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RELIABILITY-BASED TOPOLOGY OPTIMIZATION CONSIDERING MULTICRITERIA USING FRAME ELEMENTS

Katsuya Mogami, Kazuhiro Izui,
Shinji Nishiwaki, and Masataka Yoshimura
Department of Aeronautics and Astronautics,
Kyoto University
Yoshida-honmachi, Sakyo-ku,
Kyoto 606-8501 Japan
TEL: +81-75-753-5198
FAX: +81-75-753-5857
E-mail: izui@prec.kyoto-u.ac.jp

Nozomu Kogiso
Department of Aerospace Engineering
Osaka Prefecture University
Gakuencho 1-1, Sakai 599-8531
TEL: +81-72-254-9245
FAX: +81-72-254-9906
E-mail: kogiso@aero.osakafu-u.ac.jp

ABSTRACT

Since decision-making at the conceptual design stage critically affects final design solutions at the detailed design stage, conceptual design support techniques are practically mandatory if the most efficient realization of optimal designs is desired. Topology optimization methods using discrete elements such as frame elements enable a useful understanding of the underlying mechanics principles of products, however the possibility of changing prior assumptions concerning utilization environments exists since the detailed design process starts after the completion of conceptual design decision-making. In order to avoid product performance reductions due to such later-stage environmental changes, this paper discusses a reliability-based topology optimization method that can secure specified design goals even in the face of environmental factor uncertainty. This method can optimize mechanical structures with respect to two principal characteristics, namely structural stiffness and eigen-frequency. Several examples are provided to illustrate the utility of the method presented here for mechanical design engineers.

Keywords: Conceptual Design, Reliability-based Optimization, Topology Optimization, Frame Element, Multiobjective Optimization, Stiffness, Eigen-frequency

1. INTRODUCTION

Conceptual design is a critical stage in development process of mechanical products, since design factors pertaining to the most important characteristics of the product are decided in this stage. If the conceptual design process is well organized, the final products will have good performances and the product development time can be shortened, since the number of steps that need to be carried out in the detailed design stage, and repetitious steps required due to design changes can be minimized.

First Order Analysis (FOA) is a design support method for mechanical structures at the conceptual design stage that utilizes topology optimization methods to generate extremely good structural solutions [1]. While general topology optimization methods are based on continuum mechanics [2], FOA uses discrete elements to represent structural models, due to the following two reasons. The first reason is that optimal solutions using discrete elements can be easily compared with actual conceptual design solutions since the optimal solutions are represented by the allocated positions of the elements and their detailed dimensions. The second is that design verification can be carried out so that design engineers can understand the physical meaning of the generated optimal solutions, and the underlying principles of the particular structural design problem, since the formulation of the optimization problems is based on simple structural mechanics.

Such topology optimization methods that integrate FOA are effective in the conceptual design stage, but production and utilization of actual mechanical structures involves uncertain conditions such as operation loads, material properties, and dimensional variation due to manufacturing. These physical uncertainties sometimes cause violations of constraint conditions or disappointment concerning desired performances. Consequently, the reliability of a mechanical structure may decrease and even structural failures may sometimes occur.

This paper discusses a reliability-based topology optimization method (RBTO) using discrete elements, which aims to ensure that the desired performances will be consistently achieved. First, in Section 2, conventional reliability-based design optimization methods and deterministic topology optimization techniques using discrete elements are introduced. In Section 3, reliability-based topology optimization techniques using discrete elements for mechanical system design optimization problems are proposed. Then, several examples are solved in Section 4, using the proposed technique, and its effectiveness is discussed.

2. RELATED WORK

2.1 Reliability-based Design Optimization

Reliability analysis is a method for quantifying relations between variations in a range of parameters and the resulting performance insufficiency or failures. A variety of design optimization techniques based on reliability analysis have been proposed. Such techniques are called Reliability-Based Design Optimization (RBDO) techniques [3].

The Traditional Approximation Method (TAM) is one of the most basic algorithms, which optimizes design problems based on a reliability index value. TAM requires double loop calculations, where the reliability index is calculated in the inner loop and the design optimization is conducted in the outer loop. This results in an extremely high computation cost, but accurate reliability indexes can be calculated for multi failure-mode problems.

The Single Loop Single Variable (SLSV) method, the Safety-Factor Approach (SFA) and the Sequential Optimization and Reliability Assessment (SORA) method are popular techniques in which double loop problems are transformed to single loop problems to reduce calculation times. The SLSV method proposed by Chen et al. [4] enables single loop calculation while avoiding the need to calculate the reliability index during the optimization process. The SFA proposed by Wu and Wang [5] introduced the concept of the safety-factor in reliability design problems into optimization problems, and used approximately equivalent deterministic constraints. Du and Chen [6] proposed the SORA method that transforms a probabilistic design problem to an equivalent deterministic optimization problem using the inverse reliability assessment for checking the constraint feasibility. In the SFA and SORA method, the design optimization process and the reliability analysis can be strictly partitioned and conducted in reduced calculation times [7].

In this research, reliability-based design optimization methods based on RIA (the Reliability Index Approach), which is a kind of TAM, are applied to topology optimization problems of mechanical structures.

