

SPECTRAL SHIFTS IN THE NEAR ZONE OF A PARTIALLY COHERENT FIELD AFTER PASSING THROUGH A CIRCULAR APERTURE

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Abstract

In this paper, we investigate theoretically the spectral shifts in the near zone of a partially coherent field after passing through a circular aperture. The numerical results are presented for Lorentzian and Gaussian profiles of the original spectrum of the source. Various factors which have influenced on the spectral characteristics are also analyzed.

Keywords: Spectral shifts; Partially coherent field; Lorentzian profile; Gaussian profile; Circular aperture.

1. Introduction

Spectral shifts, in the near zone of a partially coherent field after passing an obstacle, have attracted considerable attention recently. In 1988, Dacic and Wolf [1] studied a mathematical analysis of the effect of spatial coherence of a source on the spectrum of the emitted light for a class of beamlike fields. Agrawal and Gamliel [2] derived the spectral shift for optical fields generated by a Gaussian Schell-model source and studied how the shift changes during transition from the near to the far zone. Earlier, Lin et al [3] investigated theoretically the spectral shifts of apertured Gaussian Schell-model beam. In a recent paper, Pu et al [4] obtained the spectral characteristics in the near zone when a certain class of partially coherent light passing through a circular aperture. They investigated an expression of the on-axis spectrum and performed a numerical calculation with a Lorentz original spectrum. On the basis of Pu et al. work, we present in this paper an explicit analytical expression of the on-axis spectrum and we perform in detail a numerical calculation by considering a Lorentzian and Gaussian original spectrum.

2. The axial intensity distribution

Following Pu et al, let us envision a typical arrangement: a circular aperture of radius a , in the plane of $z = 0$, is illuminated by a partially coherent field. The optical system is pictured in Fig. 1. We are interested in determining the on-axis spectrum $S(z, \omega)$, in the plane z , and comparing it to the spectrum of the light $S^{(0)}(\omega)$ in the aperture.

$S^{(0)}(\omega)$ is the original spectrum of the source in the plane of $z = 0$. Within the Fresnel approximation, Pu et al derived the following expression of the on-axis spectrum

$$S(z, \omega) = S^{(0)}(\omega) \left(\frac{\omega z_0}{\omega_0 z} \right)^2 \times \int_0^1 \int_0^1 \exp(E_1 r_1^2) \exp(E_2 r_2^2) I_0 \left(\frac{r_1 r_2}{\alpha^2} \right) r_1 r_2 dr_1 dr_2, \quad (1)$$

where

$$E_1 = -\frac{1}{2\alpha^2} - i\pi \left(\frac{\omega z_0}{\omega_0 z} \right) = E_2^*, \quad (2)$$

and

$$\alpha = \frac{1}{\Delta(\omega)} = \Delta_0 \frac{\omega_0}{\omega}. \quad (3)$$

Here, Δ_0 is the relative spatial correlation distance at the central frequency ω_0 of the spectrum, I_0 is the modified Bessel function of first kind and zero order and $z_0 = a^2/\lambda_0$ is the Rayleigh range associated with the aperture where λ_0 is the central wavelength of the original spectrum.

The integral in Eq. (1) can be evaluated in terms of the following infinite series involving modified Bessel functions of the first kind and the m th order I_m and using the identity derived from [5], i.e.,

$$\int_0^b \int_0^b \exp(E_1 r_1^2) \exp(E_2 r_2^2) I_0 \left(\frac{r_1 r_2}{\alpha^2} \right) r_1 r_2 dr_1 dr_2 = \Re_b(z, \omega) S_b(z, \omega), \quad (4)$$

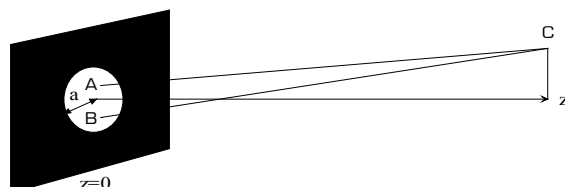


Figure 1 : Illustration of notation. A and B are two points of the aperture plane ($z = 0$). C is an observation point in the plane z .

