4, a numerical example is given for comparison of combinations of these schemes and for comparison with a Linear Quadratic Regulator. The open-loop behavior of the system is used as a guide for efficient actuator tuning of both linearized and nonlinear closed-loop systems. Conclusions are given in Section 5.

2 Mathematical Model

2.1 Beam Dynamics. The geometry of the rotating flexible beam of length L, mass m_b per unit length, and tip mass M_o is shown in Fig. 2. A detailed development of the beam dynamics is given in Panza (1989). X and Y are inertial coordinates, while x is fixed along the beam and $y_f(x, t)$ is the flexural deflection perpendicular to the beam. At $x = 0$, the beam is considered to have general boundary conditions for flexure. The unit vectors ϵ_x and ϵ_y are fixed to the rotating beam with derivatives relative to XY given by

$$\frac{d\epsilon_x}{dt} = \frac{d\theta}{dt}\epsilon_y \quad \frac{d\epsilon_y}{dt} = -\frac{d\theta}{dt}\epsilon_x \tag{1}$$

where θ is the angle of rotation of the base of the beam as if it were a rigid body. The position vectors and their derivatives for a beam element and tip mass are given by

$$\vec{r} = x\epsilon_x + y_f\epsilon_y \tag{2}$$

$$\frac{d\vec{r}}{dt} = -y_f\frac{d\theta}{dt}\epsilon_x + \left(x\frac{d\theta}{dt} + \frac{dy_f}{dt}\right)\epsilon_y \tag{3}$$

The beam and tip mass kinetic energy is given by

$$T = \frac{1}{2}\int \frac{d\vec{r}}{dt}\cdot\frac{d\vec{r}}{dt}\,dm = \frac{1}{2}\int [m_b + M_o\delta(x - L)] \times \left[y_f^2\left(\frac{d\theta}{dt}\right)^2 + \left(x\frac{d\theta}{dt} + \frac{dy_f}{dt}\right)^2\right]dx \tag{4}$$

where $\delta(x - L)$ is the Dirac delta function. The beam and tip mass gravitational potential energy and the flexural bending strain energy for a Euler-Bernoulli beam of Young's modulus E and area inertia I_B are given by

$$U = MgL^* \sin\theta + \frac{1}{2}\int_0^L EI_B\left[\frac{\partial^2 y_f}{\partial x^2}\right]^2 dx \tag{5}$$

where L^* is the center of gravity and $M = m_bL + M_o$ is the total mass. The beam dynamic flexural deflection is given as an expansion in terms of modal coordinates $q_i(t)$ and mass normalized flexural mode shapes $\phi_i(x)$

$$y_f(x, t) = \sum_{i=1}^{N} \phi_i(x)q_i(t) \tag{6}$$

where $\phi_i(x)$ are considered to satisfy orthogonality conditions given in Panza and Mayne (1994).

The equations of motion for the beam may be given by Lagrange's equations (Goldstein, 1967)

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{w}_i}\right] - \frac{\partial L}{\partial w_i} = Q_i \quad i = 1 \text{ to } N + 1 \tag{7}$$

where $L = T - U$ is the Lagrangian and $w = [q_1q_2 \ldots q_N\theta]^T$ are the independent generalized coordinates. The generalized forces Q_i are determined from the virtual work δW for an applied force $F(t)$ acting along the beam at a distance b from the base or an applied torque $T_0(t)$ acting at the base

$$\delta W = (F(t) - F_s)\delta[y(b, t)] + T_o(t)\delta[\theta_T(0, t)] \tag{8}$$

where F_s is a static force, δy is the total rigid body mode plus flexural modes virtual linear displacement at $x = b$, and $\theta_T = \partial y/\partial x$ is the total virtual angular rotation at $x = 0$. Since $y = x\theta + y_f$ and $F_s b = MgL^* \cos\theta$, the virtual work becomes

$$\delta W = \sum_{i=1}^{N} [\phi_i(b)(F(t) - F_s) + \phi_i'(0)T_0(t)]\delta q_i + [b\,F(t) - MgL^* \cos\theta + T_o(t)]\delta\theta = \sum_{i=1}^{N+1} Q_i\delta w_i \tag{9}$$

