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The Case of the Singing Vortex

A relatively high amplitude, discrete tone is radiated from fully developed tip vortex cavitation under certain conditions. The phenomenon of the "singing vortex" was first reported by Higuchi et al. (1989). This study more closely examines the singing phenomenon by varying the hydrofoil cross-section, scale, angle of attack, water quality, and cavitation number in two different facilities. Noise data were collected for each condition with visual documentation using both still photography and high speed video in an effort to explain the mechanism of vortex singing. The theory of Kelvin (1880) provides a framework for correlating all the data obtained.

Introduction

This study is part of a larger investigation of tip vortex cavitation in which both the inception physics and more developed forms of cavitation are being investigated. It was found that, under certain conditions, a discrete tone was radiated when fully developed cavitation occurred, i.e. a completely vapor filled vortex core was attached to the hydrofoil. This was noted in a previous study (Higuchi et al., 1989) but was not studied extensively.

Developed cavitation in a trailing vortex has been studied in the past by Souders and Platzer (1981), Arakeri et al. (1988), and Arndt et al. (1991). The focus of these earlier studies was on the general flow features such as the vortex trajectory and the scaling of the vapor core radius with cavitation number. No mention was made of a discrete tone, presumably due to the fact that the phenomenon takes place over a very narrow range of cavitation number and is difficult to reproduce without reasonable care. However, the data reported herein have been presented at several workshops and seminars in the last two years and since that time other laboratories have been able detect the same sound.

This study compares sound data collected in two different water tunnels in different parts of the world. A comparison is made between the results obtained in the same facility as used by Higuchi et al. (1989) and the larger water tunnel used by Arndt and Keller (1992) in Obernach, Germany. When properly normalized, the results from the two facilities agree amazingly well.

Experimental Setup

Four hydrofoils of elliptic planform with aspect ratio 3 but different cross sections were used for this study. The hydrofoil sections chosen were a NACA 66_2-415 $a = 0.8$, a modified NACA 4215 (designated herein as NACA 4215M), a NACA 16-020, and a NACA 66-012. The sectional characteristics were held constant from the root to the tip. Hence each foil had a spanwise thickness distribution that was also elliptical. Two sets of each foil were constructed. The smaller set had a root chord c_0, of 81 mm and a half span, b, of 95 mm while for the larger set $c_0 = 129.4$ mm and $b = 152.4$ mm. The small set was utilized for cavitation testing, force measurements and observation of the bubble dynamics at the St. Anthony Falls Laboratory (SAF). The larger set was used for cavitation studies and force measurements at the Versuchsanstalt für Wasserbau in Obernach, Germany. The larger foils were also used for oil flow visualization studies at SAF. These complementary studies provide a comprehensive view of the flow in the tip region and its correlation with cavitation.

Cavitation testing and force measurements were made in two water tunnels, one at SAF, which has a 190 mm square cross section (Arndt et al., 1991), and the other at Obernach with a 300 mm square cross section (Arndt and Keller, 1992). Oil flow visualizations were obtained in two wind tunnels, one at the Department of Aerospace Engineering at the University of Minnesota (Higuchi et al., 1987) and the second at SAF (originally an air model of the HYKAT facility, Wetzel and Arndt, 1994).

Cavitation tests were performed by fixing the angle of attack and velocity and then slowly lowering the pressure until fully developed cavitation occurred, with the vapor filled core just barely attached to the foil. Careful adjustments in either velocity or pressure were made until "singing" occurred. It was found in initial trials that the singing phenomenon was far more reproducible with the 66_2 series foil. Thus this study was concentrated on tests with this foil.

Oil flow data were obtained using a spray of fine droplets of an oil and titanium-oxide mixture (Maines and Arndt, 1993). The wind tunnel was run at the test velocity of 56 m/s ($Re \approx 485{,}000$). This technique highlighted the details of the boundary layer flow especially in the tip region.

