ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

A PROCEDURE FOR SOLVING INTEGER BILEVEL LINEAR PROGRAMMING PROBLEMS

A.Y. Adhami[1], and Quazzafi Rabbani[2]

Assistant Professor, Department of Mathematics, Integral University, Lucknow, India[1,2]

ABSTRACT: This paper is an extension of the K th-best approach [4] for solving bilevel linear programming problems with integer variables. NAZ cut [2] and A-T cut [3] are added to reach the integer optimum. An example is given to show the efficiency of the proposed algorithm.

Keywords: A-T cut, bilevel linear integer programming, K th-best approach, NAZ cut.

I. INTRODUCTION

Bilevel programming has been proposed for dealing with decision processes involving two decision makers with a hierarchical structure. A bilevel programming problem (BLPP) consists of two levels, namely, the first level and the second level. The first level decision maker is called the leader and the second is called the follower. The follower executes its policies after and in view of, the decisions of higher level decision maker i.e. leader. In terms of applications, bilevel programming has been used in many domains, e.g. to design optimization problems in process system engineering [6], design of transportation network [11], agricultural planning [9], management of multi-divisional firms [14].

Many researchers have designed algorithms for the solution of the BLPP [1, 4, 5, 8, 10]. However, there has been very little attention in the literature on both the solution and the application of bilevel problems involving discrete decisions. This is mainly because these problems pose major algorithmic challenges in the development of effective solution strategies. For the solution of the purely integer linear BLPP, a branch and bound type of enumerative solution algorithm has been developed by Moore and Bard [12]. Cutting plane and parametric solution approaches have been developed by Dempe [7]. Saharidis and Ierapetritou [15] gave an algorithm for the resolution of mixed integer BLPP based on Benders decomposition method.

In this paper we focus on the integer linear bilevel programming problem, in which all involved functions are linear. The aim of this paper is to present an extended K th-best approach for finding the integer solution to a bilevel programming problem by introducing A-T cut to the reduced feasible region obtained after using NAZ cut.

II. DESCRIPTION OF NAZ CUT AND A-T CUT FOR INTEGER LINEAR PROGRAMMING PROBLEMS

Consider the pure linear integer programming problem as follows:

$$\begin{aligned} &\max f(x_1, x_2) = c_1 x_1 + c_2 x_2 \\ &s.t. A_1 x_1 + A_2 x_2 \le b \qquad (1) \\ &x_1, x_2 \ge 0 \\ &x_1, x_2 \text{ are integers} \end{aligned}$$

The linear programming relaxation can be obtained by omitting the integer restrictions.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

First we solve the linear programming relaxation. Let the solution be $x^* = (x_1^*, x_2^*)$. If x^* is all integer, then the problem is solved.

Let the k^{th} component of x^* be non integer with $x_k = a_k^*$.

The nearest integer values to x_k are

$$x_k^1 = [a_k^*] \text{ and } x_k^2 = [a_k^*] + 1 = \{a_k^*\}, \quad for \;\; k = 1,2.$$

where $[t]$ is the largest integer less than or equal to t and $\{t\}$ is the smallest integer greater than or equal to t.

With such bifurcations we can find all the 2^n points in the surrounding of the non-integer solution x^*. Denote the set of indices of these 2^n points by S^0. If all these points lie outside the feasible region we move to the next integer feasible points obtained from $x_k = a_k^* - 1$.

Let the objective value at x^* be Z^*. Thus, the objective function level plane at x^* will be $cx^* = Z^*$.

Now we find the difference $d_i = Z^* - cx_i^0,\ i \in S^0$,, i.e., the difference between the objective function value at non integer solution and the objective function values at the surrounding integer points, as suggested by Rabbani and Adhami [13]. Where $x_i^0{}'s,\ i \in S^0$, are surrounding integer points around x^*.

Now we search for the feasible point x_i^0, which has a minimum positive difference from the objective function value.

Let G be the set of indices $i \in S^0$ for which $x_i^0{}'s$ are feasible.

Let $x^0 = \{x_k^0 \mid d_k = \min_{i \in G} d_i\}$

A plane passing through this integer point and parallel to the objective hyperplane will be $cx^0 = Z^0$.

Clearly $Z^0 < Z^*$

The NAZ cut is now introduced as

$$cx^0 \geq Z^0$$

which reduces the feasible region.

Here Z^0 acts as a lower bound for the integer solution to the problem.

Let x^0 be defined as:

$$x^0 = (\alpha_1^{k^0}, \alpha_2^{k^0}, \ldots, \alpha_n^{k^0}); \quad k^0 \in S^0$$

Now to find the integer optimum solution we add the A-T cut at x^0 as

$$\sum_{j=1}^{n} x_j = \sum_{\substack{k^0 \in S^0 \\ j=1}}^{n} \alpha_j^{k^0}.$$

III. THE PROCEDURE

Using the common notation in bilevel programming, the integer linear bilevel programming ILBP problem can be written as follows:

$$\max_{x_1, x_2} f_1(x_1, x_2) = c_1 x_1 + c_2 x_2, \quad \text{where } x_2 \text{ solves}$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

$$\max_{x_2} f_2(x_1, x_2) = d_2 x_2 \qquad (2)$$
$$\text{s.t. } A_1 x_1 + A_2 x_2 \le b,$$
$$x_1, x_2 \ge 0, \ x_1, x_2 \text{ are integers}$$

where c_1 is an n_1-dimensional row vector, c_2 and d_2 are n_2- dimensional row vectors, A_1 is an $m \times n_2$-matrix and b is an m-dimensional column vector . We assume that the polyhedron S defined by the common constraints is nonempty and bounded.

