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Steady MHD Free Convection Flow with Thermal Radiation Past a Vertical Porous Plate Immersed in a Porous Medium

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Abstract: The present study is concerned with thermal radiation in a steady two-dimensional MHD free convection flow through a porous vertical flat plate immersed in a porous medium. In the analysis a Darcy-Forchhemier model is considered while the fluid is taken to be gray, absorbing-emitting radiation. The non-linear governing equations have been transformed by the usual similarity transformation to a system of ordinary differential equations. These dimensionless similar equations are then solved numerically employing the Nachtsheim-Swigert shooting iteration technique along with sixth order Runge-Kutta integration scheme. Finally the effects of the pertinent parameters are examined.

Key words: Boundary layer, darcy number, free convection, magnetic induction, MHD flow, radiation and suction

INTRODUCTION

Considerable interest has recently been shown in radiation interaction with free convection for heat transfer in fluid. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small particularly in free convection problems involving absorbing-emitting fluids. One of the initiators of this problem of radiation transfer in a vertical surface is Goody (1956), who considered a neutral fluid. Cess (1966), on the other hand, considered an absorbing-emitting gray fluid with a black vertical plate. His solution was based on perturbation technique and was applicable for small values of the conduction-radiation interaction parameter. Novotny *et al*. (1974), however, made a non-gray analysis employing the method of local non-similarity and the continuous correlation of Tien and Lowder (1966) to account for the absorption. The effects of radiation on free convection flow of a gas past a semi infinite flat plate was studied by Soundalgekar and Takhar (1981) using the Cogley-Vincentine-Giles equilibrium model. Ali *et al*. (1984) studied the same effects on natural convection flow but over a vertical surface in a gray gas. Following Ali *et al*. (1984) and Mansour (1990) studied the interaction of mixed convection with thermal radiation in laminar boundary layer flow over a horizontal, continuous moving sheet with suction and injection. Alabraba *et al*. (1992) studied the same problem of free convection interaction with thermal radiation in a hydro-magnetic boundary layer taking into account the binary chemical reaction and less attended Soret-Doufour effects. By using the Rosseland diffusion approximation (Sparrow and Cess, 1978), a study of the combined unsteady free convective dynamic boundary layer and thermal radiation boundary layer at a semi-infinite vertical plate was made by Sattar and Kalim (1996). Hossain and Takhar (1996) also analyzed the same effect of radiation using the Rosseland approximation in a mixed convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with a free uniform stream velocity and surface temperature. Since suction is the best control method of boundary layer growth in the presence of radiation El-Areaway (2003) studied the effect of suction/ injection on a micropolar fluid past a continuously moving plate. Ferdows *et al*. (2004), however, considered a variable suction in a boundary layer flow at a vertical plate with thermal radiation interaction with convection. Very recently Samad and Rahman (2006) investigated the thermal radiation interaction on an absorbing emitting fluid past a vertical porous plate immersed in a porous medium. They considered an unsteady MHD flow and observed that magnetic field can control the heat transfer and radiation shows a significant effect on the velocity as well as temperature distributions.

In analogy with the above works, in the present study, the steady MHD free convection interaction with thermal radiation of an absorbing-emitting fluid along a

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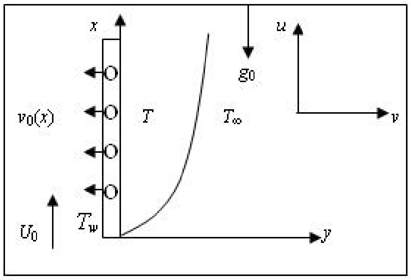

Fig. 1: Flow configuration and coordinate system

porous vertical plate immersed in a porous medium is investigated taking into account the Rosseland diffusion approximation. The investigation is based in known similarity analysis and the local similarity solutions are obtained numerically.

Mathematical formulation: Let us consider a steady two-dimensional MHD flow of a viscous incompressible and electrically conducting fluid of temperature $T\infty$ along a vertical porous plate under the influence of a uniform magnetic field with suction. The flow is assumed to be in the *x*-direction, which is chosen along the plate in the upward direction and *y*-axis normal to plate. The flow configuration and coordinate system are shown in Fig. 1.