RIA is a method that handles reliability indexes as if they were probabilistic constraint conditions. The general reliability-based design optimization problems can be formulated involving probabilistic variables as the following.

$$\underset{d}{\text{Minimize}} \quad Q(\boldsymbol{d}) \tag{1}$$

subject to

$$P(G_h(\boldsymbol{x}) \le 0) - P_{t,h} \le 0 \quad for \quad h = 1,2,\ldots,H \tag{2}$$

$$\boldsymbol{d}^L \le \boldsymbol{d} \le \boldsymbol{d}^U \tag{3}$$

where Q is the objective function, $\boldsymbol{d}$ is a design variable vector, and G_h is a limit state function. $\boldsymbol{d}^L$ and $\boldsymbol{d}^U$ stand for the lower and upper limits of the design variables. $\boldsymbol{x}$ is a random variable vector and H is the number of limit state functions. The limit state function G_h represents the effect that random variables have on the mechanical structure, as follows:

$$G_h(\boldsymbol{x}) < 0 : \text{State of failure} \tag{4}$$

$$G_h(\boldsymbol{x}) = 0 : \text{Limit state surface} \tag{5}$$

$$G_h(\boldsymbol{x}) > 0 : \text{State of safety} \tag{6}$$

Furthermore $P(G_h(\boldsymbol{x}) \le 0)$ represents the failure probability and $P_{t,h}$ is its upper limit. $P(G_h(\boldsymbol{x}) \le 0)$ can be calculated per the following equation:

$$P(G_h(\boldsymbol{x}) \le 0) = F_{G_h}(0) = \int_{G_h \le 0} \cdots \int q_{\boldsymbol{x},h}(\boldsymbol{x}) d\boldsymbol{x} \tag{7}$$

where the cumulative distribution function, and the joint probability density function of the limit state function G_h, are F_{Gh} and $q_{x,h}$, respectively.

In the First-Order Reliability Method (FORM), every random variable is transformed to a standard normal random variable. In cases where random variable x_j follows the normal distribution $N(m_j, \sigma_j^2)$, x_j is transformed using the following equation.

$$U_j = \frac{x_j - m_j}{\sigma_j} \quad for \quad j = 1,2,\ldots,J \tag{8}$$

where U_j stands for a standard normal random variable and J is the number of random variables. Therefore Eq. (2) can be written using the limit state function $G_h(\boldsymbol{U})$, which can be represented using standardized random variables as follows.

$$P(G_h(\boldsymbol{U}) \le 0) \le P_{t,h} \tag{9}$$

When the reliability index goal is $\beta_{t,h}$ and the cumulative standardized normal distribution function is Φ, Eq. (9) can be written as

$$F_{G_h}(0) \le \Phi(-\beta_{t,h}) \tag{10}$$

The inverse of this equation is obtained as

$$\beta_h \ge \beta_{t,h} \tag{11}$$

where the reliability index is β_h. β_h can be geometrically represented as minimum distance from the limit state function surface to the origin in $\boldsymbol{U}$ space. Therefore, the optimization problem to obtain the reliability index β_h can be expressed as

$$\text{Minimize } \|\boldsymbol{U}\| \tag{12}$$

subject to:

$$G(\boldsymbol{U}) = 0. \tag{13}$$

Point $\boldsymbol{U}^*$ which satisfies the optimization problem is called the Most Probable Point (MPP), and $\|\boldsymbol{U}^*\|$ is equal to the reliability index β_h, since β_h is the distance from the MPP $\boldsymbol{U}^*$ to the origin.

2.2 Topology Optimization

The concept of topology optimization is based on the introduction of a fixed design domain D and a characteristic function χ_Ω, as shown in the following equation.

$$\chi_{\Omega}(\boldsymbol{a}) = \begin{cases} 1 & if \quad \boldsymbol{a} \in \Omega_d \\ 0 & if \quad \boldsymbol{a} \in D \setminus \Omega_d \end{cases} \tag{14}$$

where fixed design domain D, which includes design domain Ω_d that would be an optimal structure, is initially assigned, and optimization problems are transformed to element allocation problems using the characteristic function χ_{Ω}.

Using χ_{Ω} in the above equation, an optimal structure can be obtained where discrete elements at position $\boldsymbol{a}$ in the fixed design domain D exist in design domain Ω_d, but this representation causes finite discontinuities concerning the discrete elements. In this paper, a design domain relaxation technique using the following equation based on the density approach and SIMP method is applied, and design optimization problems are represented using continuous design variables.

$$\rho_v{}^p \approx \chi_{\Omega}(\boldsymbol{a}) \tag{15}$$

where ρ_v is a normalized design variable subject to the following range

$$0 \le \rho_v \le 1 \tag{16}$$

and p is a penalization parameter used to emphasize the computational volume density distribution.

Murotsu et al. proposed topology optimization method based on reliability analysis using truss elements [8]. Kharmanda et al. clarified that a reliability-based design optimization method can be applied to a topology optimization problem based on continuum mechanics [9]. However, these methods are not suitable for the conceptual design of mechanical structures since design engineers cannot gain useful clues concerning the physical reasons for the optimal solutions thus obtained.