where

$$S_b(z, \omega) = \sum_{n=0}^{\infty} \left(\frac{1}{4E_1E_2\alpha^4} \right)^n \sum_{m=-n}^{\infty} \left(-\frac{1}{2E_2\alpha^2} \right)^m I_m \left(\frac{b^2}{\alpha^2} \right), \quad (5)$$

and

$$\Re_b(z, \omega) = \frac{1 - \exp(E_2 - \frac{1}{4E_1\alpha^4})b^2 - \exp(E_1 - \frac{1}{4E_2\alpha^4})b^2}{4E_1E_2 - \alpha^{-4}} + \frac{\exp[(E_1 + E_2)b^2]}{4E_1E_2}. \quad (6)$$

Substituting Eq. (4) into Eq. (1), the on-axis spectrum can be written as

$$S(z, \omega) = S^{(0)}(\omega) \left(\frac{\omega z_0}{\omega_0 z} \right)^2 \Re_{b=1}(z, \omega) S_{b=1}(z, \omega), \quad (7)$$

and the formula is true for

$$|1/2E_2\alpha^2| < 1. \quad (8)$$

On the one hand, we assume that ω_m is the central frequency at which the on-axis spectrum takes its maximum determined by

$$dS(z, \omega)/d\omega = 0, \quad (9)$$

and ω_s is the central frequency of its secondary peak. So, the relative central frequency shift can be expressed by

$$\delta\omega/\omega_0 = (\omega_m - \omega_0)/\omega_0. \quad (10)$$

On the other hand, we define the spectral symmetry parameter β_s that describes the asymmetry of the spectral shifts at the spectral switch as

$$\beta_s = \delta\omega_{s_1}/\delta\omega_{s_2} = (\omega_{s_1} - \omega_0)/|\omega_{s_2} - \omega_0|, \quad (11)$$

where ω_{s_1} and ω_{s_2} ($\omega_{s_1} > \omega_{s_2}$) are two frequencies at which the spectrum takes its maximum at the spectral switch.

3. Numerical calculation results

To illustrate the influences of the circular aperture, in the near zone, on the on-axis spectrum, we will consider two cases in which the original spectrum $S^{(0)}(\omega)$ is of Lorentz and Gauss types. In order to show the validity of the expression of the on-axis spectrum given by Eq. (7), we will give out some numerical examples.

3.1 Lorentzian profile

In this section, we consider that the original spectrum $S^{(0)}(\omega)$ is of Lorentz type, i.e.

$$S^{(0)}(\omega) = S^{(0)} \Gamma^2 / [(\omega - \omega_0)^2 + \Gamma^2], \quad (12)$$

where $S^{(0)}$ is a constant, ω_0 is the central frequency of the spectrum and Γ is the half-width at half-maximum.

The on-axis spectrum calculated from Eq. (7) is shown in Fig. 2 for three relative propagation distances z/z_0 ($= 0.23, 0.2515$ and 0.27). The spectra are normalized to have the unity value at the maximum peak of each spectrum.

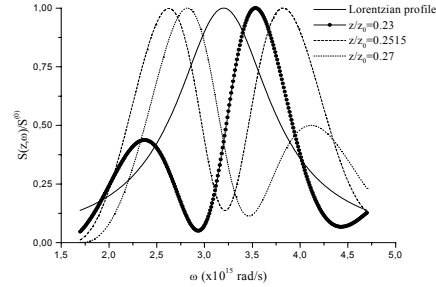


Figure 2 : The normalized on-axis spectrum for three values of z/z_0 : 0.23, 0.2515 and 0.27. The solid curve is the Lorentzian original spectrum. The parameters used in the calculation are: $\omega_0 = 3.2 \times 10^{15}$ rad/s, $\Gamma = 0.6 \times 10^{15}$ rad/s and $\Delta_0 = 1$.