The fluid is considered to be gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiation heat flux in the energy equation. The radiative heat flux in the *x*-direction is negligible to the flux in the *y*-direction. A uniform magnetic field of strength B_0 is applied normal to the plate parallel to *y*-direction. The plate temperature is initially raised to Tw ($>T\infty$) which is thereafter maintained constant.

Under the usual boundary layer and Boussinesq approximation and using the Darcy-Forchhemier model, the flow and heat transfer in the presence of radiation are governed by the following equations.

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g_0\beta\left(T-T\infty\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\upsilon u}{k}-\frac{bu^2}{k} \tag{2}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho Cp}\frac{\partial q_r}{\partial y} \tag{3}$$

where *u* and *v* are the velocity components along *x*- and *y*- directions respectively, U is the kinematic viscosity, ρ is the density of the fluid, g_0 is the acceleration due to gravity, β is the coefficient of volume expansion, σ is the electric conductivity, B_0 is the uniform magnetic field strength (magnetic induction), *k* is the permeability of porous medium, T and $T\infty$ are the fluid temperature within the boundary layer and in the free-stream respectively, q_r is the radioactive heat flux, c_p is the specific heat at constant pressure, α is the thermal diffusivity.

The corresponding boundary conditions for the above problem are given by:

$$\left.\begin{array}{ll} u=U_0; v=v_0(x); & T=T_w \ at\ y=0 \\ u=0; & T=T_\infty \ as\ y\to\infty \end{array}\right\} \tag{4}$$

where U_0 is the uniform velocity of the plate, v_0 (x) is a nonzero velocity component at the wall, T_w is the temperature of the plate and $T\infty$ is the temperature of the fluid far away from the plate. By using Rosseland approximation q_r takes the form:

$$qr=-\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ_1 is the Stefan-Boltzmann constant and K_1 is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about $T\infty$ and neglecting higher order terms, thus:

$$T^4 \cong 4T_\infty^3\, T-3T_\infty^4 \tag{6}$$

Using Eq. (5) and (6), Eq. (3) takes the form:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

where, *k* is the thermal conductivity.

In order to obtain similarity solution, for the problem under consideration, we may take the following suitable similarity variables

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}},\psi=\sqrt{2\upsilon U_0 x}\ f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{Tw-T_\infty} \tag{8}$$

where f(η) is the dimensionless stream function. Defining now the velocity components as:

$$\left.\begin{aligned} u &= \frac{\partial \psi}{\partial y} = U_0\, f'(\eta), \\ v &= -\frac{\partial \psi}{\partial x} = \sqrt{\frac{\upsilon U_0}{2x}}\left[\eta f'(\eta) - f(\eta)\right] \end{aligned}\right\} \quad (9)$$

Now introducing the similarity variables from Eq (8) and using Eq (9) into Eq (2) and Eq (6) we have

$$f''' + ff'' + 2\gamma\theta - 2\left(\frac{M}{\mathrm{Re}_x} + \frac{1}{Da\,\mathrm{Re}_x}\right) f' - 2\frac{Fs}{Da} f'^2 = 0 \quad (10)$$

$$(3N+4)\theta'' + 3N\,\mathrm{Pr}\, f\theta' = 0 \quad (11)$$

where $\gamma = xg_0\beta\Delta T/\upsilon_0^2 = Gr_x/Re_x^2$ is the local buoyancy parameter, $Gr_x = g_0\beta\Delta Tx^3/\upsilon^2$ is the local Grashof number, $Re_x = U_0x/\upsilon$ is the local Reynolds number, $M = \sigma B_0^2x^2/\rho\upsilon$ is the magnetic field parameter, $Da = k/x^2$ is the local Darcy number, $Fs = b/x$ is the Forchhemier number , $N = kk_1/4\sigma_1 T_\infty^3$ is the radiation parameter, $Pr = \rho\upsilon c_p/k$ is the Prandtl number.

As a result the corresponding boundary conditions take the form

$$\left.\begin{aligned} f = fw,\ f' = 1,\ \theta = 1 \quad at \quad \eta = 0 \\ f' = 0,\ \theta = 0 \quad as \quad \eta \to \infty \end{aligned}\right\} \quad (12)$$

where

$$f_w = -v_0(x)\sqrt{\frac{2x}{\upsilon U_0}}$$

is the porosity parameter or suction parameter.