This paper applies topology optimization techniques utilizing ground structure approaches and using frame elements as discrete elements to represent mechanical structures in optimization problem configurations. This paper discusses two important design criteria, namely stiffness, eigen-frequency, since these two criteria are the most important fundamental characteristics at the conceptual design stage.

3. RELIABILITY-BASED TOPOLOGY OPTIMIZATION TECHNIQUE FOR STRUCTURAL SYSTEM SYNTHESIS

3.1 Problem Formulations for Deterministic Problems

Consider that an elastic mechanical structure Ω consisting of I elastic frame elements is fixed at boundary Γ_d, as shown in Fig. 1. This structure is subjected to a static load $\boldsymbol{f}$ at point P_k. Body forces applied to the structure are assumed to be ignored for simplicity in the formulation. Let $\boldsymbol{f}$ and $\boldsymbol{u}$ be the load and displacement vectors, respectively. Here, the mean compliance l [2][10] is a measure of the stiffness at point P_k and is defined as

$$l = \boldsymbol{f}^T \boldsymbol{u} = \boldsymbol{u}^T \boldsymbol{K} \boldsymbol{u} \tag{17}$$

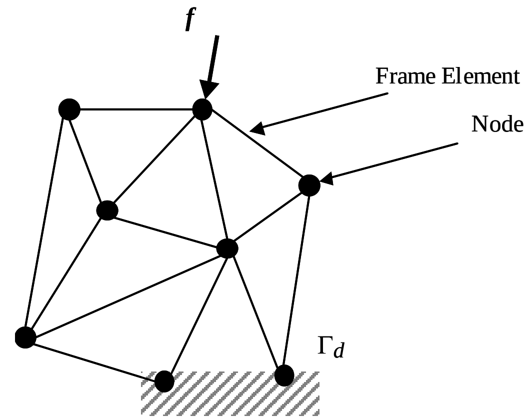


Figure 1. Configuration of design domain for stiffness maximization problems

where $\boldsymbol{K}$ is the global stiffness matrix of the entire structure. A topology optimization problem concerning structural stiffness is generally formulated as follows.

$$\underset{\rho_{A,i}}{\text{Minimize}} \quad l \tag{18}$$

subject to:

$$V = \sum_{i}^{n} \rho_{A,i} A_{\max} L_i \le V^U \tag{19}$$

$$0 \le \rho_{A,i} \le 1 \quad for \quad i = 1,2,\ldots,I \tag{20}$$

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \tag{21}$$

where $\rho_{A,i}$ is the cross-sectional area of the i-th frame element, L_i is the length of the i-th frame element, V is the total volume of the frame elements, and V^U is upper limit of the total volume V.

Fig. 2 illustrates a configuration of the eigen-frequency maximization problem. As with the stiffness maximization problem, the mechanical structure Ω consists of I frame elements and certain nonstructural mass points, and is fixed at

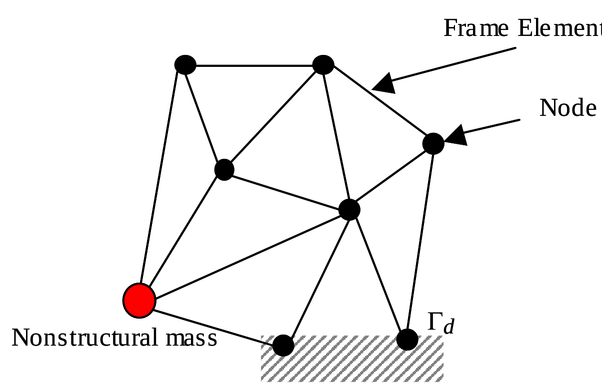


Figure 2. Configuration of design domain for eigen-frequency maximization problems