The solid curve in Fig. 2 is the original spectrum. We observe that the on-axis spectrum is split into two peaks (maximum and secondary peaks). A rapid transition is observed, when the two peaks reach the same height, at $z/z_0 = 0.2515$. This value differs from that of Pu et al [4] which is equal to $z/z_0 = 0.2449$ (see Table 1). This value of z/z_0 is a critical point beyond which a spectral switch occurs. Fig. 3 shows the dependence of the relative central frequency shift $\delta\omega/\omega_0$ on the relative propagation distance from $z/z_0 = 0.15$ to $z/z_0 = 1.2$ for two coherence states of the source: $\Delta_0 = 1$ case and $\Delta_0 = 10$ case. We give the positions at which spectral switches occur such as : 0.167, 0.251 and 0.508 for $\Delta_0 = 1$ and 0.166, 0.249 and 0.499 for $\Delta_0 = 10$. When $z/z_0 = 1.2$ then $\delta\omega/\omega_0 = 1.88 \times 10^{-2}$ for $\Delta_0 = 1$, and $\delta\omega/\omega_0 = 1.25 \times 10^{-2}$ for $\Delta_0 = 10$. We can see clearly that as z/z_0 increases from 0.15, the spectral shift gradually changes. With our numerical simulation, when z/z_0 decreases, the spectral switch corresponding to $\Delta_0 = 1$ occurs before the spectral switch of $\Delta_0 = 10$ (see Fig. 3), contrary to the results of Pu et al [4].

In Table 1, we give some special spectral parameters near $z/z_0 = 0.25$. In this Table, where ω_m and ω_s denote the frequencies of the maximum peak and the secondary peak respectively, H_s the height of the secondary peak of the spectrum and $\delta\omega/\omega_0$ the spectral shift, we compare our numerical results to those of Pu et al. Besides it shows that our results

differ from those of Pu et al. From Fig. 3, we can see that as z/z_0 increases, the blue shift occurs for $\Delta_0 = 1$ and $\Delta_0 = 10$ cases. As z/z_0 continues to increase, the blue shift gradually becomes small. For the curve of $\Delta_0 = 1$, when $z/z_0 = (z/z_0)_1 = 0.167$, a first spectral switch occurs which corresponds to a transformation from the blue shift to a red one.

So we can derive the positions of the spectral switches, for $\Delta_0 = 1$ and $\Delta_0 = 10$ cases, and the spectral symmetry parameters β_s ($s = 1, 2$ and 3).

From Fig. 3, we deduce that for $\Delta_0 = 10$ the spectral shift is always symmetrical but it is not available for $\Delta_0 = 1$. The spectral symmetry parameters β_s are: 1.07, 1.09 and 1.27 for $\Delta_0 = 1$ and 0.98, 0.97 and 0.99 for $\Delta_0 = 10$.

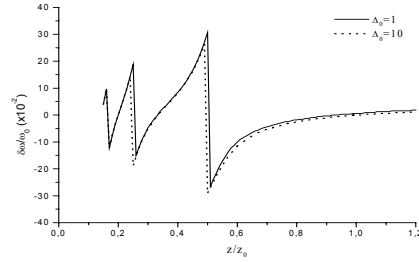


Figure 3 : Relative frequency shifts $\delta\omega/\omega_0$ (called the spectral shifts) versus z/z_0 from 0.15 to 1.2. $\Delta_0 = 1$ (solid line), $\Delta_0 = 10$ (dashed line), $\omega_0 = 3.2 \times 10^{15}$ rad/s and $\Gamma = 0.6 \times 10^{15}$ rad/s.