Observations of cavitation were made with either conventional still photography or with high speed video. Only still photography, using a standard Nikon 6006 camera, was used for observations at SAF. A new Kodak video camera, with the possibility of framing rates as high as 40,500 fps, was used at Obernach in conjunction with still photography, also using a standard Nikon camera. Video observations were made at a framing rate of 4500 fps, which was more than adequate for observing phenomena that had a frequency less than 600 Hz. Data were collected over a range of lift coefficients, velocities and water quality in both facilities.

Radiated sound was measured in the SAF tunnel with an hydrophone positioned above the hydrofoil tip in a tank of quiescent water that was separated from the test section by a thin plate of plexiglas (Higuchi et al., 1989). The hydrofoil was mounted at the floor of the test section and the thin plexiglass plate formed the roof of the test section. A similar setup was used in Obernach, except that the hydrofoil was mounted in the roof of the test section and a single hydrophone was mounted in a tank of water that was positioned against one of the side windows of the test section. Therefore the observation angle differed by approximately 90° in the two test facilities. The significant differences in the acoustic path for measurements in the two facilities precluded comparison of amplitude data. Only frequency data were compared. A serious attempt was made to acoustically calibrate the Obernach tunnel. However, an accurate calibration procedure was not possible in the frequency range of interest, because of the complex acoustic response of the water tunnel.

Uncertainty Limits

An error analysis for the measurement of cavitation number and lift in the Obernach facility was given in Arndt and Keller

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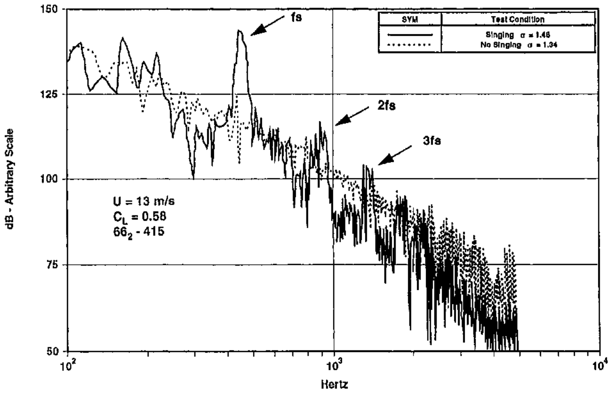

Fig. 1 Comparison of noise spectra with and without singing, U = 13 m/s. (66_2 − 415 hydrofoil in the Obernach facility)

(1992). These measurements can be made within an accuracy of approximately 1 percent.

All the video and photographic data collected at both laboratories was reduced at SAF. A detailed discussion of the measurements made at SAF is presented in the dissertation of Maines (1995). Errors in measuring velocity were found to be less than 1 percent for velocities greater than 8 m/s. Errors in lift coefficient varied greatly with velocity but were in the range 0.2–2.2 percent in the velocity range 16 m/s to 5 m/s, the largest error occurring at the lowest velocity. The errors in cavitation number were about the same as the lift coefficient over the same velocity range, again the largest error was associated with the lowest velocity.

There is considerable difference in the errors associated with measuring core radius using still photography and video analysis. The error in analyzing the still photographs is quite good, typically less than 5 percent. Unfortunately, limitations on the spatial resolution of the video can create quite large errors (approximately 25 percent) at diameters less than 1 mm. Diameters greater than 2 mm can be resolved with error that is less than 10 percent.

Experimental Results

A sample comparison of the measured sound spectra with and without singing is shown in Fig. 1. These data were obtained in the Obernach tunnel by holding velocity constant at 13 m/s and slowly lowering σ until singing occurred. The difference in σ for the two spectra is very small. What is apparent is that when singing occurs it shows up in the spectrum as a very intense peak of about 25 dB above the background at a discrete frequency.