Combining Eqs. (1)–(9), Lagrange's equations of motion results in a system of $N + 1$ nonlinear ordinary second-order differential equations

$$M\frac{d^2w}{dt^2} + Kw = f_u \tag{10}$$

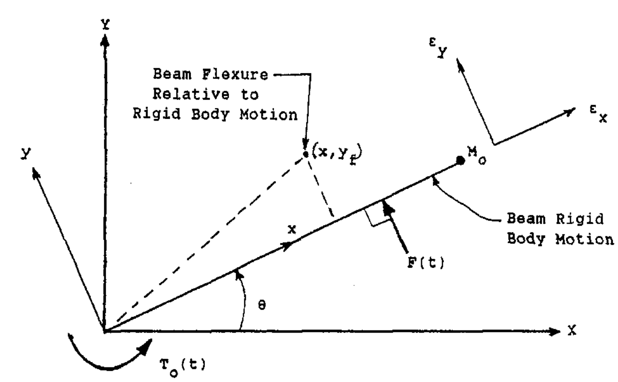

Fig. 2 Schematic of rotating flexible beam

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where inertia matrix M, stiffness matrix K, and input vector f_u are given by

$$M = \begin{bmatrix} \underset{(N \times N)}{I} & \underset{(N \times 1)}{symmetric} \\ \underset{(1 \times N)}{\int_0^L [m_b + M_o\delta(x - L)]x\phi_i(x)dx} & J + \sum_{i=1}^{N} q_i^2 \end{bmatrix}$$

$$K = \begin{bmatrix} \underset{(N \times N)}{\omega_{bi}^2} & \underset{(N \times 1)}{0} \\ \underset{(1 \times N)}{0} & 0 \end{bmatrix} \quad (11)$$

$$f_u = \begin{bmatrix} \underset{(N \times 1)}{\phi_i(b)F(t) + \phi_i'(0)T_0(t) + \left(\frac{d\theta}{dt}\right)^2 q_i - \phi_i(b)F_s} \\ bF(t) + T_o(t) - 2\frac{d\theta}{dt}\sum_{i=1}^{N} q_i \frac{dq_i}{dt} - MgL^* \cos\theta \end{bmatrix}$$

where ω_{bi} are the beam natural frequencies and J is the total mass moment of inertia about the base. The first N equations are associated with beam flexural modal coordinates while the last equation is associated with beam rigid body motion. The off-diagonal terms in the mass matrix account for inertial coupling between the rigid body motion and beam flexural modes. These terms provide the orthogonality condition between the rigid body mode x and the flexural mode $\phi_i(x)$. For an actuator input $T_o(t)$, the local $x = 0$ boundary condition becomes that for a cantilevered beam and these terms are non-zero. For an actuator input $F(t)$, the local $x = 0$ boundary condition becomes that for a hinged beam and these terms are zero. $F(t)$ supplied by a hydraulic actuator is the case investigated in this paper. Nonlinear terms appear in the input vector f_u in the form of centrifugal and Coriolis type forces due to the interaction of flexural and rigid body coordinates. The combination of slow beam dynamic response and low flexural vibrations generally makes these nonlinear terms small.

Rayleigh beam damping may be included by adding the term $D\,dw/dt$ to the left side of Eq. (10). The D matrix has the same form as the K matrix in Eq. (11) but with the $N \times N$ diagonal matrix consisting of elements $2\zeta_{bi}\omega_{bi}$ where ζ_{bi} are beam modal damping factors (Meirovitch, 1990).

2.2 Actuator Dynamics. From Fig. 1 the actuation force for a rigid piston rod and piston of area A_p is $F(t) = pA_p \times \cos\alpha$ where the angle α is a function of θ given as

$$\cos\alpha = \frac{\sin(\theta + \gamma)}{\sqrt{1 + \left(\frac{b}{d}\right)^2 - 2\left(\frac{b}{d}\right)\cos(\theta + \gamma)}} \quad (12)$$

For large motions of the mechanism, this geometric nonlinearity represents the most significant nonlinearity in the overall system. The hydraulic cylinder and flow line dynamics from servovalve to cylinder have been derived by Panza and Mayne (1989) and for the case of negligible fluid inertia are given by

$$C\frac{dp}{dt} + \frac{1}{R}p + A_p\frac{dy_p}{dt} = \frac{1}{R}p_v \quad (13)$$

where A_p is the piston area, C is the fluid capacitance of cylinder and flow lines, and R is the fluid resistance for the flow lines plus resistance control valves. The piston velocity dy_p/dt may be expressed in terms of the beam velocity at $x = b$

$$\frac{dy_p}{dt} = \frac{dy(b,t)}{dt}\cos\alpha = \left(b\frac{d\theta}{dt} + \sum_{i=1}^{N}\phi_i(b)\frac{dq_i}{dt}\right)\cos\alpha \quad (14)$$

The piston velocity couples the hydraulic cylinder dynamics to the beam dynamics.