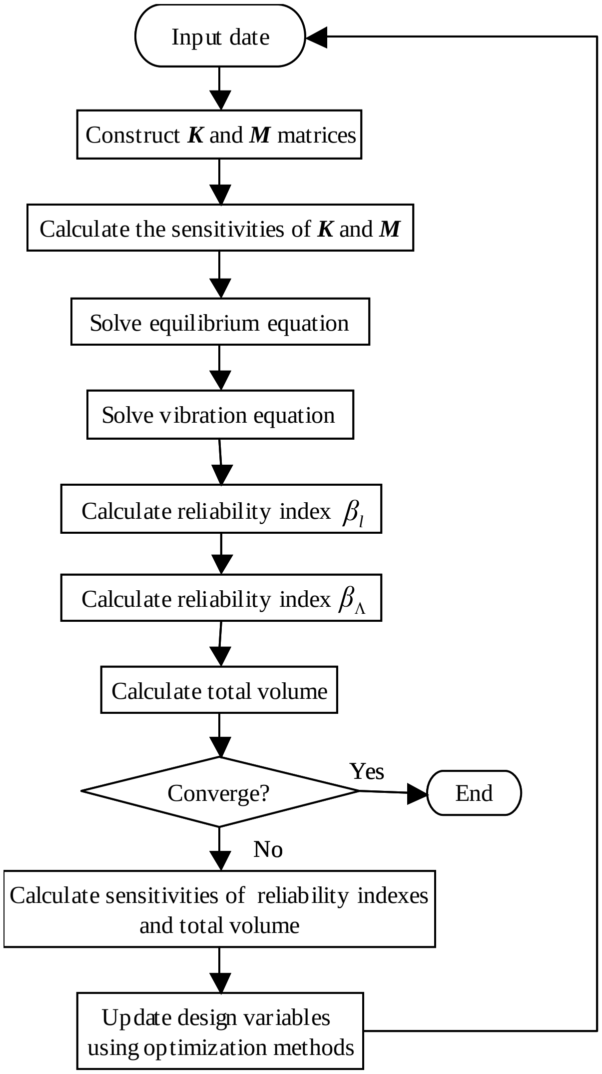


Figure 3. Flowchart of optimization procedure

boundary Γ_d. A vibration equation can be formulated as

$$(\boldsymbol{K} - \lambda_m \boldsymbol{M})\varphi_m = 0 \tag{22}$$

where $\boldsymbol{M}$ is the global mass matrix of the entire mechanical structure, λ_m (m = 1,2…, M) is m-th eigen-value, and φ_m (m = 1, 2,…, M) is an eigen-vector. The eigen-frequency of the mechanical structure ω_m (m = 1,…, M) can then be obtained using the following equation.

$$\lambda_m = \omega_m^2 \tag{23}$$

In usual mechanical system design situations, the first eigen-frequency is maximized to avoid resonance phenomenon, but solving eigen-frequency maximization problems is more difficult than solving stiffness maximization problems since the order of the eigen-frequencies often changes during the optimization process. To avoid this problem, this paper applies the mean eigen-value as an objective function [11], represented as follows.

$$\Lambda = 3\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}\right)^{-1} \tag{24}$$

The eigen-frequency maximization problem can now be formulated as the follows.

$$\underset{\rho_{A,i}}{\text{Maximize}} \quad \Lambda \tag{25}$$

subject to:

$$V = \sum_i^n \rho_{A,i} A_{\max} L_i \le V^U \tag{26}$$

$$0 \le \rho_{A,i} \le 1 \quad for \quad i = 1,2,\ldots,I \tag{27}$$

$$(\boldsymbol{K} - \lambda_m \boldsymbol{M})\varphi_m = 0 \quad for \quad m = 1,2,\ldots,M \tag{28}$$

While the above two optimization problems are deterministically defined, mechanical system design optimization problems involve uncertainty concerning physical parameters such as external loads, material properties, and actual physical dimensions, due to variable operating environments and manufacturing processes. Furthermore, mechanical structure design problems have a number of design criteria, and during the conceptual design stage, design engineers must take into account multiple failure modes of the mechanical system, hence reliability-based topology optimization techniques that can consider multiple failure modes and uncertain factors should be developed.

3.2 Formulation of Reliability-based Topology Optimization

3.2.1 Two Formulation Types

This paper discusses mechanical structure design optimization using reliability-based topology optimization (RBTO) techniques as shown in Fig. 3. The reliability calculation procedures based on the RIA for structural stiffness and eigen-frequency are shown in Fig.4 and 5. However, detailed optimization problem formulations depend on the situation, since structural design processes vary and the most important criterion usually depends on the design target. This paper proposes two types of optimization problem formulations: a reliability-constrained formulation and a reliability maximization formulation.

3.2.2 Reliability-constrained Formulation

When design goal values for mechanical structure reliability are assigned, the reliability index of the total mechanical structure system is set as constraint condition and different criteria are set as objective functions. The optimization problem here can thus be formulated as follows.

$$\underset{\rho_{A,i}}{\text{Minimize}} \quad V \tag{29}$$

subject to:

$$\beta_s \ge \beta_{st} \tag{30}$$

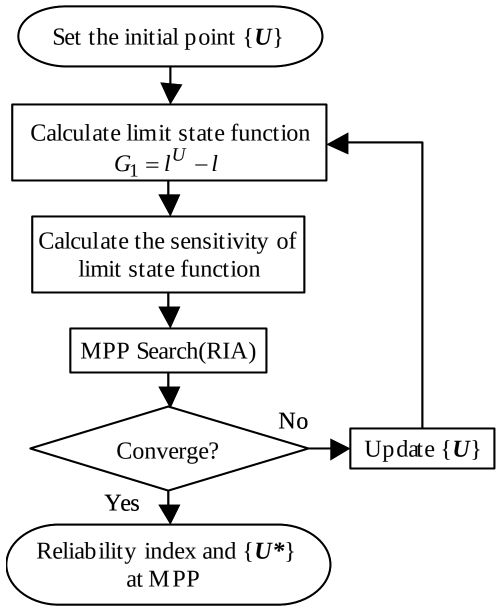


Figure 4. Reliability calculation procedure for structural stiffness

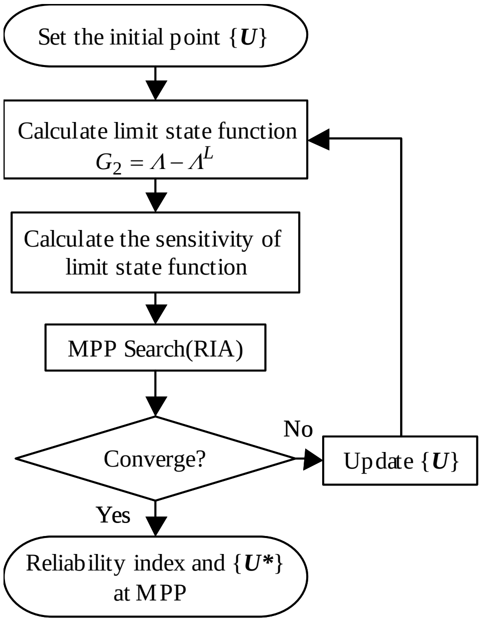


Figure 5. Reliability calculation procedure for eigen-frequency

$$0 \le \rho_{A,i} \le 1 \quad for \quad i = 1,2,\ldots,I \tag{31}$$

$$\boldsymbol{Ku} = \boldsymbol{f} \tag{32}$$

$$(\boldsymbol{K} - \lambda_m \boldsymbol{M})\varphi_m = 0 \quad for \quad m = 1,2,\ldots,M \tag{33}$$

Limit state function:

$$G_1(\boldsymbol{x}) = l^U - l(\boldsymbol{x}) \tag{34}$$

$$G_2(\boldsymbol{x}) = \Lambda(\boldsymbol{x}) - \Lambda^L \tag{35}$$

where the total volume of the structure is an objective function. The reliability index of the total system β_s is a constraint condition, and β_{st} is its lower limit. l^U and Λ^L are the upper and lower limitation of the mean compliance and the mean eigen-value, respectively, that are assigned by design engineers.

In general cases, reliability judgments are based on whether or not failures occur in the system. However, this paper considers reliability from the standpoint of satisfying the design goal values formulated by equations (34) and (35). If the probability that the design goal values are not satisfied is lower than a specified probability goal, this formulation judges that the system will have the desired degree of reliability. The converse is also true, so that when the probability that the design goal values are not satisfied is higher than the specified probability goal, the system will be deemed to have inadequate reliability. This approach was based on firsthand experience with conceptual design stage situations, where the detailed design process starts after conceptual design decision-making, despite the possibility of change in product utilization environments that were previously assumed. In order to avoid product performance reduction due to such environmental changes, this paper uses reliability measures to secure specified design goals even in the face of environmental factor uncertainty.

The above equations involve two failure modes, concerning the mean compliance and the mean eigen-value. In cases where multiple failure modes need to be considered, the probability of the structural system's failure is equal to the probability of the sum event of each failure event. Therefore, the structural system failure probability P_s can be written as the following.

$$P_s = P[F_l] + P[F_\Lambda] - P[F_l \cap F_\Lambda] \tag{36}$$

where F_l represents a situation where the mean compliance is larger than its upper limit and, F_Λ represents a situation where the mean eigen-value lower than its lower limit. $P[F_l]$ and $P[F_\Lambda]$ are the F_l and F_Λ occurrence probabilities which can be calculated using the corresponding reliability indexes. The term $P[F_l \cap F_\Lambda]$ is the probability in which the F_l and F_Λ situations simultaneously occur, and can be calculated using the following procedure.

$$\begin{aligned} P[F_l \cap F_\Lambda] &= \Phi_2(-\beta_l, -\beta_\Lambda; \Theta_{l\Lambda}) \\ &= \int_{-\infty}^{-\beta_l} \int_{-\infty}^{-\beta_\Lambda} \phi_2(x, y; \Theta_{l\Lambda}) dxdy \\ &= \int_{-\infty}^{-\beta_l} \int_{-\infty}^{-\beta_\Lambda} \frac{1}{2\pi\sqrt{1-\Theta_{l\Lambda}{}^2}} \exp\left(-\frac{x^2 - 2\Theta_{l\Lambda} xy + y^2}{2(1-\Theta_{l\Lambda}{}^2)} \right) dxdy \end{aligned} \tag{37}$$

where Φ_2 is a two-dimensional standardized normal cumulative distribution function, ϕ_2 is a two-dimensional standardized normal density function, and $\Theta_{l\Lambda}$ is a correlation coefficient between random variables.

Finally, the reliability index of the total system β_s can be obtained via the following equation.

$$\beta_s = -\Phi^{-1}(P_s) \tag{38}$$

3.2.3 Reliability Maximization Formulation

While Section 3.2.2 discussed a RBTO formulation when the reliability index is assigned as a constraint condition, when the goal values of fundamental characteristics such as structural stiffness, eigen-frequency, and total volume are important, these criteria are handled as constraint conditions and the reliability index is set as the objective function, as shown below.

$$\underset{\rho_{A,i}}{\text{Minimize}} \quad \beta_s \tag{39}$$

subject to:

$$V \le V^U \tag{40}$$

$$0 \le \rho_{A,i} \le 1 \quad for \quad i = 1,2,\ldots,I \tag{41}$$

$$\boldsymbol{Ku} = \boldsymbol{f} \tag{42}$$

$$(\boldsymbol{K} - \lambda_m \boldsymbol{M})\varphi_m = 0 \quad for \quad m = 1,2,\ldots,M \tag{43}$$

Limit state function:

$$G_1(\boldsymbol{x}) = l^U - l(\boldsymbol{x}) \tag{44}$$

$$G_2(\boldsymbol{x}) = \Lambda(y) - \Lambda^L \tag{45}$$

4. EXAMPLES

Several numerical examples are presented to examine and verify the proposed optimization method. In the following examples, Young's modulus, Poisson's ratio, and the penalization parameter are set to 209GPa, 0.3, and 1.0, respectively. The Convex Linearization Method (CONLIN) and the HL-RF method [12, 13] is applied to the topology optimization and MPP search, respectively.