The parameters of engineering interest for the present problem are the skin friction coefficient and local Nusselt number which indicate physically wall shear stress and rate of heat transfer respectively. The skin-friction coefficient is given by:

$$C_f\left(\frac{\mathrm{Re}_x}{2}\right)^{\frac{1}{2}} = f''(0) \quad (13)$$

And the local Nusselt number may be written as:

$$Nu\left(\frac{\mathrm{Re}_x}{2}\right)^{-1/2} = -\theta'(0) \quad (14)$$

Thus the values proportional to the skin-friction coefficient and Nusselt number are f(0) and – θ'(0) respectively. In Eq (13) and (14), the gradient values of f and θ of the surface are evaluated when the corresponding differential equations are solved satisfying the convergence criteria.

Numerical computation: The numerical solutions of the nonlinear differential Eq. (10) and (11) under the boundary conditions (12) have been performed by applying a shooting method namely Nachtsheim and Swigert (1965) iteration technique (guessing the missing value) along with sixth order Runge-Kutta integration scheme. We have chosen a step size of $\Delta\eta = 0.01$ to satisfy the convergence criterion of 10^{-6} in all cases. The value of η_∞ was found to each iteration loop by $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ to each group of parameters γ, f_w, M, N, Pr, Gr, Da and Fs was determined when the value of the unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-6}. In order to verify the effects of the step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta = 0.01$, $\Delta\eta = 0.005$, $\Delta\eta = 0.001$ and in each case we found excellent agreement among them. Figure 2 & 3 show the velocity and temperature profiles for different step sizes respectively.

RESULTS AND DISCUSSION

For the purpose of discussing the results, the numerical calculations are presented in the form of non-dimensional velocity and temperature profiles. Numerical computations have been carried out for different values of the buoyancy parameter (γ), suction parameter (f_w), Magnetic field parameter (M), radiation parameter (N), Prandtl number (Pr), Darcy number (Da) and Forchhemier number (Fs).

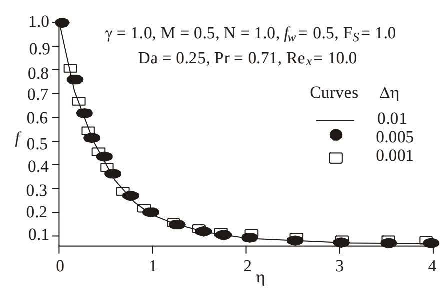

Fig. 2: Velocity profiles for different step sizes

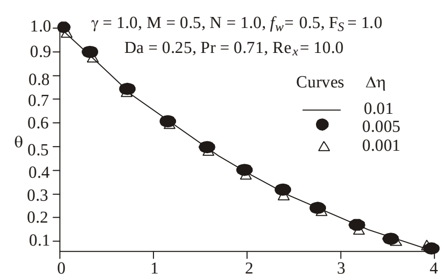

Fig. 3: Temperature profiles for different step sizes

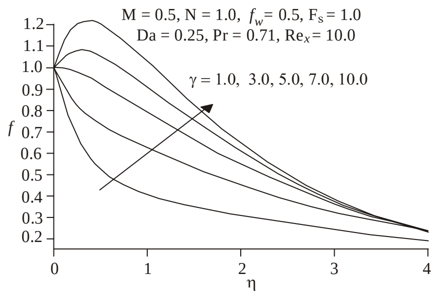

Fig. 4: Velocity profiles for various values of buoyancy parameter (γ)

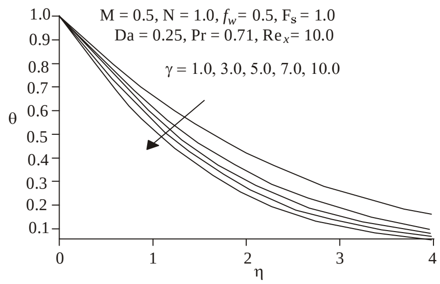

Fig. 5: Temperature profiles for various values of buoyancy parameter (γ)