Firstly we solve the linear programming LP relaxation for leader's problem associated with (2) using simplex method i.e., we solve,

$$\max f_1(x_1, x_2) = c_1 x_1 + c_2 x_2$$
$$\text{s.t. } A_1 x_1 + A_2 x_2 \le b, \qquad (2a)$$
$$x_1, x_2 \ge 0$$

Let the solution be $x^\bullet$. If the solution is non integer we add the NAZ cut $c_1 x_1 + c_2 x_2 \ge c_1 x_1^0 + c_2 x_2^0 = z^0$, which passes through x^0, where $x^0 = (x_1^0, x_2^0)$ is the integer point inside the feasible region and z^0 is the value of leader's problem at x^0.

Now to find the integer optimum solution we add the A-T cut $\sum_{j=1}^{n} x_j = \sum_{\substack{k^0 \in S^0 \\ j=1}}^{n} \alpha_j^{k^0}$ at x^0.

Let $x_{[1]}^\bullet, x_{[2]}^\bullet, \ldots x_{[N]}^\bullet$ denote the N ordered basic feasible solutions to the ILBP for (2a) such that $cx_{[i]}^\bullet \ge cx_{[i+1]}^\bullet, (i = 1, \ldots, N-1)$.

Let S_1 be the projection of S onto the leader's decision space. For each $x_{[1]}^\bullet \in S_1$, a feasible solution to the ILBP problem (2) is obtained by solving the following integer linear programming problem:

$$\max_{x_2} d_2 x_2$$
$$\text{s.t. } A_2 x_2 \le b - A_1 x_{[1]}^\bullet, \qquad (2b)$$
$$x_2 \ge 0 \text{ and integer.}$$

For the above problem also we can find the integer optimum by using NAZ cut and A-T cut. Let $M(x_{[1]}^\bullet)$ denote the set of optimal solution to (2b). We assume that for any fixed choice of leader, follower has some room to respond, i.e., $M(x_{[1]}^\bullet) \ne \phi$. Hence, the feasible region of the leader, called the inducible region IR, is

IR$=\{(x_1, x_2) : x_1 \in S_1, x_2 \in M(x_{[1]}^\bullet)\}$.

With the above extensions in the Kth-Best algorithm we can find the integer optimum solution for the bilevel programming problems.

The procedure can be summarized in the following steps:

IJIRSET

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

Step 1. Set $i=1$. Solve (2a) with the simplex method. If the solution is non integer then add NAZ cut and A-T cut to obtain integer optimum solution as $x^{\bullet}_{[1]}$. Let $W=(x^{\bullet}_{[i]})$ and $T=\phi.$ Go to Step 2.

Step 2. Solve (2b) for integer optimum solution using NAZ cut and A-T cut. Let this solution be denoted by $\tilde{x}$. If $\tilde{x}=x^{\bullet}_{[i]}$, stop; $x^{\bullet}_{[i]}$ is the global optimum to (2). Otherwise, go to Step 3.

Step 3. Let $W_{[i]}$ denote the set of adjacent extreme points x of $x^{\bullet}_{[i]}$ such that $cx \le cx^{\bullet}_{[i]}$.

Let $T=T\cup(x^{\bullet}_{[i]})$ and $W=(W\cup W_{[i]})\cap T^{c}$. Go to step 4.

Step 4. Set $i=i+1$ and choose $x^{\bullet}_{[i]}$ so that $cx^{\bullet}_{[i]}=\max_{x\in W}(cx)$. Go to step 2.

IV. NUMERICAL EXAMPLE

Consider the following ILBP problem:

$$\max_{x_1} f(x_1,x_2)=18x_1+22x_2$$

where x_2 solves:

$$\max_{x_2} f_1(x_1,x_2)=2x_1+x_2 \qquad (3)$$

$$\text{s.t.}\quad 17x_1+24x_2\le 102$$

$$84x_1+76x_2\le 399$$

$$x_1,x_2\ge 0\ ,\ x_1,x_2 \text{ are integers.}$$

The first step of the above procedure is to solve the linear programming problem

$$\max f(x_1,x_2)=18x_1+22x_2$$

$$\text{s.t.}\quad 17x_1+24x_2\le 102 \qquad (3a)$$

$$84x_1+76x_2\le 399$$

$$x_1,x_2\ge 0$$

We get the non integer solution as $x^{\bullet}_1=2.66$, $x^{\bullet}_2=2.36$ and $f(x^{\bullet}_1,x^{\bullet}_2)=99.96$