4.1 Two-dimensional structural model

Figure 6 shows a ground structural model of example 1 in a rectangular design domain with a fixed support boundary on the left-hand side. A vertical load x is applied at the center of the right-hand side. The optimal configurations are obtained by minimizing the total volume under the stiffness reliability constraint, and the mean eigen-value is ignored in order to simplify the problem, which is expressed as follows.

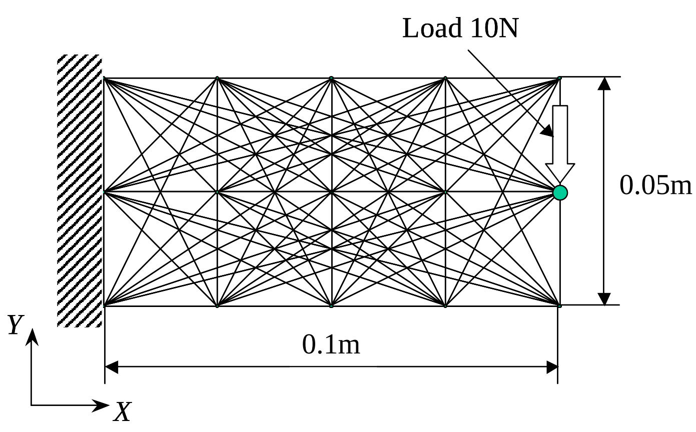


Figure 6. Two-dimensional design domain for example 1

$$\text{Minimize } V \tag{46}$$

subject to:

$$\beta_l \ge \beta_{lt} \tag{47}$$

$$0 \le \rho_{A,i} \le 1 \quad for \quad i = 1,2,\ldots,I \tag{48}$$

$$\boldsymbol{Ku} = \boldsymbol{f} \tag{49}$$

Limit state function:

$$G(x) = l^U - l(x) \tag{50}$$

where the random variable in this example is the amount of applied load x. The average and standard deviation of the vertical load x is set to 10N and 1N, respectively. The maximum diameter of each frame element cross-section is set to 0.01m.

In order to compare the proposed method with a deterministic approach, the deterministic optimization problem for the structural stiffness shown in section 3.1 is solved where the volume constraint V^U is 1% of the total volume. The mean compliance l obtained via this deterministic problem is 2.497 x 10^{-5} J, and is used as the assigned goal value l^U in Eq. (50).

Figure 7 illustrates the results of the optimization. Fig.7 (a) is the result in the case where the deterministic approach is applied. Figs.7 (b) and (c) show the results of the proposed reliability-based topology optimization method where the required reliability index was set to 3 and 5, respectively.

Note that these three optimal solutions show different topologies. As the required reliability index value increases, the total volume of the solutions increases to accommodate larger applied loads, as shown in Table 1. Therefore the reliability constraint formulation used in these examples is suitable in cases where reliability is the most important factor and must absolutely be secured. However, clear goal values, or reliability indexes for goal values of the physical characteristics, cannot be specified during the conceptual design stage. Therefore, in such cases, this formulation may occasionally be insufficient.

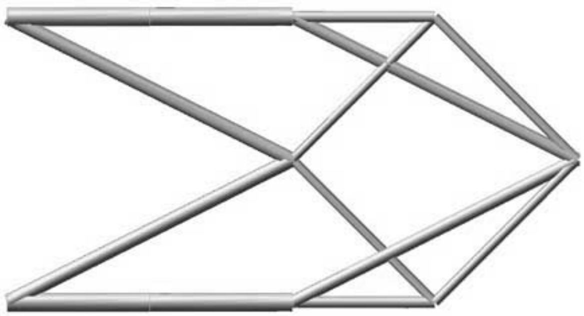
Table 1 Total volume of each solution

Problems	Total volume
Deterministic approach	2.858 x 10^{-6} m^3
$\beta_{lt} = 3$	4.729 x 10^{-6} m^3
$\beta_{lt} = 5$	6.242 x 10^{-6} m^3

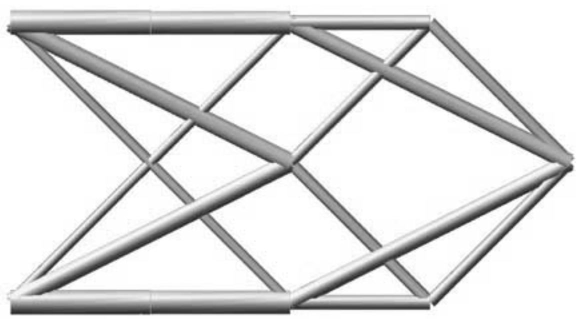
4.2 Reliability Maximization Problem

In example 2, the reliability maximization formulation proposed in Section 3.2.3 is applied, and Fig.8 shows its configuration. In this example, the direction of the applied load is an uncertain factor. The both X and Y components are uncertain, and the average applied loads of each component of the applied loads are set to 10 N and 0 N and standard deviations are 1 N and 5 N, respectively. As before, the mean eigen-value is ignored in this problem.

