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SQUARE CONCRETE SECTION CONFINED BY FRP: ULTIMATE STRENGTH PREDICTION

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ABSTRACT

During the last ten years, several studies have been devoted to understanding the behaviour of circular concrete sections confined by FRP. Less studies have dealt with square and rectangular sections, even if the problem is equally important, given the large amount of buildings with such sections that need to be retrofitted due to poor performance under vertical and horizontal actions.

A numerical model was already proposed for the prediction of the stress-strain relationship. In order to implement such model in a design code framework, a mechanics-based equation has been developed and preliminary results are presented for the prediction of the ultimate strength of square sections confined by FRP. Since the model considers the corner radius influence, circular sections are automatically included as a particular case. The simplified model has been calibrated by means of a set of experimental values available in the literature.

KEYWORDS

FRP, confinement, concrete, square section, experimental investigation, modelling.

INTRODUCTION

Research on FRP-confined circular concrete sections has nowadays developed theoretical considerations that gather a certain consensus in the scientific community. However, though square and rectangular sections are more common in RC buildings, comparatively less advance has been made in defining their behaviour. The experimental database, even if the early tests date back to 1998 (Mirmiran et al., 1998), is not exhaustive due to the lack of tests on full columns. Besides, except in some cases, the experiments regard square specimens. In this framework the authors planned an experimental campaign (Monti et al., 2007a,b) devoted to the assessment of FRP-confined square and rectangular concrete section. Based on some preliminary results of those tests, a numerical model was developed (Monti and Nisticò, 2007a,b) and verified in the prediction of stress-strain relationships: an important issue of such model was the definition of the lateral *vs.* axial strain relationship, based on a preliminary estimation of the confined strength. If path-independence (Imran and Pantazopoulou, 1996) is assumed, and a triaxial failure criterion is adopted for concrete, it is possible to define its ultimate axial strength, based on the confinement stress field corresponding to an ultimate transverse configuration. Following a simplified analytical approach, this paper presents an equation for predicting the ultimate strength.

EXPERIMENTAL RESULTS AND PHENOMENOLOGICAL ASPECT

Experimental investigations on square sections date back to the tests proposed in (Mirmiran et al., 1998) where the effects of the corner radius were clearly described, influencing both strength and ultimate strain. After that work, other contributions focussed either on the prediction of ultimate parameters or on the development of constitutive models by means of tests on small specimens.

In this framework, de Diego et al. (2007) recently presented tests on thirty specimens of square (150x150 mm) reinforced concrete columns (600 mm tall); all specimens were characterized by rounded corners with curvature radius of 25 mm. The tested specimens are grouped based on the confinement devices that are: 1) wrapping, 2) prefabricated shell, and 3) prefabricated shell combined with wrapping. For each group, two different composite materials (glass and carbon) were adopted, and five concrete cylinder unconfined strength ranging between 8.8 and 17.5 MPa were used.

The effects of the corner rounding radius has been extensively investigated also in (Wang and Wu, 2007**),** where compression tests were performed on 108 concrete specimens (300 mm tall). The section of all specimens can be considered as inscribed in a square section (150x150 mm) with six values of the corner radius: 0, 15, 30, 45, 60, and 75 mm (so that circular sections have been included in the tests). Two different concrete types (C30 and C50) and two carbon jacket thicknesses (0.165 and 0.33 mm) have been considered, so that 36 classes of specimens have been tested (including the unwrapped specimens) and for each class 3 tests have been performed.