The dynamics of the servovalve from the control input u_v to the output p_v are generally nonlinear. For this study, where the focus is on how actuator-beam interaction affects the control, a simple first order linear model may be appropriate. Panza (1989) has shown that a high gain pressure feedback scheme may be used to linearize the servovalve and represent its dynamics as a first order system. The resistance control valves are proposed to be adjustable in-line flow control valves with resistance continuously variable via input u_R simultaneously applied to both valves. In general, flow control valve dynamics are higher order and may cause additional oscillations. To be consistent with the scope of this study, these oscillations are considered insignificant and a linear first order model is used. These active in-line valves make the resistance R in the actuator Eq. (13) time dependent with an initial value equal to the resistance used for small motion studies. The first-order linear models for servovalve dynamics and for the fluid resistance time dependence resulting from the active resistance control valves are given by

$$\tau_v\frac{dp_v}{dt} + p_v = K_v u_v \quad (15a)$$

$$\tau_R\frac{dR}{dt} + R = K_R u_R \quad (15b)$$

where τ's, K's, and u's represent valve time constants, gains, and control inputs, respectively. Since the flow lines including resistance control valves from servovalve to cylinder are symmetrical, the one Eq. (15b) may be used to represent the dynamics of the total hydraulic resistance in the system. The general form of the complete beam-actuator-valve system equations may be written as

$$\frac{dx}{dt} = f(x) + Bu \quad (16)$$

where f is a $2N + 5$ element nonlinear vector function of a $2N + 5$ element state vector x

$$x = \left[q_1 - - - q_N \theta \frac{dq_1}{dt} - - - \frac{dq_N}{dt}\frac{d\theta}{dt} p p_v R\right]^T,$$

u is a 2×1 input vector $u = [u_v u_R]^T$, and B is a $(2N + 5) \times 2$ element matrix. Use will be made of a linearized system about the starting point x_o defined for $q_i = \theta = dq_i/dt = d\theta/dt = 0$, $p = p_o$, $p_v = p_{vo}$, and $R = R_o$.

$$\frac{dx}{dt} = Ax + Bu \quad (17)$$

where

$$A = \left.\frac{\partial f}{\partial x}\right|_{x=xo}$$

3 Actuator Tuning and Control

A reference dynamic system may be defined as one with a rigid beam, negligible fluid capacitance, no gravitational force, and a fast servovalve with negligible response time. Applying

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the linearized Eqs. (17) gives a first-order system in the beam angular velocity ω with valve pressure p_v as input

$$J \frac{d\omega}{dt} + RA_p^2 \cos^2 \alpha b^2 \omega = A_p \cos \alpha b p_v \qquad (18)$$

A dimensionless actuator-flexible beam interaction parameter K_B is defined as the ratio of the fundamental beam flexural natural frequency of vibration to the corner frequency of the reference dynamic system in Eq. (18)

$$K_B = \frac{\omega_{b1}}{RA_p^2 \cos^2 \alpha b^2/J} \qquad (19)$$

The open-loop behavior of a linear flexible beam-hydraulic actuator system with respect to K_B has been reported in Panza and Mayne (1994), where particular values of K_B have been shown to lead to good dynamic response with small beam vibrations.

A simple position control scheme consisting of feedback of the total angle of rotation at the base is used as a basis for control design via actuator tuning

$$u_v = K_P(\theta_d - \theta_T) \qquad (20)$$

where θ_d is the desired angular position, K_P is the amplifier gain, and the actual beam angle is given by

$$\theta_T = \theta + \sum_{i=1}^{N} \phi_i'(0) q_i$$

The use of actuator tuning to obtain good closed-loop behavior is based on having a certain effective K_B value during system transients. For small motions modeled with a linearized system, α is considered constant and a constant R may be selected to provide a desirable constant K_B value which provides for good behavior.

If for the linear case, a desirable K_B is not possible due to constraints in parameter selection, then the addition of a minor loop feedback u_m to the servovalve input may provide an additional parameter leading to achieving an effective desirable K_B value. Feedback of the total velocity at the actuator attachment point is one possibility for this minor loop. The motivation for velocity feedback can be seen by considering actuator Eqs. (13)–(15) for the case of a fast servovalve ($\tau_v = 0$) and a servovalve control input $u_v = K_P(\theta_d - \theta_T) + u_m$

$$RC \frac{dp}{dt} + p + RA_p \cos \alpha \frac{dy(b, t)}{dt} - K_v u_m = K_v K_P(\theta_d - \theta_T) \qquad (21)$$

Since the reference system corner frequency from Eqs. (18) and (19) originates from the term in Eq. (21) with fluid resistance coefficient R, a control u_m with the same form as this term can provide an effective new fluid resistance for a desirable actuator-flexible beam interaction parameter K_B. Thus u_m can be considered as

$$u_m = \pm \frac{R_m A_p \cos \alpha}{K_v} \frac{dy(b, t)}{dt} = \pm K_m \frac{dy(b, t)}{dt} \qquad (22)$$

whereby

$$K_B \sim \frac{1}{R \mp R_m}$$

The minor loop feedback gain K_m is proportional to an effective additional resistance R_m and is adjusted as required to provide the desirable effective constant K_B for small motions. If the actual resistance is too low such that the K_B value is larger than desired, then negative velocity feedback may be used to decrease K_B. If the actual resistance is too large such that the K_B value is smaller than desired, then positive velocity feedback may be used to effectively reduce the resistance and increase K_B. Recall, of course, that this assumes that valve dynamics do not significantly effect the results.