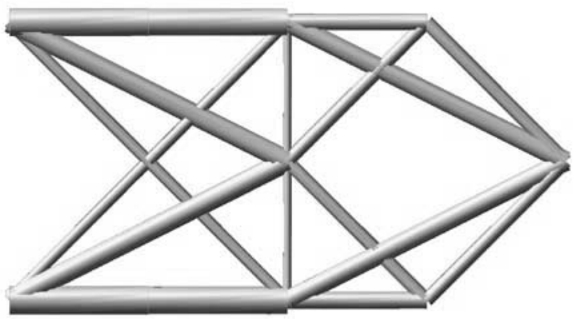
Here, the mean compliance goal value l^U is set to 1.5 times the goal value when the deterministic problem was solved, i.e. 3.745 x 10^{-5} J, and this figure is used in the limit state function. The upper limit structural volume V^U is assigned as a constraint condition of this optimization problem, and its value is set to 1% of the total volume.



(a) Deterministic approach



(b) $\beta_{lt} = 3$



(c) $\beta_{lt} = 5$

Figure 7. Optimal configurations for example 1

Figure 9 shows the optimal structure. Comparison with the result of the deterministic approach shown in Fig. 7 (a) shows that a horizontal frame was added to the structure to support the horizontal load and the reliability index β of this solution were 1.778.

4.3 Three-dimensional structural model

In example 3, the reliability maximization formulation is applied to three-dimensional structure problems and Figure 10 shows the configuration. A fixed design domain is supported at a boundary on the left-hand side. This problem simultaneously considers two structural characteristics, namely the stiffness

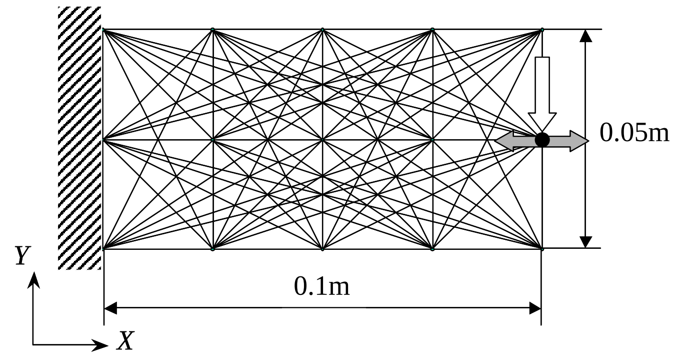


Figure 8. Initial design domain configuration for example 2

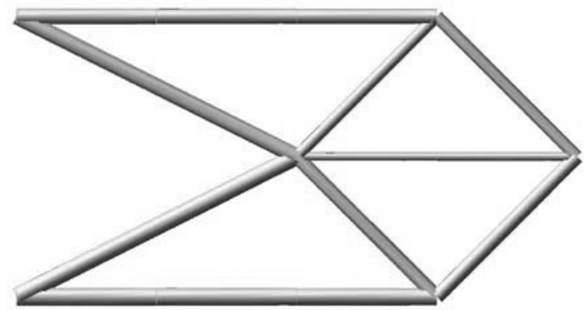


Figure 9. Optimal configurations for example 2

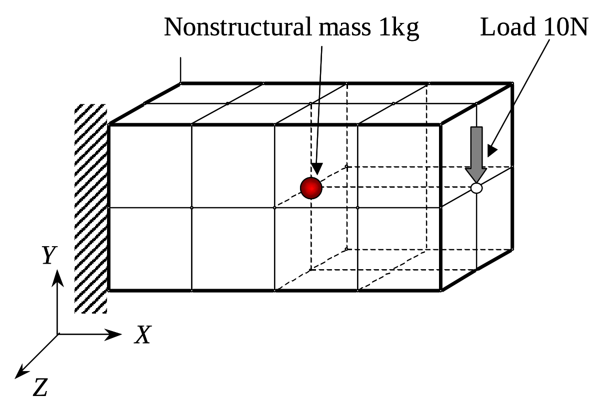
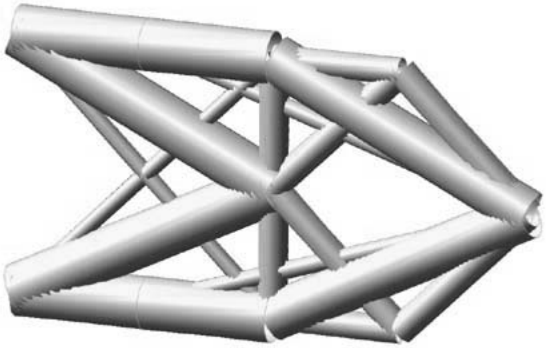


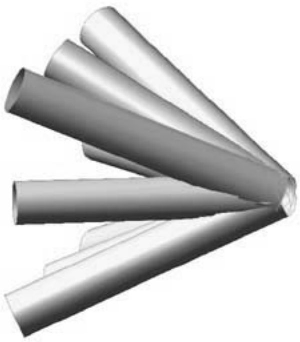
Figure 10. Three-dimensional design domain for example 3

and the eigen-frequency. A nonstructural mass is positioned at the center of this structure, and a load is applied at the center of the right-hand side. For this problem, the amount of the mass and the load were uncertain variables.