In general, the tests on small specimens could be considered exhaustive by including, among others, the results discussed in (Rochette and Labossiére, 2000; Parvin and Wei Wang, 2001; Wang and Restrepo, 2001; Tastani et al., 2006). General conclusions can be: 1) the ultimate strength increase is evident if the radius corner is appropriate, 2) the ultimate strain increase is evident even if statistically it is difficult to find a strong dependence of that increase on both mechanical and geometrical properties of the confining device. Even if the test database can be considered exhaustive, the lack of systematic tests on large scale specimens (columns) is felt. In this framework, the authors (Monti et al, 2007a) performed preliminary compression tests on nine specimens of square (200x200 mm) and rectangular (200x300 and 200x400 mm) concrete columns (1400 mm tall). All specimens had rounded corners with curvature radius of 20 mm. For each typical section, three different configurations were considered: 1) unwrapped, 2) fully wrapped, and 3) fully wrapped with steel L-shaped angles placed along the corners. The performed tests highlighted that: 1) the axial strain cannot be considered as uniformly distributed along the column, 2) the ultimate global strain is generally significantly lower than the maximum strain, 3) the damage is generally localized at the column top and is in general not smeared but tends to be concentrated along an inclined crack, and 4) in general a concentration of damage along the section sides is not observed, which implies a weak arching effect.

ULTIMATE STRENGTH PREDICITON

The classical approach proposed in the literature for the ultimate strength prediction is based on the modification of expressions obtained for circular sections having equal confinement stress (f_{lu}). The classical approach can be schematically described through Equation 1, where the strength incremental increase is obtained introducing the multiple product (Π) of n modification factor (k_i) in order to modify the increase $F(f_{lu})$ obtainable in case of equal confinement stress:

$$f_{cc} = C_1 \cdot f_{co} + \prod_{i=1}^{n} k_i \cdot F\left(f_{lu}\right) \tag{1}$$

where f_{co} is the unconfined peak strength and C_1 a modification factor of it.

The previous approach is proposed in different works, see among others (Vintzileou and Panagiotidou, 2007; Toutanji et al, 2007), as well as in design guidelines (National Research Council, 2004). Even if its predictions appear satisfactory, its mechanical justification could be misleading: modification factors are usually based on the concept that a confined zone exists where the circular section model can be applied, whose radius is evaluated based on a supposedly known *arching effect*. However, the latter is not so evident from experimental tests.

The introduction of the above-mentioned modification factors can be regarded as a shortcut to encapsulate the mechanical behaviour in a simple expression, usable for both circular and square/rectangular sections. Nonetheless, a sound mechanical justification is needed. In this work, a simple yet robust analytical expression is developed starting from some numerical investigations performed on square sections with different corner radii, as explained in the following sections.

Numerical approach

The model is based on the approach proposed in (Monti and Nisticò, 2007a,b), where a methodology is presented to predict the section's stress-strain relationship, based on the following steps: 1) the non-uniform Confinement Stress Field (CSF) applied by the confining device are numerically evaluated, 2) the local (position-dependent) axial stress-strain relationships are evaluated at each discretized point within the section, and 3) they are appropriately combined to obtain the global axial stress-strain curve of the section.
Under the assumption of path-independence, if one knew the CSF at an ultimate condition (*e.g.*, FRP failure), then the corresponding ultimate strength can be evaluated in each region of the section, based on a classical

strength criterion (*e.g.*, Ottosen, 1977). By discretizing the section in *n* regions, the section strength can be evaluated as:

$$f_{cc} = \frac{\sum_{i=1}^{n} f_{cc,i} \cdot A_i}{\sum_{i=1}^{n} A_i} \tag{2}$$

where A_i and $f_{cc,i}$ are the area and ultimate strength of each region, respectively.

The definition of the CSF is a difficult yet fundamental step. It can be assumed (based on numerical analyses) that the CSF is independent of the mechanical properties of the section material (Young's modulus and Poisson's coefficients of concrete). This implies that the *shape* of the CSF found from a linear analysis is maintained also in the nonlinear range. Actually, the CSF problem can be considered as linear only if the flexural stiffness of the confinement device is negligible, which is however the case for FRP confining devices. Thus, given the CSF for a known FRP axial strain field (E), the proportionality principle applies as follows:

$$CSF\left(c \cdot E\right) = c \cdot CSF\left(E\right) \tag{3}$$

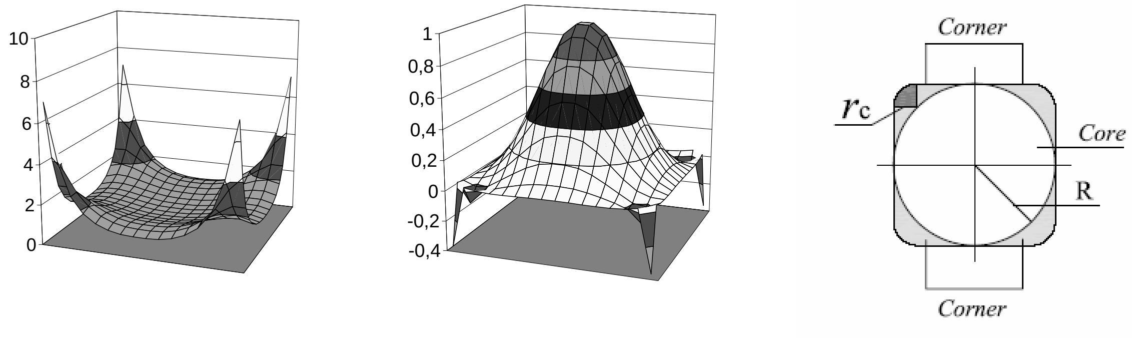
Though the strain field E varies along the section side, parametric numerical analyses have shown that the maximum/average ratio varies less than 10%. Thus, a control point at mid-side is chosen that allows to reduce the problem to a one-parameter one.

Figure 1 focuses on the Principal Stress Field (PSF) numerically evaluated: Figure 1(a) shows a typical distribution of the minimum PSF, while Figure 1(b) shows the min/max Principal Stresses Ratio (PSR). Note that the former has been normalized to the confinement stress theoretically attained in a circular section inscribed in the square one, given by:

$$f_l = 2\frac{t_f}{L} E_f \varepsilon_l \tag{4}$$

where L = square section side length, t_f = FRP thickness, E_f = FRP Young's modulus, ε_l = FRP strain at mid-side (control point).

If two sub domains are introduced (Fig 1c), one inside the inscribed circle (*core*) and remaining part (*corner*) outside the *core*, it can be assumed that: 1) inside the *core* the normalized minimum PSF is practically constant and equal to 1 (Fig. 1a), while the min/max PSR is 1 at the *core* centre and tends to 0 close to the *core* boundaries (Fig. 1b); 2) in the *corner* regions the PSF value increases approaching the section edge (Fig. 1a); 3) both PSF and PSR tend to 1 if the *corner* radius (r_c) tends to the *core* radius (R). These conclusions, based on parametric analyses, are independent on the section size.



(a) Normalized Minimum Principal Stress Field (PSF) | (b) Min/max Principal Stresses Ratio (PSR) | (c) Section with *core* and *corner* regions

Figure 1. Confinement Principal Stress Field (PSF):
the minimum principal stresses are normalized to the stress obtained in the inscribed circular section.

Analytical Approach

Alternatively, the section strength can be evaluated based on an analytical closed-form expression. The formulation here proposed is based on a simplified strength criterion that expresses the strength increase of a concrete element in terms of the PSF, as follows (the *linearity assumption* of Eq. 3 holds):

$$f_{cc} = C_1 \cdot f_{co} + F\left(\sigma_{min}\right) \cdot \frac{\sigma_{min}}{\sigma_{max}} \tag{5}$$

where σ_{min} is the minimum Principal Stress (PS), $\sigma_{min} / \sigma_{max}$ is the min/max PSR, and $F(\sigma_{min})$ is the function giving the strength increase when $\sigma_{min} = \sigma_{max}$ (i.e., in circular sections).
By integrating the previous expression over the section area, the global ultimate strength can be found. In order to obtain a viable expression for it, some simplifications are introduced in the definition of the PSF.

Evaluation of the minimum confinement stress

The minimum PS is considered as constant inside the *core (*see Figure 1a). The minimum PS in the *corner* regions reaches its max absolute value at the section edge and its min absolute value at the core boundaries. For both *core* and *corners* the PSF depends on the ratio between the *corner* rounding radius r_c and the *core* radius R (half of the section side: $R = 0.5\ L$).
The *core* minimum PSF $(\sigma_{min})_{core}$ is assumed equal to the stress field in a circular section (see Equation 4).
The *corner* minimum PS $(\sigma_{min})_{corner}$ depends on the corner radius as follows:

$$\left(\sigma_{min}\right)_{corner} = \left(2 - \frac{r_c}{R}\right)^{\beta} \cdot \left(\sigma_{min}\right)_{core} \tag{6}$$

The previous expression states that: 1) if $r_c = R$, the square section becomes circular and σ_{min} is constant all over it, 2) if $r_c < R$, σ_{min} depends on the parameter ß, which can be obtained from statistical elaboration of experimental data.