For large motions, the geometric nonlinearity tends to change the actuator-flexible beam interaction parameter K_B with movement. Since K_B is changing during operation of the general nonlinear system, the linear system schemes above may be effective only from $\theta = 0$ to some θ where $\cos \alpha$ rapidly diverges from its value at $\theta = 0$. The implementation of a variable resistance strategy via an active control valve may compensate for this geometric nonlinearity and provide for a near constant K_B throughout the dynamic process. The desired variable resistance for constant K_B is obtained from Eq. (19) and is given by

$$R(\theta) = R(0) \frac{\cos^2 \alpha(0)}{\cos^2 \alpha(\theta)} \qquad (23)$$

whereby the resistance control valves input for Eq. (15*b*) is given by

$$u_R = \frac{R(\theta)}{K_R} \qquad (24)$$

Except for the short (i.e., with small τ_R) control valve transient, the effective actuator-flexible beam interaction parameter becomes $K_B = K_B\ (\theta = 0)$, which may be determined as in the linear system model. The resulting interactive beam-actuator-control system is nonlinear but contains active actuator tuning designed for the purpose of providing an effectively linear and well behaved dynamic response according to the proper K_B value.

4 Numerical Results

A 1.22 m long steel beam with 4.76 mm by 38.1 mm cross section, inertia $J = 0.934$ kg-m^2, and pinned-free boundary conditions (i.e., $T_o(t) = 0$, $M_o = 0$) is oriented in a horizontal plane and excited by a 18.0 mm diameter hydraulic actuator. The geometry of the actuator from Fig. 1 is $b = 0.114$ m, $d = 0.326$ m, and $\gamma = 1.21$ rad, which gives $\alpha = 0$ for $\theta = 0$. This is the same beam and actuator geometry used for the experimental investigation in Panza and Mayne (1994). The fundamental natural frequency of vibration for the pinned-free beam is 11.2 Hz and the beam modal damping factors are assumed to be 0.01. Valve time constants are considered small at 0.003 seconds and servovalve gain is 15 psig/volt. Outputs of interest include the tip flexural deflection $y_f(L, t)$ for four beam flexural modes, and the total motion of the tip of the beam defined as the sum of arc length produced by the rigid body component $\theta(t)$ and the tip flexural deflection $y_f(L, t)$

$$Y_{TIP} = L\theta(t) + y_f(L, t)$$

where

$$y_f(L, t) = \sum_{i=1}^{4} \phi_i(L) q_i(t) \qquad (25)$$

The total tip motion defined above is presented instead of the total beam angle to provide a direct assessment of the effect of beam vibration on tip positioning. To demonstrate how actuator tuning may improve dynamic response, a transient response goal of minimal overshoot with the fastest possible response for Y_{TIP} is considered.

Baseline open loop responses of the linearized system for a suddenly opened servovalve are shown in Fig. 3 for three K_B values and hydraulic parameters given in Table 1. The simulations are for four beam flexural modes which was considered accurate for open-loop responses (Panza, 1989). It is clear that the best combination of fast angular velocity ($\omega_T = d\theta_T/dt$) response and low rapidly decaying tip vibration is for the intermediate value of $K_B = 6$. At this value of K_B, the beam is

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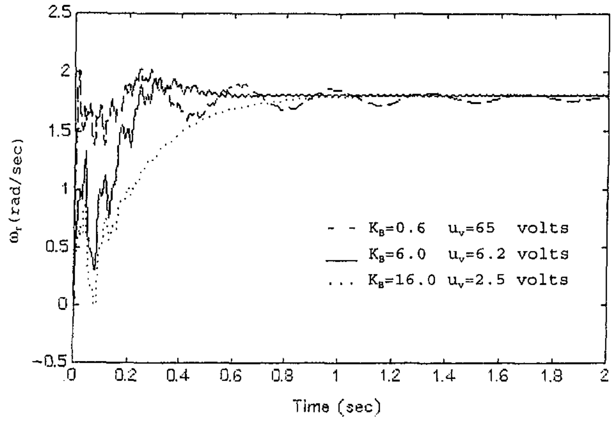