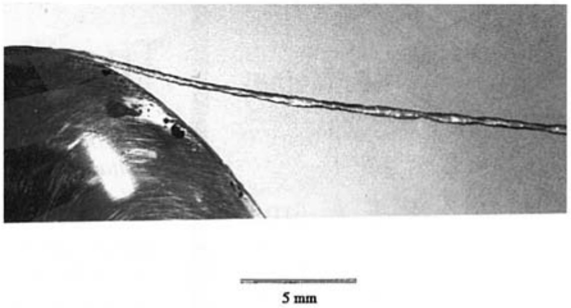

Fig. 2 Photograph of developed tip vortex cavitation without singing (NACA 66_2 − 415)

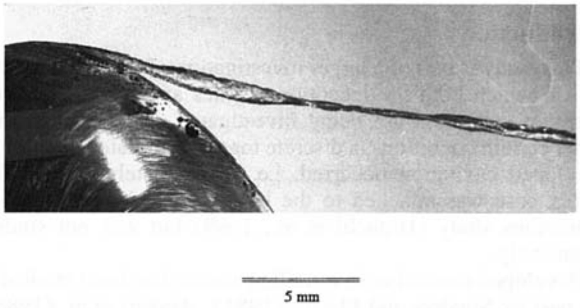

Fig. 3 Photograph of singing vortex with enlarged core near the tip (NACA 66_2 − 415)

The singing vortex has been observed primarily on the NACA 66_2-415 hydrofoil. Singing was achieved with the larger scale NACA 4215M at the Obernach facility but only at very low amplitudes. Singing was not observed with the smaller scale 4215M in the SAF facility. In general, the phenomenon has the appearance of a standing wave superimposed on the surface of the hollow vortex core. Figures 2 and 3 are photographs of a cavitating core without and with singing respectively. In Fig. 2, the cavitation number has been raised just enough to suppress the singing. In Fig. 3, σ has been lowered to the point that the vortex begins to sing. Note the thickening of the core just downstream of the tip followed by a very thin core. This pattern is repeated downstream. As the oscillation progresses through one cycle, the radius of the thick portion near the tip decreases while the thin segment expands radially.

The driving mechanism for this phenomenon is still unknown. However, flow induced vibration is discounted, in agreement with Higuchi et al. (1989). The natural frequency of both sets of foils was measured using a laser vibrometer. At the SAF facility, measurements were taken with the foil outside the tunnel. Thus the measured frequency is slightly higher than it would be submerged in water. The natural frequency of the hydrofoil at the Obernach facility was measured while submerged in the water tunnel test section under static conditions. Hydrofoil vibration was then monitored with the laser vibrometer over a wide range of flow velocity with and without cavitation and with and without singing. Under all conditions the predominant vibrational frequency was the same as that measured under static conditions. In both cases the foil natural frequency was found to be much lower than the observed singing frequencies.

Throughout the test program singing was only observed when the hollow core was attached to the tip. Thus a more likely mechanism for singing is the result of a complex interaction between the tip boundary layer and the attached cavity. High speed video observations indicate that the attached tip cavity oscillates with the same frequency as the pulsating vortex core.

Nomenclature

a = core radius, m
b = half span, m
c_o = root chord, m
C_1, C_2, C_3 = constants
C_L = lift coefficient
f = frequency, Hertz
k = wave number, $2\pi/\lambda$, 1/m
K_1, K_1' = modified Bessel functions of the second kind
N = function of ka, defined in the text
U = freestream velocity, m/s
Γ = circulation, m^2/s
λ = wavelength, m
Ω = core rotational speed, radians/s
σ = cavitation number
σ_s = cavitation number when singing occurs

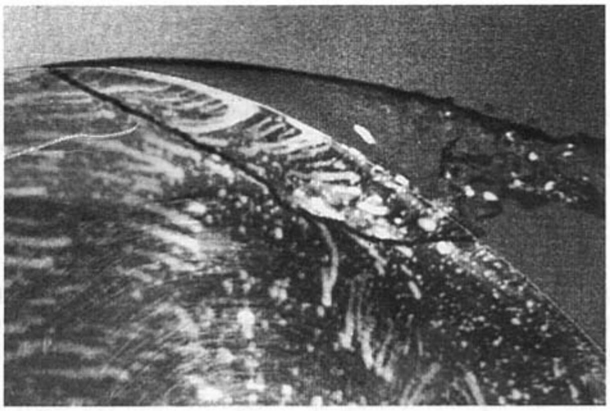

Fig. 4 Superposition photograph of the tip cavity and boundary layer characteristics