Evaluation of min/max PSR

Inside the *core*, the min/max PSR (ratio between min and max principal stresses) can be assumed as inversely proportional to the distance of the considered point from the *core* centre (having an axially symmetric field), according to the following expression:

$$\alpha_{core} = \frac{\sigma_{min}}{\sigma_{max}} = 1 - \frac{d}{R} \cdot \left(1 - \frac{r_c}{R}\right) \tag{7}$$

The previous expression implies that α_{core} is equal to 1 (equal confinement stresses) on the centre of the *core* or everywhere if $r_c = R$ (that is the case of the circular section).
Inside the *corners*, the min/max PSR is assumed to depend on the *corner*/*core* radii ratio, as:

$$\alpha_{corner} = 1 - \gamma \cdot \left(1 - \frac{r_c}{R}\right) \tag{8}$$

The previous expression implies that: 1) if $r_c = R$, α_{corner} is equal to 1 (equal confinement stresses); 2) if $r_c = 0$, α_{corner} is equal to $(1 - \gamma)$.
Having defined the PSF, Equation 5 is then applied to *core* and *corner* regions and integrated over the relevant regions in order to obtain the section strength. It holds:

$$f_{cc} = \frac{\left(F_{cc}\right)_{core} + \left(F_{cc}\right)_{corners}}{A} \tag{9}$$

where:

$$\left(F_{cc}\right)_{core} = \int_{A_{core}} \left[C_1 \cdot f_{co} + F\left(\sigma_{min}\right)_{core} \cdot \alpha_{core}\right] \cdot dA = A_{core} \cdot \left[C_1 \cdot f_{co} + \frac{1}{3} F\left(\sigma_{min}\right)_{core} \left(1 + 2 \cdot \frac{r_c}{R}\right)\right] \tag{10a}$$

$$\begin{aligned}\left(F_{cc}\right)_{corner} &= \int_{A_{corner}} \left[C_1 \cdot f_{co} + F\left(\sigma_{min}\right)_{corner} \cdot \alpha_{corner}\right] \cdot dA = \\ &= A_{corner} \cdot \left[C_1 \cdot f_{co} + F\left(\sigma_{min}\right)_{corner} - F\left(\sigma_{min}\right)_{corner} \cdot \gamma \cdot \left(1 - \frac{r_c}{R}\right)\right]\end{aligned} \tag{10b}$$

$$A_{core} = \pi \frac{L^2}{4} \tag{10c}$$

$$A_{corner} = \left(L^2 - 4 \cdot r_c^{\,2}\right) \cdot \left(1 - \frac{\pi}{4}\right) \tag{10d}$$

Further on, $F(\sigma_{min})$ can be assumed as proportional to the minimum principal confinement stress (f_{lu}) as:

$$F\left(\sigma_{\min}\right) = C_2 \cdot f_{lu} \cdot \left(\frac{r_c}{R}\right)^{C_3} \tag{11}$$

where $(r_c/\mathrm{R})^{C_3}$ represents a reduction factor accounting for the ultimate strain reduction when considering coupon test failure strains (for sharp corner radius, $r_c = 0$, the confinement is assumed as inefficient).

RESULTS AND DISCUSSIONS

Both the above-presented numerical and analytical approaches have been calibrated based on the experimental results of 14 tests reported in (de Diego et al., 2007) and 24 tests reported in (Wang and Wu, 2007). The accuracy of the models has been evaluated based on: 1) the average absolute errors (AE), 2) the average ratio (AR) between experimental and predicted values, 3) the R^2 value.