$\frac{z}{z_0}$	ω_m $\times 10^{15}$ rad/s (this work)	ω_m $\times 10^{15}$ rad/s ([4])	ω_s $\times 10^{15}$ rad/s	H_s (this work)	H_s ([4])	$\frac{\delta\omega}{\omega_0}$ (this work)	$\delta\omega/\omega_0$ ([4])
0.26	2.720	2.705	3.953	0.720	0.523	-0.150	-0.155
0.255	2.665	2.655	3.876	0.873	0.647	-0.167	-0.170
0.2515	3.823		2.626	1.		0.195	
0.25	3.801	2.595	2.609	0.941	0.803	0.188	-0.191
0.2449	3.728, 2.550	3.675, 2.54	2.550	0.770	1.	0.165, -0.203	0.148, -0.206
0.24	3.661	3.615	2.492	0.636	0.816	0.144	0.130
0.235	3.596	3.555	2.432	0.525	0.665	0.124	0.111
0.23	3.534	3.5	2.373	0.437	0.548	0.104	0.094

Table 1: Special parameters corresponding to the on-axis Lorentzian spectrum with $\omega_0 = 3.2 \times 10^{15}$ rad/s, $\Gamma = 0.6 \times 10^{15}$ rad/s and $\Delta_0 = 1$.

To illustrate how the source spectrum width affects the spectral switch, we present in Fig. 4 the relative spectral shift versus z/z_0 for three spectrum widths of the source: $\Gamma = 0.6 \times 10^{15}$ rad/s, 0.4×10^{15} rad/s and 0.2×10^{15} rad/s.

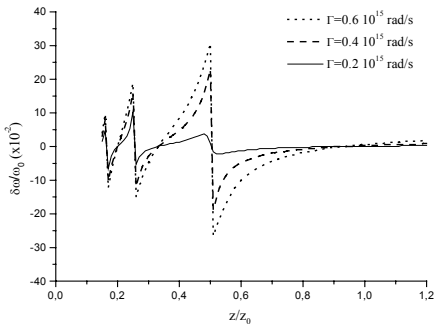


Figure 4 : Relative frequency shifts $\delta\omega/\omega_0$ versus z/z_0 for different source spectrum widths. $\Delta_0 = 1$ and $\omega_0 = 3.2 \times 10^{15}$ rad/s.

It can be deduced from this figure, that at the positions at which spectral switches occur, the frequency shift decreases with the spectrum width of the source but the positions of the spectral switches remain invariant. This figure shows also that the slope of the curves tends to the horizontal when the spectrum width decreases importantly.

$\Gamma \times 10^{14}$ rad/s	6.	4.	2.
$(\delta\omega/\omega_0)_{z=1.2}$ $\times 10^{-2}$	1.875	0.938	0.313
β_1	1.07	1.08	0.97
β_2	1.09	1.04	1.03
β_3	1.27	1.23	1.71

Table 2 : Spectral switch parameters for different spectrum widths of the Lorentzian source relatively to Fig. 4.

Relatively to Fig. 4, we give in Table 2 the frequency shift at $z/z_0 = 1.2$ for the three values of the spectrum width, which decreases with it, from 1.88×10^{-2} to

0.31×10^{-2} and the spectral symmetry parameters β_s ($s = 1, 2$ and 3) for each value of Γ . It is shown from this Table that, for $s = 1$ and 2 , $\beta_s \approx 1$, but for $s = 3$, the spectral switch is not symmetrical for each value of Γ .

From Fig. 5, we can see clearly that the coherence Δ_0 takes an important role on frequency shift of the source in the region of the spectral switch. For example, near $z/z_0 = 0.5$ we give three curves of $\delta\omega/\omega_0$ as a function of Δ_0 from 1 to 4. For $z/z_0 = 0.490$, the relative spectral shift remains positive (i.e. blue shift occurs) but for $z/z_0 = 0.495$ it becomes negative for $\Delta_0 \in [1.43, 1.92]$ (red shift).

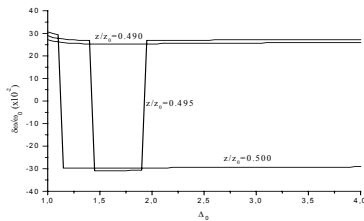


Figure 5 : Plots of the relative frequency shifts $\delta\omega/\omega_0$ for a Lorentzian profile versus Δ_0 from 1 to 4 near $z/z_0 = 0.500$. The parameters used in the calculation are the same as in Fig. 2.