Fig. 3(*a*) Open-loop angular velocity

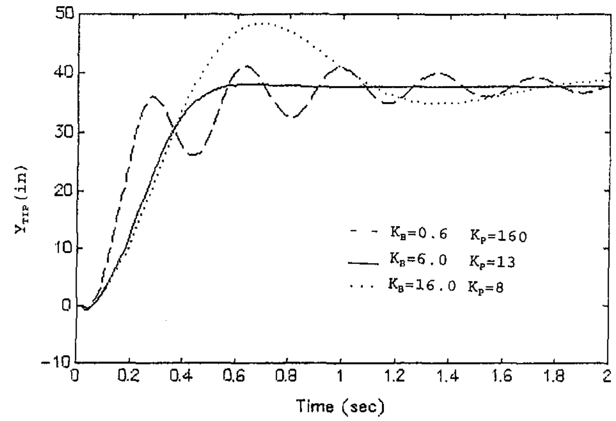

Fig. 4(*a*) Total tip motion for simple position control

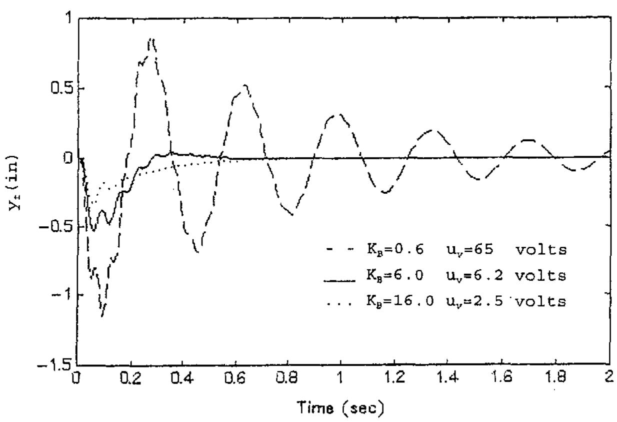

Fig. 3(*b*) Open-loop tip flexural deflection

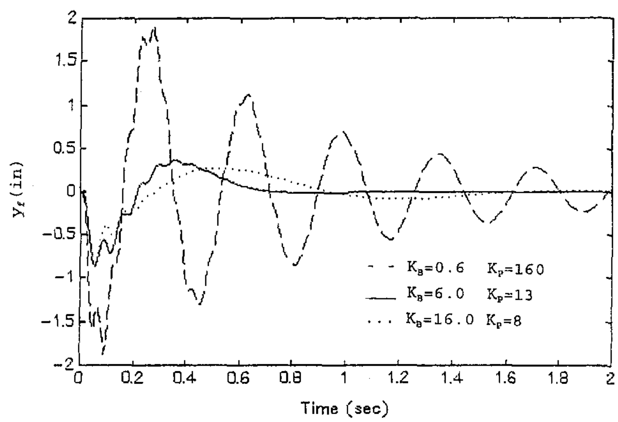

Fig. 4(*b*) Tip flexural deflection for simple position control

coupled to the actuator in a manner which permits beam vibrations to be readily damped in the dissipation mechanism of the actuator while still providing for a relatively fast dynamic response. This open-loop behavior for a linear system has been thoroughly investigated in Panza and Mayne (1994). The effect of K_B in actuator tuning for closed-loop tip position control is now presented for the numerical example.

4.1 Linearized System Control. Figure 4 gives a comparison of the simple closed-loop position control of Eq. (20) for the three K_B values and a desired position of $\pi/4$ radians modeled as a step input command. Several iterations indicated that four beam flexural modes were also appropriate for this closed loop study. The K_P values were chosen by an iterative procedure such that all three tip motion responses have similar rise times in an average sense. However, only the intermediate $K_B = 6$ response occurs without overshoot. Additionally, $K_B = 6$ provides the lowest vibration amplitude and fastest decay of beam vibrations. The oscillations for the $K_B = 0.6$ case occur at approximately 2.8 Hz which is similar to the fixed-free beam fundamental natural frequency rather than the pinned-free frequency of 11.2 Hz. The effect of actuator dynamics effectively changing the beam vibration frequency in the overall system has also been reported by Sah and Mayne (1990). The transient behavior of the total beam angle θ_T for the three K_B cases is

Table 1 Hydraulic parameter values

K_B	R (N-s/m^5)	C (m^5/N)
0.6	13 e 10	1.72 e − 13
6.0	1.23 e 10	8.88 e − 13
16.0	0.49 e 10	8.59 e − 13

similar to the total tip motion Y_{TIP}. The linearized system is a reasonable model for $\theta \le \pi/4$ since from Eq. (12), $\cos \alpha(0) = 1$ and $\cos \alpha(\pi/4) = 0.77$. This results in a 30 percent increase in K_B which would not significantly affect the selection of $K_B = 6$ as a means of tuning the actuator to provide for an overall favorable system response.