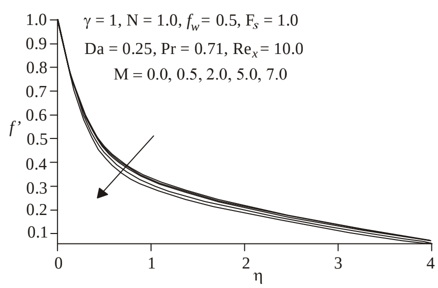

Fig. 6: Velocity profile for various values of the magnetic field parameter (M)

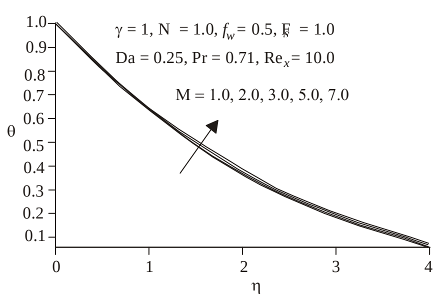

Fig. 7: Temperature profile for various values of the magnetic field parameter (M)

Figure 4 and 5 express the effects of buoyancy parameter γ on velocity and temperature profiles. From Fig. 4 we see that velocity increases with the increase of γ. The temperature profiles decreases with the increase of γ as seen in Fig. 5.

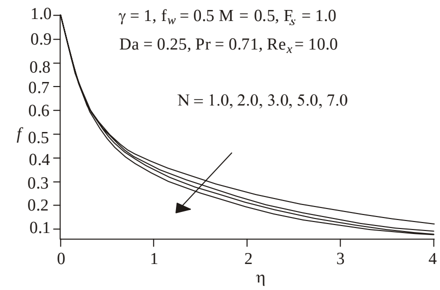

Fig. 8: Velocity profiles for various values of the radiation number (N)

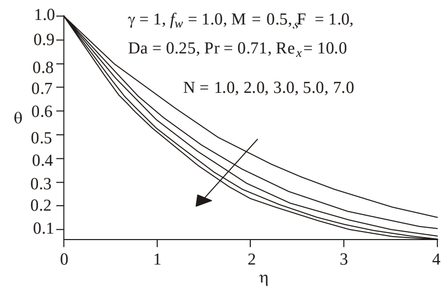

Fig. 9: Temperature profiles for various values of the radiation number (N)

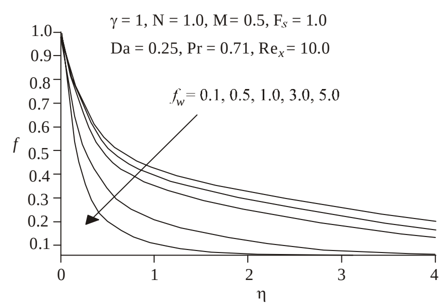

Fig. 10: Velocity profiles for various values of suction parameter (f_w)

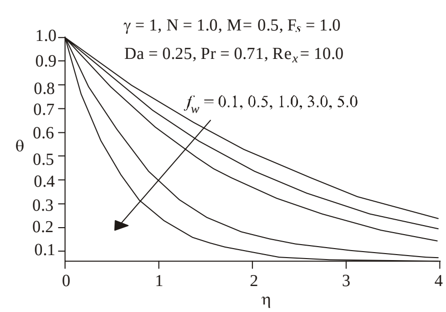

Fig. 11: Temperature profiles for various values of suction parameter (f_w)

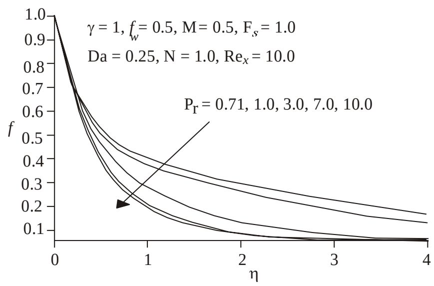

Fig. 12: Velocity profiles for various values of Prandtl number (Pr)

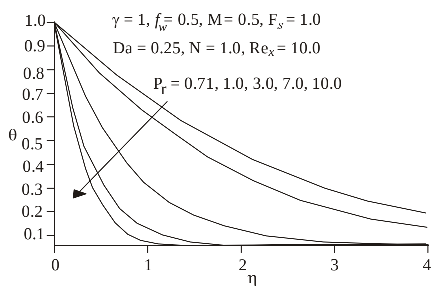

Fig. 13: Temperature profiles for various values of Prandtl number (Pr)