We round off the non integer solution to the nearest four integer points as (2, 2), (2, 3), (3, 2) and (3, 3). The respective differences are

$$99.96-80=19.96;\ 99.96-102=-2.04;\ 99.96-98=1.96;\ 99.96-120=-20.04$$

We are left with only one feasible point (2, 2), which gives the minimum positive difference. Now the NAZ cut and A-T cut passing through the integer point (2, 2) can be derived respectively as

$$18x_1+22x_2\ge 80$$

and

$$x_1+x_2=4$$

Now solving the problem (3) with these additional constraints we obtain the integer optimum solution as

$$x^{*}_1=0,\ x^{*}_2=4 \text{ and } f(x^{*}_1,x^{*}_2)=88$$

Let $x^{\bullet}_{[1]}=(0,4)$, the first best solution. Set $W=\{(0,4)\}$ and $T=\phi.$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 1, January 2014

To determine if $x^{\bullet}_{[1]}$ is an element of $M(x^{\bullet}_{[1]})$ we solve

$$\begin{aligned} &\max f_1(x_1,x_2) = 2x_1 + x_2 \\ &\text{s.t.} \quad 17x_1 + 24x_2 \le 102 \\ &\qquad 84x_1 + 76x_2 \le 399 \\ &\qquad x_1, x_2 \ge 0 \\ &\qquad x_1 = 0 \\ &\qquad x_1, x_2 \text{ integer.} \end{aligned} \tag{3b}$$

After adding the required NAZ cut and A-T cut we get the integer optimal solution as $\tilde{x} = (0,4)$. Hence, $\tilde{x} = x^{\bullet}_{[1]}$

Therefore, $x^* = (0,4)$ is the global optimal solution to ILBP problem (3).

V CONCLUSION

We have extended the *K*th-best algorithm for solving linear bilevel programming problems with the help of NAZ cut for integer programming along with the A-T cut. This algorithm gives us the integer solution for bilevel programming problems with much computational ease.

REFERENCES

[1] J.F. Bard, "An Algorithm for Solving the General Bilevel Programming Probem". Mathematics of Operations Research. Vol. 8, No.2, pp. 260-272, 1983.

[2] A. Bari, and Q. S. Ahmad, "NAZ cut for Integer Programming". Pure and Applied Mathematika Sciences. Vol LVII, No.1-2, 87-94, 2003.

[3] A. Bari, and T. Alam, "Search for Integer Optimum after Adding NAZ cut". Pure and Applied Mathematika Sciences. Vol. LXII, No. 1-2, pp. 77-83, 2005.

[4] W. F. Bialas, and M. H. Karwan, "Two Level Linear Programming", Management Science, Vol. 30, No. 8, pp.1004-1020, 1984.

[5] M Campelo and S. Scheimberg, "A Simplex Approach for Finding Local Solutions of a Linear Bilevel Program by Equilibrium Points" Annals of Operations Research.
Vol. 138, pp. 144-157, 2005.

[6] P. A. Clark and A.W.Westerberg, "Bilevel Programming for Steady-State Chemical Process Design – I. Fundamentals and Algorithms". Comput. Chem. Eng. Vol. 14, **No.** 1, pp. 87-97 , 1990.

[7] S. Dempe, "DiscreteBilevelOptimizationProblems".http;//www.mathe.tufreiberg.de/ dempe, TU Chemnitz.

[8] N.P. Faísca, V. Dua, B. Rustem, P.M. Saraiva, E.N. Pistikopoulos, "Parametric Global Optimization for Bilevel Programming". Journal of Global Optimization. Vol. 38, pp. 609-623, 2007.

[9] J. Fortuny-Amat, B. McCarl, "A Representation and Economic Interpretation of a Two-Level Programming Problem". Journal of Operations Research Society. Vol. 32, **No.** 9, pp. 783-792, 1981.

[10] J. Judice, and A. Faustino, "A Sequential LCP method for Bilevel Linear Programming". Annals of Operations Research. Vol. 34, pp. 89-106, 1992.

[11] L.J. LeBlanc, and D.E. Boyce, "A Bilevel Programming Algorithm for Exact Solution of Network Design Problem with User-Optimal Flows". Trans. Res.-Part B, Vol. 20, pp.259-265, 1985.

[12] J.T. Moore, and J.F. Bard. "The Mixed Integer Linear Bilevel Programming Problem". Operations Research. Vol. 38, pp. 911-921, 1990.

[13] Q. Rabbani, and A.Y. Adhami, "A Note on NAZ cut for Integer Progrmming" Pure and Applied Mathimatika Sciences, Vol. LXVIII, No. 1-2, pp. 75-77, 2008

[14] J. Ryu, V. Dua, E.N. Pistikopoulos, "A Bilevel Programming Framework for Enterprise-Wide Process Networks under uncertainty". Comput. Chem. Eng. Vol. 28, pp. 1121-1129, 2004.

[15] G.K. Saharidis, and M. G. Ierapetritou, "Resolution Method for Mixed Integer Bilevel Linear Problems Based on Decomposition Technique". J. Glob. Optim. 2008.