A deeper insight into the concept and performance of actuator tuning may be obtained by comparing the tuned actuator case with an untuned actuator optimally controlled with a state feedback Linear Quadratic Regulator (Kirk, 1970). Defining a position error $e = \theta_d - \theta$ as a state variable instead of θ, the cost functional

$$J_o = \int_0^\infty (x^T Q_o x + u_v^T R_o u_v) dt \tag{26}$$

is minimized by solving the algebraic Matrix Ricatti equation

$$PA + A^T P - PBR_o^{-1} B^T P + Q_o = 0 \tag{27}$$

for the control $u_v = -R_o^{-1} B^T P x$. Q_0 and R_0 are selected to provide a total tip motion close to the $K_B = 6$ case in Fig. 4(*a*) while also having well damped tip vibrations similar to Fig. 4(*b*). Figure 5 gives a comparison of the passively tuned $K_B = 6$ case of simple position feedback with the case of an optimally controlled state feedback untuned $K_B = 0.6$ system. Also given in Fig. 5 is the case of an originally untuned $K_B = 0.6$ system with simple position control plus the positive velocity feedback of Eq. (22) designed with $R_m = 0.9$ to provide an effective $K_B = 6$ system. The three systems in Fig. 5 are designed to give essentially the same total tip motion as the tuned $K_B = 6$ case in Fig. 4(*a*). The tip flexural vibrations shown in Fig. 5(*a*) are also similar. However, Fig. 5(*b*) shows that the control input for both the optimal state control and the positive velocity feedback

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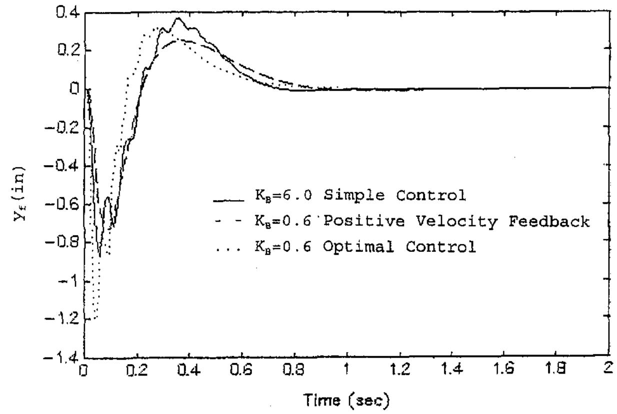