Prior to reliability-based optimization, the deterministic problems were solved where the volume constraint V^U is 1% of the total volume and the two optimal configurations are shown in Fig. 11. The mean compliance for problem (a) was 1.897 x



(a) Mean compliance minimization



(b) Eigen-frequency maximization

Figure 11. Results of deterministic approach

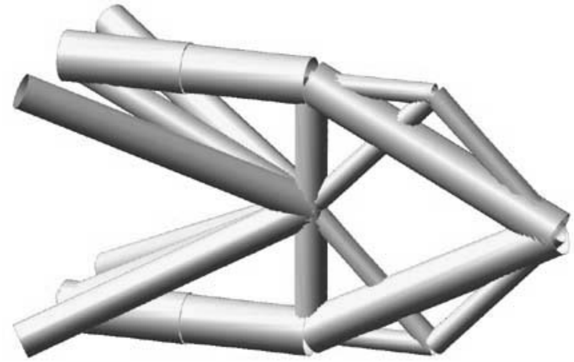
10^{-6} J and the mean eigen-frequency for problem (b) was 97.13 Hz.

The goal values for the reliability-based optimization problems were configured using a mean compliance value l^U 3 times larger and an eigen-frequency value Λ^L 0.3 times smaller than the respective values achieved in the deterministic solutions. Three different variance cases concerning the uncertain variables of load and nonstructural mass were examined. The standardized deviations of the load σ_l and the mass σ_m and the resulting reliability index β for each case are shown in Table 2. Figure 12 illustrates the optimal configurations achieved by the reliability-based optimization.

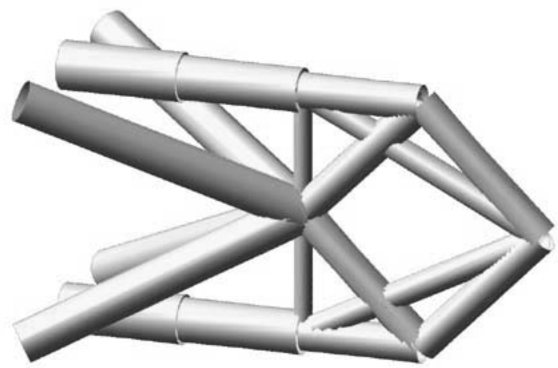
When the variance of the load is comparatively larger than the nonstructural mass, the cross-sectional areas of frame elements in the central vertical plane area are large since the load is vertically applied in this structure. On the other hand, when the variance of the mass is larger than the load, frame elements in the space between the nonstructural mass center and the fixed boundary have increased cross-sectional areas, to reduce movement of the nonstructural mass. Since the eigen-mode of this particular structure has three-dimensional components, the obtained structure has a three-dimensional distribution of the frame elements that reflects the influence of the central non-structural mass.

Table 2. Problem settings and resulted reliability index

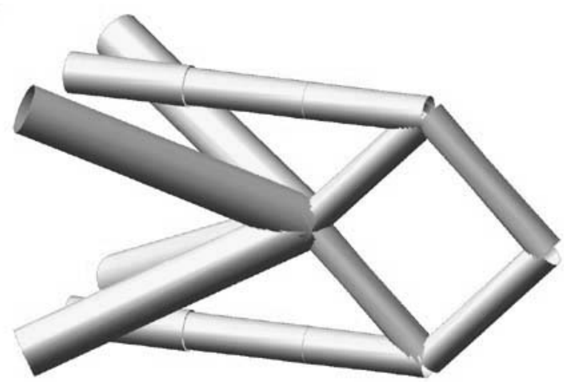
Problems	σ_l	σ_m	β
(a)	3 N	0.1 kg	2.176
(b)	2 N	0.2 kg	3.027
(c)	1 N	0.3 kg	4.431



(a) $\sigma_l = 3$, $\sigma_m = 0.1$



(b) $\sigma_l = 2$, $\sigma_m = 0.2$



(c) $\sigma_l = 1$, $\sigma_m = 0.3$

Figure 12. Optimal configurations for example 3

5. CONCLUDING REMARKS

In this paper, reliability-based topology optimization methods for mechanical structures were proposed. Two different optimization formulations were presented for various design goal situations, namely reliability-constrained problems and reliability maximization problems. Two principal mechanical structures characteristics, the structural stiffness and the eigen-frequency, were simultaneously considered. Ultimately, a multiple failure mode reliability analysis technique was applied in the optimization formulations.

The proposed methods were applied to three structural design optimization examples. Consideration of reliability indexes under a range of uncertain load and mass conditions resulted in different, and useful, topological solutions. Mechanical design engineers should find such solutions reasonable, since the proposed methods use discrete elements to represent the design space, and the physical reasons for the obtained solutions are extremely clear.

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