Finally for $z/z_0 = 0.500$, the spectral shifts begin positive and become negative from $\Delta_0 = 1.13$. The spectral symmetry parameters β_s are: 0.87 and 0.88 for $z/z_0 = 0.495$ and 0.98 for $z/z_0 = 0.500$. Note that for this last value, the spectral switch is symmetrical.

3.2 Gaussian profile

Suppose now that the original spectrum has a Gaussian profile given by [6]

$$S^{(0)}(\omega) = S^{(0)} e^{-(\omega - \omega_0)^2 / 2\delta^2}, \quad (13)$$

where $S^{(0)} (= \frac{1}{\delta\sqrt{2\pi}})$, ω_0 and $\delta < \omega_0$ are

positive constants. In order to show the influences of the profile on the on-axis spectrum, we will give out some numerical examples in the case of the Gaussian profile by using Eq. (7).

The calculated on-axis spectrum, shown in Fig. 6 where the solid curve is the original spectrum, is performed with $\omega_0 = 3.887 \times 10^{15}$ rad/s ($\lambda_0 = 4.861 \text{ \AA}$), $\delta = 9.57 \times 10^{14}$ rad/s and $\Delta_0 = 1$. For this profile, the values of the relative frequency shifts $\delta\omega/\omega_0$ are: 0.135, (0.233, -0.204), -0.186 for $z/z_0 = 0.23, 0.2534$ and 0.26 respectively (see Table 3).

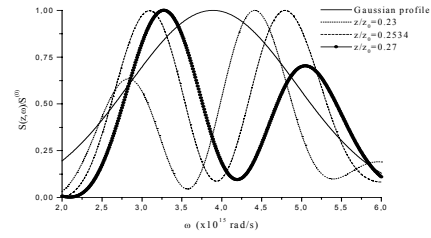


Figure 6 : The normalized on-axis spectrum for three values of z/z_0 : 0.23, 0.2534 and 0.27. The solid curve is the Gaussian original spectrum.

As in the above section, we present in Fig. 7 the plots of the relative frequency shifts for the Gaussian profile of the original spectrum as a function of z/z_0 from 0.15 to 1.2 for the two following values of Δ_0 (1 and 10).

The values of $\delta\omega/\omega_0$ for $\Delta_0 = 1$ and $\Delta_0 = 10$ when $z/z_0 = 1.2$ are: 7.02×10^{-2} and 3.94×10^{-2} respectively. From this figure, we can see that the positions of the spectral switches are: 0.168, 0.253 and 0.510 for $\Delta_0 = 1$ and 0.166, 0.249 and 0.499 for $\Delta_0 = 10$. We note that in the case of $\Delta_0 = 10$, we obtain the same value as in the Lorentzian profile. In this case, the spectral symmetry parameters β_s are: 1.154, 1.147 and 1.126 for $\Delta_0 = 1$ and 0.977, 0.98 and 0.988 for $\Delta_0 = 10$.

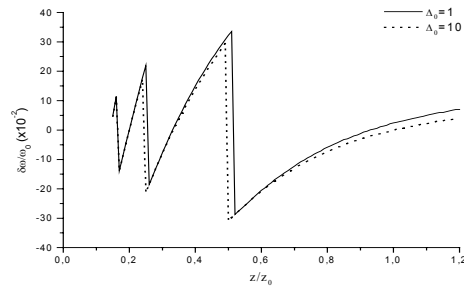


Figure 7 : Relative frequency shifts $\delta\omega/\omega_0$ for a Gaussian original spectrum versus z/z_0 from 0.15 to 1.2 for two values of Δ_0 , $\Delta_0 = 1$ (solid line) and $\Delta_0 = 10$ (dashed line).