Fig. 5(*a*) Tip flexural deflection for three controllers

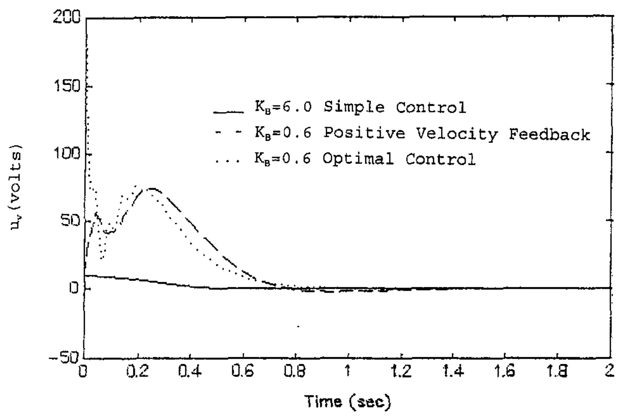

Fig. 5(*b*) Control input for three controllers

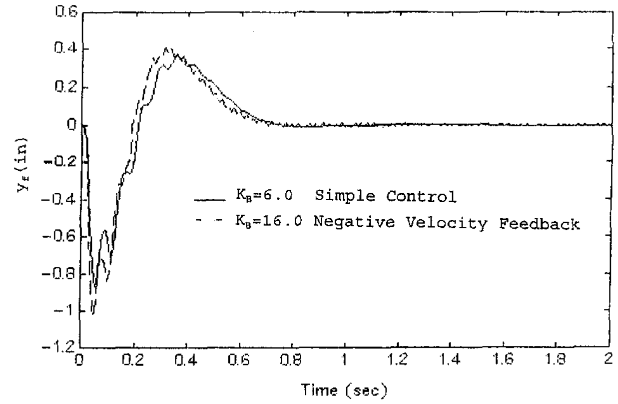

Fig. 6(*a*) Tip flexural deflection for high K_B case

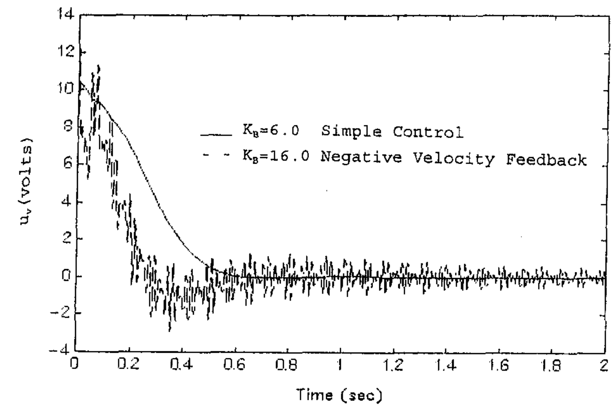

Fig. 6(*b*) Control input for high K_B case

control acting on the $K_B = 0.6$ system is much higher than that for the simple control acting on the tuned $K_b = 6$ system. The results in Fig. 5 also indicate that if the actuator cannot be tuned by hydraulic resistance selection and high control effort can be tolerated, then simple velocity feedback may be an effective alternative to full state feedback control. Additionally, if the original system has a low K_B value, then positive velocity feedback designed for effective actuator tuning rather than the normally used negative velocity feedback is the means of providing the proper amount of system damping for good overall performance.

For the case where $K_B = 16$, negative velocity feedback may be combined with a simple position control to provide a tip response similar to the $K_B = 6$ case. The negative velocity feedback has the effect of effectively decreasing K_B via Eq. (22). Figure 6 gives a comparison of the $K_B = 16$ system (position gain $K_P = 16$ and velocity gain $K_m = 6.7$) and the $K_B = 6$ system with just position control. The implication is that the negative velocity feedback may be sufficient for the high K_B system because the magnitude of the control effort shown in Fig. 6(*b*) is similar to the $K_B = 6$ system. A concern may be that this control effort contains a potentially harmful high frequency oscillatory component that may result from the poorly damped beam vibration due to little actuator interaction in this high K_B region, especially for higher-order modes (Panza and Mayne, 1994).

4.2 Nonlinear System Control. The effect of the nonlinear beam dynamic terms in Eqs. (10–11) and the nonlinear actuator force angle cos α in Eq. (12) are evaluated for the passively tuned (constant resistance control valve setting) actuator case of $K_B = 6.0$ in the simple position control scheme. Active actuator tuning (continuously varying resistance control valve) may be implemented via the variable resistance control given in Eq. (24) designed to provide constant $K_B = 6$ value during movement. For the $K_B = 6$ case, the beam flexural responses q_i are well damped and with the moderate response speed for the numerical example, the beam dynamics nonlinear terms are small compared to the actuator angle term. Figure 7 gives the behavior of cos α as a function of positioning angle θ. For the case of positioning to a small desired angle of $\pi/4$, the overall effect of the nonlinear terms on tip transient motion is small as shown in Fig. 8. The case of positioning to a large desired angle of 0.9 $\pi/2$, where the actuator angle significantly differs from its initial value at $\theta = 0$ is shown in Fig. 9. The drastic effect of nonlinear terms on both tip flexural vibration and tip positioning is evident after a short period of time. The

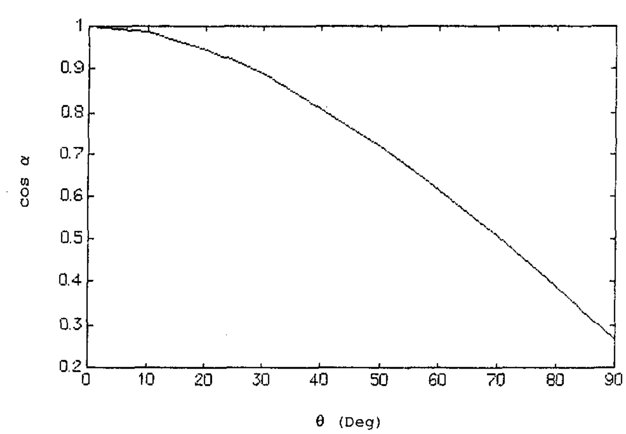

Fig. 7 Actuator/beam angle α versus beam position angle θ

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dynamics is unstable relative to convergence to the desired equilibrium position. As θ increases from 0 to θ_d, cos α decreases and thus K_B increases from $K_B = 6$ to $K_B = 15.8$. The larger K_B values result in much less actuator damping for flexural vibrations as can be seen in Fig. 9(*b*). These vibrations prevent the tip motion from settling at the desired equilibrium as seen in Fig. 9(*a*). The resulting tip arc motion is so large that the instability would in practice cause the cylinder piston to use up its entire stroke and bottom out in the cylinder. However, the variable resistance controller maintains $K_B = 6$ and is shown to be very effective in actively tuning the actuator to the flexible beam such that the well behaved desirable linear system model performance is obtained. Additionally, the overall control scheme is based on the low control effort K_B system. The variable resistance portion is essentially a form of partial feedback linearization specifically designed to counteract the nonlinear terms from cos α and to maintain a new constant $K_B = 6$ throughout the motion.