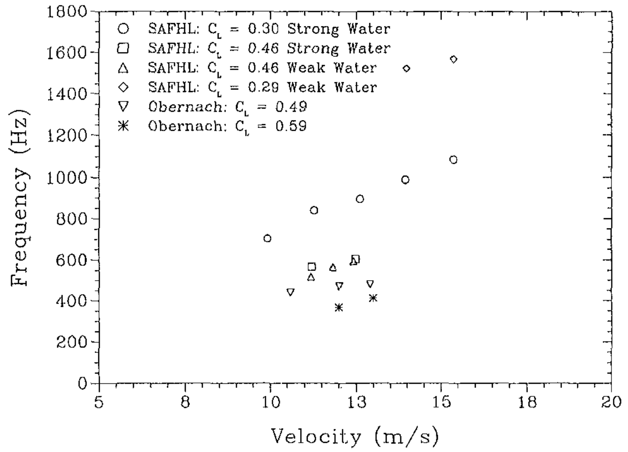

Fig. 6 Variation of singing frequency with test section velocity

Figure 4 highlights the relationship between the tip cavity and the average boundary layer characteristics. This picture was created by superimposing a single image of a tip cavity at a cavitation number slightly higher than σ_s with a photograph of oil film streaklines taken at an equivalent condition in a wind tunnel. Note that cavity detachment correlates well with separation of the boundary layer in the tip region at this phase of the cycle.

The conditions at which singing is observed to occur are limited. For a given free-stream velocity and angle of attack, singing only occurs within a narrow band of cavitation number, $\Delta\sigma \approx \pm 0.15$ from the test condition, σ_s. While varying σ through this narrow band, the frequency of the tone is observed to decrease with decreasing sigma. In addition, the amplitude of the tone peaks at σ_s. Figure 5 is a plot of the cavitation number for singing, σ_s, versus velocity for different lift coefficients obtained in both water tunnels. Note that σ_s at a given lift coefficient remains constant with velocity. Testing in the Obernach facility showed some variation of σ_s with velocity which may be due to water quality effects. Tests at Obernach also indicated that water quality, i.e., air content, of the tunnel can greatly affect the cavitation number at which singing occurs. The water quality is difficult to maintain for long periods of time in this facility. The value of σ_s was relatively unaffected by water quality in the SAF facility. In general, it was found that the vortex sings most easily and with the greatest amplitude at an optimum value of lift coefficient and σ_s.

The frequency of the tone was found to vary with cavitation number around a given σ_s. In addition, the frequency also varies with freestream velocity in a linear fashion, as noted previously by Higuchi et al. (1989). This is illustrated in Fig. 6, which is a plot of frequency versus velocity at σ_s for various conditions in both the SAF and Obernach facilities. The frequency range in the SAF studies was approximately 650 Hz to 1600 Hz. By comparison the frequency range in the larger Obernach tunnel was significantly lower, approximately 300 Hz to 600 Hz.

High speed video images provided a convenient method to verify the frequencies measured with the hydrophones and confirm that the noise is produced by the undulations of the vortex core. A plot of frequency measured from the video images versus that obtained with the hydrophones is shown in Fig. 7. Very good agreement is observed. As noted in Fig. 6, the frequencies measured at the Obernach facility are consistently lower than those observed in the SAF facility. This scale effect on singing appears to be related to the radius of the cavitating core. This is confirmed by the data in Fig. 8 which contains a plot of the variation of frequency at σ_s with core radius. Clearly there is an inverse relationship between frequency and core radius. It should also be noted that core radius varies with cavitation number (see Arakeri et al., 1989) which suggests a complex relation between frequency, σ and core radius.