Fig. 8 shows the dependence of the relative frequency shifts versus z/z_0 for a Gaussian profile for three source spectrum widths: $\delta = 10 \times 10^{14}$ rad/s, 8×10^{14} rad/s and 6×10^{14} rad/s. It is shown that as δ increases, the relative frequency shifts decrease and the positions, where the spectral switches take place, decrease gradually, contrary to Lorentzian profile where these positions remain invariant.

z/z_0	$\omega_m \times 10^{15}$ rad/s	$\omega_s \times 10^{15}$ rad/s	H_s	$\delta\omega/\omega_0$
0.26	3.166	4.895	0.873	-0.186
0.255	3.111	4.817	0.969	-0.200
0.2534	4.792,3.094	3.094	1	0.233,-0.204
0.25	4.738	3.056	0.931	0.219
0.245	4.658	3.000	0.843	0.198
0.24	4.577	2.944	0.764	0.178
0.235	4.495	2.887	0.695	0.156
0.23	4.412	2.830	0.634	0.135

Table 3: Special parameters corresponding to the on-axis Gaussian spectrum with $\omega_0 = 3.887 \times 10^{15}$ rad/s and $\delta = 9.57 \times 10^{14}$ rad/s and $\Delta_0 = 1$.

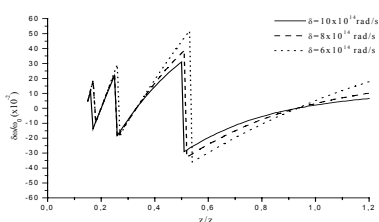


Figure 8 : Relative frequency shifts $\delta\omega/\omega_0$ for a Gaussian original spectrum versus z/z_0 for different source spectrum widths. $\Delta_0 = 1$ and $\omega_0 = 3.887 \times 10^{15}$ rad/s.

For this case, we can see from Fig. 8 and Table 4 that the width of the source affects the relative frequency shifts and the positions of the spectral switches. As Fig. 5, we give in Fig. 9 the relative frequency shifts as a function of Δ_0 from 1 to 4 near $z/z_0 = 0.5$. This figure shows that for $z/z_0 = 0.490$, $\delta\omega/\omega_0$ is still positive. For $z/z_0 = 0.495$, $\delta\omega/\omega_0$ begins positive and becomes negative between $\Delta_0 = 1.31$ and $\Delta_0 = 2.31$. But for $z/z_0 = 0.500$, $\delta\omega/\omega_0$ becomes negative from $\Delta_0 = 1.13$ as the Lorentzian profile.

$\delta \times 10^{14}$ rad/s	10.	8.	6.
$(\delta\omega/\omega_0)_{z/z_0=1.2} \times 10^{-2}$	6.51	10.11	17.83
$(z/z_0)_1$	0.168	0.170	0.174
β_1	1.154	1.327	1.806
$(z/z_0)_2$	0.253	0.255	0.263
β_2	1.137	1.218	1.575
$(z/z_0)_3$	0.509	0.515	0.539
β_3	1.112	1.199	1.498

Table 4: Spectral switch parameters for different spectrum widths of the Gaussian source relatively to Fig. 8.

The spectral symmetry parameters β_s in this case are: 0.93 for $z/z_0 = 0.495$ and 0.997 for $z/z_0 = 0.500$.

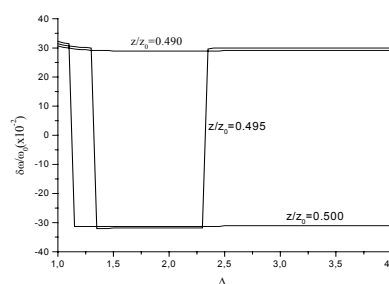


Figure 9 : Plots of the relative frequency shifts $\delta\omega/\omega_0$ for a Gaussian profile versus Δ_0 from 1 to 4 near $z/z_0 = 0.5$. The parameters used in the calculation are identical to those of Fig. 6.

4. Conclusion

In this paper, we have derived an explicit expression in the near zone for the on-axis spectrum within the Fresnel approximation when a partially coherent light is incident upon a circular aperture. We have investigated the normalized on-axis spectrum and the relative frequency shifts for two profiles of the original spectrum of the source: Lorentz and Gauss types. We have also shown that the on-axis spectrum and the relative frequency shifts are dependent on the relative spatial correlation distance at the central frequency and the original spectral parameters.

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