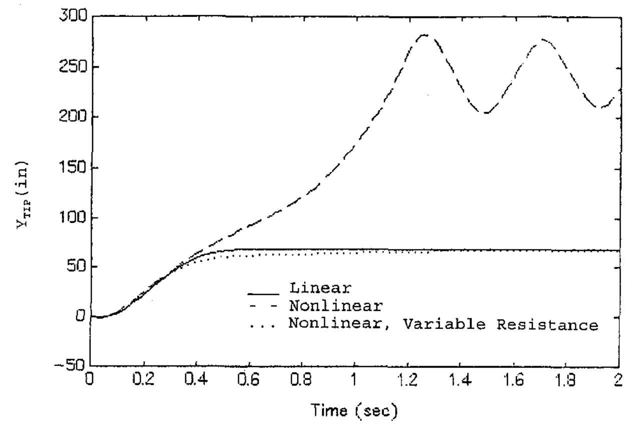

Fig. 9(*a*) Effect of variable resistance controller on total tip motion for simple position control to 0.9π/2 rad

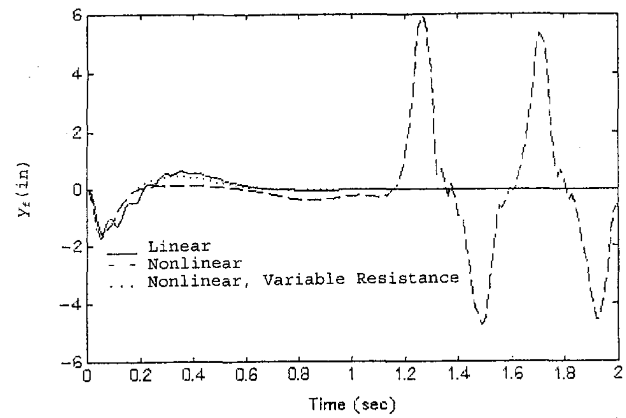

Fig. 9(*b*) Effect of variable resistance controller on tip flexural deflection for simple position control

5 Conclusions

A method of tuning a hydraulic cylinder actuator to the dynamics of a rotating flexible beam mechanism is proposed to provide a simple low effort, relatively fast, and accurate positioning control scheme. An integrated complete system model consisting of general nonlinear multi-modal flexible beam dynamics and actuator geometry coupled to hydraulic actuator and control dynamics provides the basis for system analysis and design. Actuator tuning is accomplished via both passive (constant resistance control valve setting) and active (continuously varying resistance control valve) implementation of hydraulic resistance in the context of a simple angular position feedback scheme. Active tuning is applied via a variable hydraulic resistance and used to cancel the negative effect of major nonlinear dynamics. Numerical results show that actuator tuning based on obtaining a particular dimensionless actuator-flexible beam interaction parameter leads to a well behaved dynamic response with well damped and suppressed beam vibration. Additionally, the results indicate that a tuned actuator-load system with a simple control scheme provides a much lower control effort than a linear optimal control based on a more complex state feedback scheme. The results for passive actuator tuning in linearized systems are in general agreement with electromagnetic cases in Sah et al. (1993).

Finally, the results for the continuously variable resistance controller imply that active actuator tuning to match the flexible member dynamics may be an effective means of improving the quality of motion and stabilizing nonlinear dynamic effects in mechanisms with flexible members. With the use of a fully integrated system model, such improvements may be obtained through considerations of actuator-load interaction rather than from direct complex control methods. The authors cover only a hydraulic actuation system but suggest that the tuning concepts may also be applied to an electromagnetic actuator system.

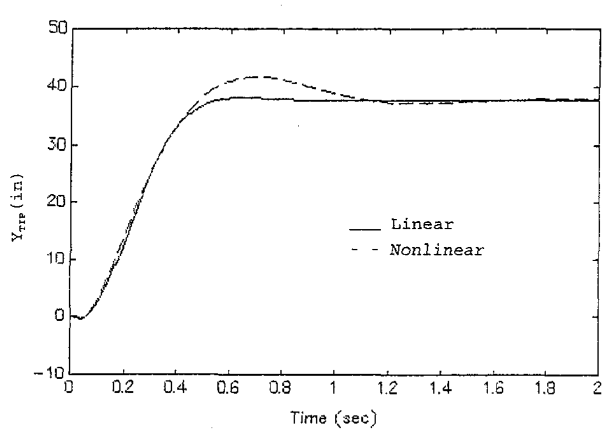

Fig. 8 Linear versus nonlinear total tip motion for simple position control to π/4 rad

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