The final parameter measured from the video images and still photographs is the wavelength of the disturbance. Figure 9 is a graph of wavelength versus core radius. In the case of the Obernach data, it appears that wavelength falls along two different curves. Careful analysis of the high speed video images revealed that two general types of wave oscillations exist. Both types of

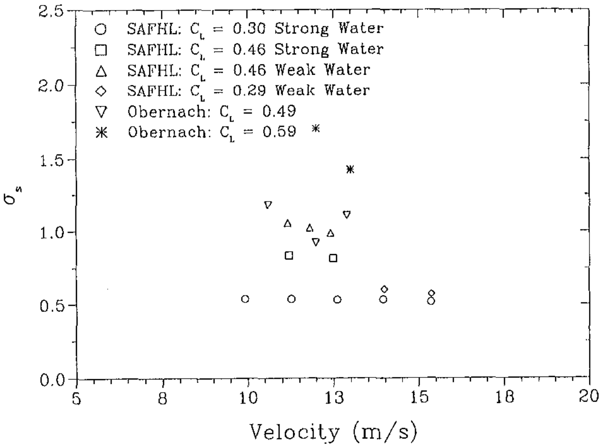

Fig. 5 Cavitation number at singing versus velocity for various conditions

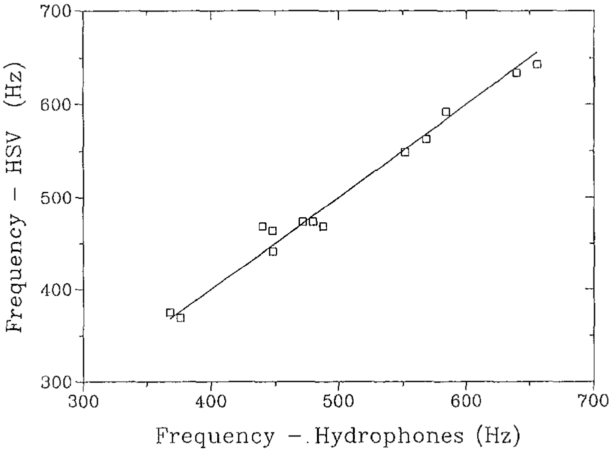

Fig. 7 Comparison of measured frequency with observed frequency in the high speed videos

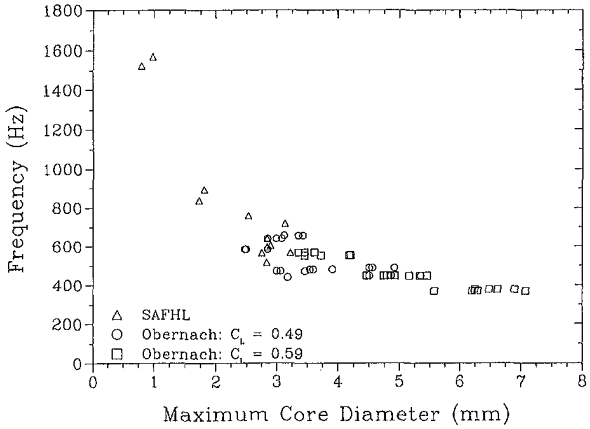

Fig. 8 Variation of singing frequency with core diameter

oscillations are possible at a given condition. However, they do not occur simultaneously. Type I oscillations are distinguished by stationary node points whereas for Type II, the nodes oscillate in the axial direction. Type I oscillations more closely resemble a standing wave. Wavelength measurements for Type II oscillations are therefore more difficult to determine and must be regarded as approximate. The SAF data have the same trend as the Obernach Type I data. However, the trend is followed in steps, rather than linearly. It is unclear at present whether the nonlinear relationship between wavelength and core radius is real or is an artifact of the experimental procedure. Since only random still photographs were available from the SAF testing, it was difficult to determine at which point in the cycle the photograph was taken. It is much easier to determine core diameter than wavelength and since diameter measurements appear to be consistent when compared to the Obernach data in Fig. 9, it is likely that the measured wavelengths from the SAF still photographs are in error.