The numerical approach does not fit well the experimental results and the best fitting is obtained when considering the coupon ultimate strain instead of the greatest measured strain of the wrapped concrete specimens; improved results are obtained by reducing the predicted values by a factor of 0.72, thus obtaining: AE = 0.35, AR = 0.96, R^2 = 0.48. A possible reason for such low accuracy might be the inapplicability of the path-independence assumption, which assumes that the peak strength does not depend on the confinement stress history.

The analytical approach fits well (AE = 0.17, AR = 0.99, R^2 = 0.8) the experimental results, by assuming the coupon ultimate strain and adopting: $\beta = 1.7$, $\gamma = 1$, $C_1 = 0.8$, $C_2 = 3$; $C_3 = 0.5$. In Figure 2a the comparison between experimental and predicted values (normalized to the unconfined strength) is shown. In Figure 2b the comparison between predicted *core* strength and predicted section strength (normalized to the unconfined strength) is shown, in order to show that their ratio is almost unitary so that the section strength can be evaluated by means of Equation 10a, substituting the obtained parameter values. Thus, the equivalent strength for the square section can be evaluated according to the following expression (adopting the approach formally expressed through Equation 1):

$$f'_{cc} = \frac{(F_{cc})_{core}}{A_{core}} = C_1 \cdot f_{co} + \left[\frac{1}{3}\cdot\left(\frac{r_c}{R}\right)^{C_3}\cdot\left(1+2\cdot\frac{r_c}{R}\right)\right]\cdot\left[C_2 \cdot f_{lu}\right] = C_1 \cdot f'_{co} + k_e \cdot \left(C_2 \cdot f_{lu}\right) \tag{12}$$

where the value of k_e (efficiency factor) is:

$$k_e = \left[\frac{1}{3}\cdot\left(\frac{r_c}{R}\right)^{C_3}\cdot\left(1+2\cdot\frac{r_c}{R}\right)\right] \tag{13}$$

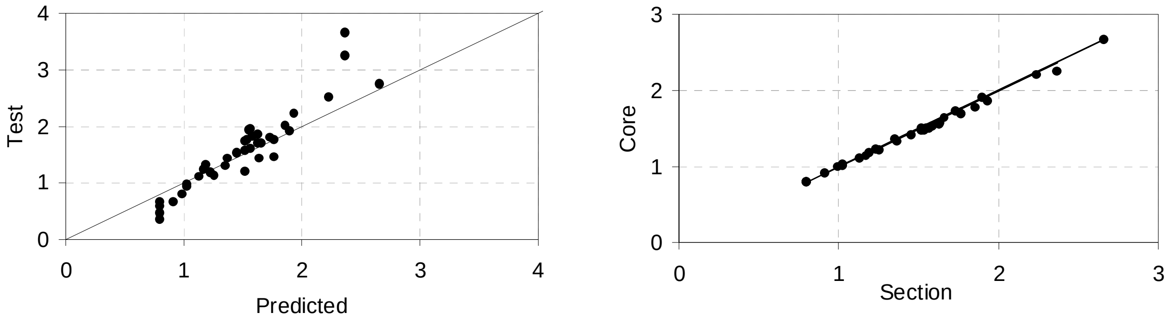


Figure 2. (a) Test *vs.* Predicted results; (b) Predicted results: Core *vs.* Section strength.

CONCLUSIONS

This paper has presented a numerical and an analytical model conceived and implemented to predict the ultimate strength of FRP-confined square sections with different corner rounding radii. Both models are based on the *path independence* assumption: *the ultimate strength of the confined concrete depends only on the confinement stress field acting at failure*. The numerical model, by means of FEM analysis: 1) evaluates the non-uniform confinement stress field at the ultimate FRP strain, and 2) by applying the Ottosen criterion, it evaluates the ultimate strength. The analytical model, based on a simplified analytical description of the non-uniform confinement stress field, evaluates the ultimate strength by means of a simple strength criterion calibrated on the results of 38 selected experimental tests. Based on the obtained results, the calibrated proposed analytical approach performs well and better than the numerical approach, thanks to the introduced parameters that allow to minimize the error between experimental and predicted results.

Further studies will regard a validation of the analytical model against other experimental tests and an extension of the proposed approach to rectangular sections.

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