# Optimal placement of sensors to detect delamination in composite beams

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The paper describes an approach for the optimal placement of sensors in composite beam structures for online detection of damage. The ability to identify damage is based on establishing a mapping between the charactgeristics of specific damage mechanisms (location and extent) such as delamination, fiber breakage, and matrix cracking, and strain measurements at the selected sensor locations; a trained neural network is proposed as a tool to generate this mapping. The design problem considered in the present paper was to place the least number of sensors in the structure so that the ability of the neural network to predict the extent and location of damage is not compromised. The optimization problem involved a mix of discrete and integer variables, and a genetic algorithm was used as the search tool.

## INTRODUCTION

A "smart" structure, instrumented with sensors and actuators, and responding in an intelligent manner to a dynamically changing environment, is an intriguing concept. The key ingredients in the realization of such a system include an adequate instrumentation of the structure, ability to rapidly analyze measured data and correlate to the existing state of the system, and to limit adverse structural behavior by providing real-time reaction in response to the evaluated state of the system. The present paper focusses on the use of strain measurements to detect delamination damage in composite beams. The approach can be extended to include other commonly encountered damage mechanisms in composites such as fiber breakage and matrix cracking, and analytical models relating the location and size of damage to strain fields in the structure under an applied load, are presented in (Teboub & Hajela, 1992). Since the damage can be in more than one place, and furthermore, there can be multiple modes of damage present at the same time, the identification space in such a problem is often nonunique. Artificial neural network (ANN) based classifiers present themselves as a logical tool for relating specific strain response to damage type and location. Once trained, these networks can rapidly generalize new strain measurements into an estimated state of the structure, and are therefore ideal for online damage detection systems.

An adequate instrumentation of the structure, however, continues to be a pivotal problem. The least number of sensors is clearly desirable from a standpoint of complexity of hardware. However, a sufficient number must be placed to resolve problems of nonunique identification and to have a robust system that is relatively insensitive to partial failures in the sensor array. The problem of optimally locating the least number of sensors that would identify damage over some admissible range of degradation and location, is explored in subsequent sections of this paper. Placement of sensors at some predefined grid in the structure is a discrete optimization problem, and computationally burdensome to handle using traditional branch-and-bound methods in nonlinear programming. The use of a genetic algorithm (Hajela 1993) is adopted in the present work, as this method is naturally amenable to search in a discrete space.

Once the placement of the sensors is known, a neural network can be trained to develop the mapping between the characteristics of damage, and the strain measurements at the sensor locations. However, to determine the optimal location of sensors by genetic search requires that a very large number of function evaluations be performed. Such function evaluations would involve determining the strain state for many different sizes and locations of damage, and then varying the number and locations of sensors to find the optimal distribution of the sensors in the structure to successfully identify various occurrences of damage. This is clearly a computationally intensive procedure, and in the present work, a trained neural network was used as an approximate analysis tool. Note that in optimizing for sensor locations, each strain reading may correspond to a totally different set of sensor locations. A novel hybrid neural network and a

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counterpropagation (CP) neural network (Szewczyk and Hajela, 1992) were used to construct approximations to the function information required in the optimization process. The characterization of delamination damage and analysis of the damged beam, statement of the optimal sensor placement problem, and the solution strategy are described in subsequent sections of this paper.

## DELAMINATION IN COMPOSITES

Delamination is a commonly encountered mode of damage in laminated composites, and appears as debonding of adjoining plies. It may result from interlaminar stresses created by impact, eccentricities in structural load paths, discontinuities in the structure, or during manufacturing. Delamination can result in reductions in composite strength of up to 65 % (Ramkumar 1983, Rosenfeld and Gauss 1981). The reduction in load bearing capacity, and degradation of structural integrity and stiffness, has been the subject of concern in engineering applications of composite materials.

The presence of delamination prevents proper load distribution between plies, and the composite is reduced to a number of independent longitudinal plies acting in parallel to support the applied load. The weakest of these plies fails, and may trigger failure of the remaining longitudinal plies. The effect of internal damage on macroscopic material response is observed only when the frequency of internal damage is sufficiently high. Since normal inspection practice relies on visual examinations, it is possible that internal delaminations ( e.g. caused by low velocity impacts) may go undetected for some length of time. It is of some importance to assess whether such internal damage has the potential for catastrophic growth. The model of delamination behavior described in this section extends the work of (Tracy 1989), to include unsymmetric laminates and the effects of shear deformation. It is assumed that the delamination divides the beam into four regions and extends over the entire width as shown in Figure 1. The delamination is located along a plane z=constant, and the thickness-to-span, and, delamination to length ratios are assumed to be small.

Consider a laminated composite beam of length L, width b, and height h, made of layers of orthotropic materials. The principal material axes of a layer may be oriented at an arbitrary angle with respect to the x-axis. The following assumptions are made in developing the equilibrium equations:

1) All layers behave elastically.
2) Displacements, rotations and strains are small relative to the beam thickness.
3) Perfect bonding exists between non delaminated layers.
4)The laminate of each region is equivalent to a single anisotropic layer.