A Proposed Correlation

The experimental observations suggest a standing wave on the surface of the hollow vortex core. Kelvin (1880) studied the wave pattern on a stationary, irrotational hollow core vortex. He found two dominate helical modes, one rotating with the same sense as the vortex and the other rotating and propagating in the opposite direction. The rotational speeds, $2\pi f$, are given by

$$\frac{2\pi f}{\Omega} = [1 \pm \sqrt{N}] \tag{1}$$

where Ω is the rotational speed of the vortex and N is a function of wave number based on core radius, $ka = 2\pi a/\lambda$, and is numerically greater than unity:

$$N = \frac{kaK_1'(ka)}{K_1(ka)} \tag{2}$$

As suggested by Keller and Escudier (1980), a standing wave is possible when the vortex is superimposed on a uniform axial flow. This can occur when the celerity of the counter-rotating mode is equal and opposite to the freestream velocity, U (negative frequency in Eq. (1)). This occurs when

$$\frac{f\lambda}{U} = 1 \tag{3}$$

In all cases tested, the condition $f\lambda/U = 1$ corresponded to Type I oscillations.

The negative root of Eq. (1) can be approximated by C_1ka or

$$\frac{2\pi fa}{U} = C_1 ka \times \left\{\frac{\Omega a}{U}\right\} \tag{4}$$

By assuming the vortex to be irrotational, the use of the Bernoulli equation yields $(\Omega a/U) = \sqrt{\sigma}$. Using this result in combination with Eqs. (3) and (4) yields

$$\sqrt{\sigma_s} = \frac{1}{C_1}, \Rightarrow \sigma_s = \frac{1}{C_1^2} \tag{5}$$

This suggests the surprising result that a standing wave will occur at only a single value of σ. This finding is in qualitative agreement with the observation that singing only occurs over a very narrow range of cavitation number.

The assumptions leading up to Eqs. (4) and (5) imply a fixed value of lift coefficient as well. By assuming an elliptical loading of the hydrofoil and noting that $\Gamma = 2\pi\Omega a^2 = (1/2) C_LUc_o$, a relationship between lift coefficient, C_L and σ can be obtained:

$$\frac{\Gamma}{Ua} = 2\pi\left\{\frac{\Omega a}{U}\right\} = 2\pi\sqrt{\sigma_s} \tag{7}$$

or

$$C_L = 4\pi\left(\frac{a}{c_o}\right)\sqrt{\sigma_s} \tag{8}$$

This result is also qualitatively consistent with observations, since singing is only observed over a relatively narrow range of lift coefficient as well.

Discussion

The simple theory outlined above only provides a rough framework for analysis of the data. Singing does not actually occur at a fixed value of ka and σ_s. As previously stated, a linear fit to the data in Fig. 9 corresponds to $ka = 0.5$ (assumed equal to $2\pi fa/U$). However, it was found that $2\pi fa/U$ can vary from approximately 0.25 to 0.65. The variation in ka can be correlated with variations in σ_s as shown in Figure 10 which is a graph of $2\pi fa/U$ versus $\sigma_s^{0.5}$. This implies a slightly different dispersion relation than expressed by Eq. (4):

$$\frac{2\pi fa}{U} = C_2\sqrt{\sigma_s} \tag{9}$$

where $C_2 \approx 0.45$.