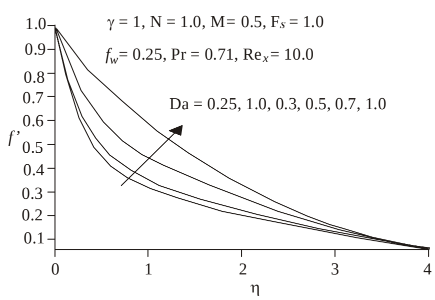

Fig. 14: Velocity profiles for various values of Darcy number (*Da*)

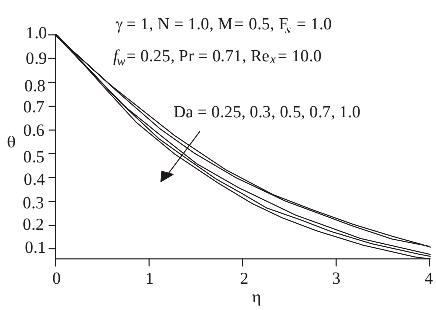

Fig. 15: Temperature profiles for various values of Darcy number (*Da*)

Figure 6 and 7 show the effects of magnetic field parameter *M* on the velocity and temperature profiles. From these figures we see that velocity decreases with the increase of the magnetic parameter *M*. The magnetic field

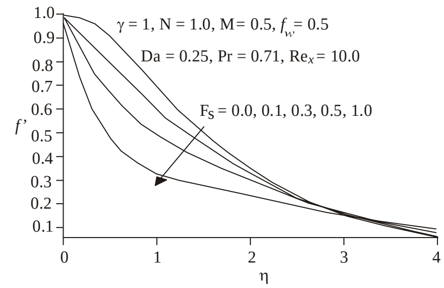

Fig. 16: Velocity profiles for various values of Forchhemier number (*Fs*)

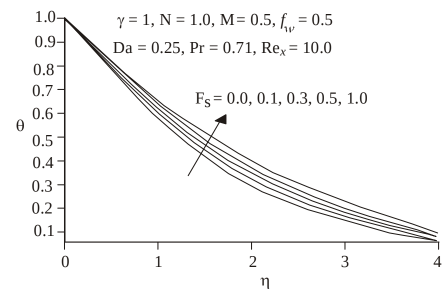

Fig. 17: Temperature profiles for various values of Forchhemier number (*Fs*)

lines act like a string and tend to retard the motion of the fluid. The consequence of which is to increase the rate of heat transfer temperature variation is very negligible as seen in Fig. 7.

Figure 8 and 9 show the velocity and temperature profiles for different values of radiation parameter (*N*). From Fig. 8 it is seen that radiation *N* has decreasing effect on the velocity field with the increase of *N*. On the other hand the effect of *N* on the temperature profiles (Fig. 9) are very much prominent showing that temperature decreases with the increase of *N*. For large *N*, the decrease of the temperature is found to be more rapid. From these two figures it is apparent that radiation can control the flow characteristics particularly the temperature distribution.

The effects of the suction parameter (f_w) on the velocity and temperature profiles are shown in Fig. 10 & Fig. 11. From Fig. 10 we see that the velocity decreases uniformly with the increase of suction. Figure 11 reveals that temperature profiles decreases rapidly with the increase of the suction velocity or mass transfer parameter.

Figure 12 and 13 show the effect of Prandtl number (Pr) on the velocity as well as temperature profiles. Since Prandtl number does not arise directly in the momentum Eq (9), its effects on the velocity profiles are negligible very close to the wall. Temperature decreases with the increase of Pr as seen in Fig. 13.