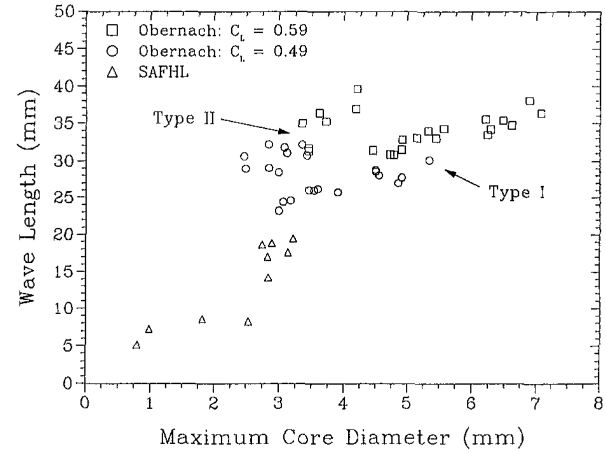

Fig. 9 Variation of wavelength with core diameter

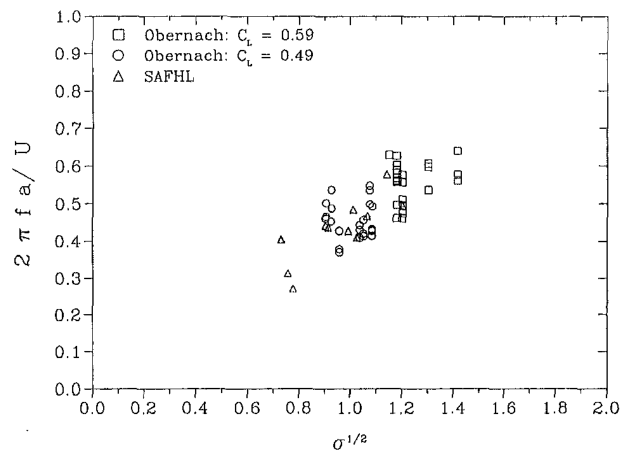

Fig. 10 Experimental dispersion relation

Equations (4) and (8) are only consistent for a fixed value of ka. As noted earlier, the Type I oscillation data can be fitted by a nearly constant value of $ka = 0.5$. If C_1 is calculated from $C_2/C_1 = ka$ and $ka \approx 0.5$, the cavitation number at which singing should occur is calculated to be to $\sigma_s = 1.2$, which is in reasonable agreement with experiment. The approximate center of the cluster of the data in Fig. 10 corresponds to $ka = 0.5$ and $\sigma_s = 1.2$. Noise amplitude did vary for each data point, but could not be accurately measured in this frequency range. Had it been possible to accurately measure amplitude, it may have been possible to fit the data in Fig. 10 with iso-contours of amplitude that would be elliptical in shape with their major axes aligned at a slope of 0.45.

Kelvin's theory is actually matched by $C_1 = 0.336$, $\sigma_s = 8.85$, and $ka = 1$, which is a contradiction with the observations. However, the experimental trends are in qualitative agreement with the theory if a slightly modified dispersion relation is used with a measured value of $ka = 0.5$. This analysis clearly has limitations which could cause the observed discrepancies between theory and experiment. First, as mentioned previously, the dispersion relation in Eq. (1) is based on a linearized analysis. Observed displacements of the vortex core are on the order of twice the core radius under nonsinging conditions.

The assumption of the potential flow around the core is also questionable. Simple analytical models, e.g., Arndt and Keller (1992), indicate that potential flow can exist outside the core, but Escudier et al. (1980) suggests that a viscous layer surrounds the hollow core. In addition, it is known that a single phase vortex deviates from the simple Rankine vortex used to model the rotational flow. Actual pressures in the core were not measured and therefore the assumption of vapor pressure may also be invalid. Unfortunately, attempts to directly measure the velocity in the liquid surrounding the vapor filled core were also not successful. In addition, the correlation with C_L is based on measured values of this parameter in noncavitating flow.

The validity of assuming noncavitating values of C_L and of assuming potential flow was investigated by Levy (1995). Using the 66_2-415 hydrofoil, the lift coefficient and core radius was measured over a wide range of σ for two values of angle of attack and velocity that bracket the conditions where singing is observed. In these experiments sigma was lowered until lift decreased. Depending on the angle of attack, the lift was found to increase by as much as 20 percent with decreasing σ before the lift decreased. Singing occurred at values of σ where the lift was equal to the noncavitating value. However, measured values of core radius did not correlate with measured values of lift coefficient as predicted by Eq. (7) in the form

$$\frac{a}{c_o} = \frac{C_L}{4\pi\sqrt{\sigma}} \tag{10}$$

This is shown in Fig. 11 in which the measured core radius in normalized form, a/c_o, is compared with the measured lift coefficient in the form $C_L/4\pi\sqrt{\sigma}$. The core radius is consistently under-predicted by this equation. Data from the Obernach experiments are in qualitative agreement with the SAF data shown in Fig. 11.

These results indirectly indicate that the Bernoulli equation is not valid which is one of the assumptions in the Kelvin theory. How inaccurate this assumption is open to debate. If the flow were irrotational, the circulation at the edge of the core would be equal to the total circulation in the vortex, i.e.,

$$\frac{2\pi\Omega a^2}{\frac{1}{2}C_L U c_o} = 1 \tag{11}$$

which was assumed in deriving Eq. (7). When the data in Fig. 11 are replotted as $(4\pi a\sqrt{\sigma}/C_L c_o) = f(\sigma/\sigma_C)$, where σ_C is the value of sigma at the lift breakpoint, it is found that $(4\pi a\sqrt{\sigma}/C_L c_o) \to 1$ as $(\sigma/\sigma_C) \to 1$. However, this occurs at values of sigma well below that for singing to occur.

Lastly, Kelvin's model was developed for an infinitely long vortex core and predicts two helical waves. In a uniform flow, when the reverse wave speed is equal to the freestream velocity, a standing wave should occur. For these experiments, a standing wave does occur for Type I oscillations but, as shown in Fig. 3, has radial undulations. This motion could be due to imprecise

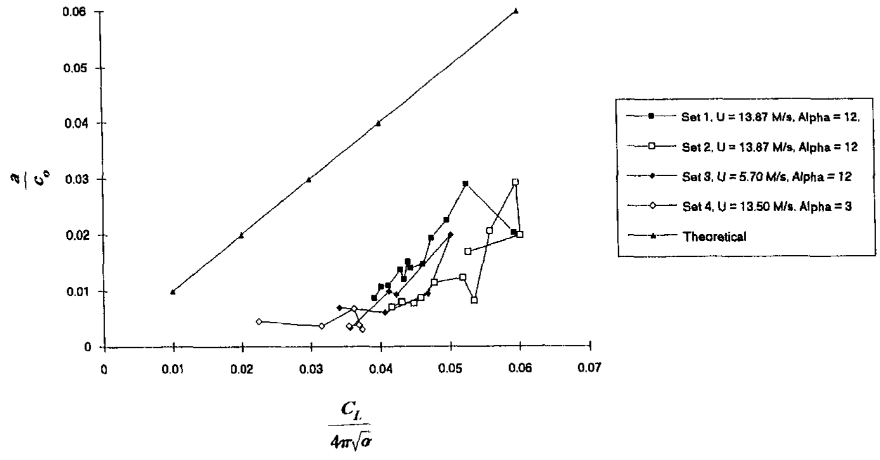

Fig. 11 Correlation of core radius with lift coefficient and σ_s

lock-in. However, it is most likely related to the fact that the experimental vortex core is not infinite but originates at the tip of the hydrofoil. Regardless of these limitations, Kelvin's dispersion relation does seem to describe the gross behavior observed experimentally.

Conclusions

Vortex singing has been observed at different facilities under varying conditions. The frequency varies inversely with the size of the hydrofoil being tested and the phenomenon is very sensitive to cavitation number. A relatively simple linear theory predicts that singing will only occur at a one fixed value of sigma that is unrealistically high and at a fixed value of lift coefficient. Experimentally, singing was observed over a range of lift coefficient and cavitation number. However, singing only occurred over a very narrow range of cavitation number for each given value of lift coefficient.

In spite of the limitations of the theory, correlation of the frequency with the measured core radius was quite good. The most likely value is $fa/U = 0.5$ at $\sigma_s \approx 1.2$. However, imprecise lock-in can occur of a range of a and σ_s given by

$$\frac{2\pi fa}{U} = 0.45\sqrt{\sigma_s}, \quad \left[0.25 < \frac{2\pi fa}{U} < 0.65\right] \qquad (12)$$

The exact mechanism for singing is not understood. It appears that a standing wave on the vapor-liquid interface of the vortex core is in phase with sheet cavity oscillations when this phenomenon occurs.

Singing is observed most readily with the $66_2 - 415$, but singing is also observed with the 4215M and 16-020 sections. Singing appears to occur more readily with the larger scale hydrofoils in Obernach.

Acknowledgments

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