Figure 14 and 15 show the effects of Darcy number (*Da*) on the velocity and temperature profiles. We have

Table 1: Skin-friction and rate of heat transfer coefficients for different values of γ and Pr

γ	c_f	Nu_x	Pr	c_f	Nu_x
1	-2.10509430	0.37894696	0.71	-2.10509430	0.37894696
3	-1.01379703	0.44418012	1.0	-2.12956881	0.48051119
5	-0.04796932	0.48276912	3.0	-2.25323867	1.09127570
7	0.84833800	0.51148535	7.0	-2.39071591	2.17162748
10	-2.10683099	0.54513244	10.0	-2.45053379	2.93170349

Table 2: Skin-friction and rate of heat transfer coefficients for different values of M and Fs

M	c_f	Nu_x	Fs	c_f	Nu_x
0.0	-2.08255532	0.38051636	0.0	0.02669003	0.46322738
0.5	-2.10509430	0.37894696	0.1	-0.37172382	0.44603047
1.0	-2.12747444	0.37736860	0.3	-0.93531565	0.42221145
3.0	-2.21553639	0.37128694	0.5	-1.34929305	0.40574961
5.0.	-2.30133693	0.36554589	1.0	-2.10509430	0.37894696

Table 3: Skin-friction and rate of heat transfer coefficients for different values of N and Da

N	c_f	Nu_x	Da	c_f	Nu_x
1.0	-2.10509430	0.37894696	0.25	-2.10509430	0.37894696
2.0	-2.12909259	0.47848831	0.3	-1.84914100	0.38873197
3.0	-2.14103365	0.52968448	0.5	-1.22111800	0.41500555
5.0	-2.15300074	0.58225672	0.7	-0.87053170	0.43082345
7.0	-2.15900985	0.60916349	1.0	-0.54826956	0.44580472

Table 4: Skin-friction and rate of heat transfer coefficients for different values of fw

f_w	c_f	Nu_x
0.1	-1.96236311	0.29801872
0.5	-2.10509430	0.37894696
1.0	-2.30647563	0.49049387
3.0	-3.43242691	1.00696938
5.0	-5.06867527	1.57721623

found that velocity increases with the increase of *Da*. Darcy number is the measurement of the porosity of the medium. As the porosity of the medium increases, the value of *Da* increases. Figure 15 reveals that temperature profiles decreases with the increase of Da.

The effects of Forchhemier (*Fs*) number on the velocity and temperature fields are shown in Fig. 16 and 17. It is observed from those figures that Fs has decreasing effect on the velocity field but far away from the plate these profiles overlap while temperature increases with the increases of *Fs* as seen in Fig. 17.

The effects of the above mentioned parameters on the skin-friction coefficient and Nusselt number are shown in Table 1-4. These effects as observed from the Tables are found to agree with the effects on the velocity and temperature profiles hence any further discussions about them seem to be redundant.

NOMENCLATURE

B_0	Uniform magnetic field strength
Pr	Prandtl number
q_r	Rosseland approximation
c_f	Skin friction coefficient
c_p	Specific heat at constant pressure
q_w	Local heat flux
Da	Darcy number
T	Temperature within the boundary layer
Fs	Frochhemier number
T_w	Temperature of the fluid at the plate
f	Dimensionless stream
T_∞	Temperature of the fluid far away from the plate
f_w	Suction parameter
U_0	Uniform velocity
Gr_x	Local Grashof number
u	Component of velocity in the x-direction
g_0	Acceleration due to gravity
v	Component of velocity in the y-direction
k	permeability constant
$v_0(x)$	Suction velocity
M	Magnetic field parameter
x	Coordinate along the plate
N	Radiation parameter
y	Coordinate normal to the plate
Nu_x	Local Nusselt number

Greek symbols:

η	Similarity parameter
Ψ	Stream function
$\Delta\eta$	Step size
γ	Buoyancy parameter
θ	Dimensionless temperature
ρ	Density of the fluid
σ	Electric conductivity
μ	Coefficient of dynamic viscosity
β	Coefficient of volume expansion
υ	Coefficient of kinematic viscosity
κ	Thermal conductivity

CONCLUSION

In this study we have studied the thermal radiation interaction with steady MHD boundary layer flow past a continuously moving vertical porous plate immersed in a porous medium. From the present study we can make the following conclusions:

- The suction stabilizes the boundary layer growth.
- The velocity profiles increase whereas temperature profiles decrease with the increase of the free convection current.
- Using magnetic field we can control the flow and heat transfer characteristics.
- Radiation has significant effects on the velocity as well as temperature distribution.
- Large Darcy number leads to the increase of the velocity profiles.

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