The assumed displacement field for the composite beam, based on the first order shear deformation theory, is given as follows:

$$u_1(x,z) = u(x) + z\psi(x) \tag{1}$$

$$u_3(x,z) = w(x) \tag{2}$$

Here u, w are the displacements of a point on the midplane, and $\psi$ is the rotation of the normal to the midplane about x axis. The strain-displacement relations are given as follows,

$$\varepsilon_x = u_{,x}(x) + z\psi_{,x}(x) \tag{3}$$

$$\gamma_{xz} = \psi(x) + w_{,x}(x) \tag{4}$$

$$\kappa_x = \psi_{,x}(x) \tag{5}$$

where, the comma indicates differentiation with respect to x. The constitutive equations of a laminate are given by the classical lamination theory,

$$\begin{bmatrix} N_x \\ M_x \\ Q_{xz} \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & 0 \\ B_{11} & D_{11} & 0 \\ 0 & 0 & A_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \kappa_x \\ \gamma_{xz} \end{bmatrix} \tag{6}$$

where, $N_x$ is the resultant stress and $M_x$ is the resultant moment. The coeficients $A_{11}$, $B_{11}$, and $D_{11}$ are the extensional, coupling, and bending stiffness coefficients, respectively; $A_{55}$ is the shear stiffness coefficient. These coefficients are computed as follows:

$$\left(A_{11}, B_{11}, D_{11}, A_{55}\right) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left(\bar{Q}_{11}, \bar{Q}_{11}z, \bar{Q}_{11}z^2, K\bar{Q}_{55}\right)_k dz \tag{7}$$

In the above equation, $\bar{Q}_{55}, \bar{Q}_{11}$ are the transformed elasticity constants, N is the number of layers in the composite laminate, and K is the shear correction factor. This factor is necessary in first order shear deformation theory to account for the effects of nonuniform shear strain distribution. Methods to obtain values of the shear correction factors for different laminates have focussed on matching gross response predicted by fisrt order shear deformation theory with that of 3-D elasticity. For rectangular cross sections and low to intermediate thickness, a value of K=5/6 is widely adopted, and was used in the present work; $\varepsilon_x^o$ and $\kappa_x$ are the midplane strain, and the bending curvature, respectively.

For the delaminated beam structure, the equations of equilibrium can be developed for each of the four distinct sections i=1,4 shown in Figure 1 From the principle of minimum potential energy, we obtain the following system of equations,

$$\left(A_{11}\right)_i u_{i,xx} + \left(B_{11}\right)_i \psi_{i,xx} = p_1 \delta(x-a) \tag{8}$$

$$\left(A_{55}\right)_i \left(\psi_{i,x} + w_{i,xx}\right) = q(x) \tag{9}$$

$$\left(B_{11}\right)_i u_{i,xx} + \left(D_{11}\right)_i \psi_{i,xx} - \left(A_{55}\right)_i\left(\psi_i + w_{i,x}\right) = \left(m_1 - m_2\right)\delta(x-a) \tag{10}$$

where i denotes the beam segment under consideration. These equations can be integrated to obtain the following solution for the axial displacement u(x), transverse displacement w(x), and rotation $\psi(x)$.

$$u_i(x) = -\frac{\left(B_{11}\right)_i}{6D_i}qx^3 - \frac{\left(B_{11}\right)_i C_{i1}x^2}{\left(A_{11}\right)_i \, 2} + C_{i2}x + C_{i3} + \frac{1}{D_i}\left[\left(D_{11}\right)_i p_1 - \left(B_{11}\right)_i\left(m_1 - m_2\right)\right](x-a)H(x-a) \tag{11}$$

$$w_i(x) = \frac{qx^2}{2\left(A_{55}\right)_i} - \frac{q\left(A_{11}\right)_i x^4}{24D_i} - \frac{C_{i1}x^3}{6} - \frac{C_{i4}x^2}{2} + \left(\frac{C_{i1}D_i}{\left(A_{55}\right)_i\left(A_{11}\right)_i} - C_{i5}\right)x + C_{i6} - \frac{1}{2D_i}\left[-\left(B_{11}\right)_i p_1 + \left(A_{11}\right)_i\left(m_1 - m_2\right)\right](x-a)^2 H(x-a) \tag{12}$$

$$\psi_i(x) = \frac{\left(A_{11}\right)_i}{6D_i}qx^3 + \frac{C_{i1}x^2}{2} + C_{i4}x + C_{i5} + \frac{1}{D_i}\left[-\left(B_{11}\right)_i p_1 + \left(A_{11}\right)_i\left(m_1 - m_2\right)\right](x-a)H(x-a) \tag{13}$$

Where $D_i$ is obtained as follows:

$$D_i = \left(A_{11}\right)_i\left(D_{11}\right)_i - \left(B_{11}\right)_i^2 \tag{14}$$

The constants of integration are obtained upon substitution of the appropriate boundary conditions, the kinematic continuity conditions, and continuity of forces and moments. These are defined as follows:

**Boundary conditions**

$$u_1(0) = 0 \qquad w_1(0) = 0 \tag{15}$$

$$\psi_1(0) = 0 \qquad N_4(L) = 0 \tag{16}$$

$$M_4(L) = 0 \qquad V_4(L) = 0 \tag{17}$$

**Kinematic continuity condition**

$$w_1\left(x_1\right) = w_2\left(x_1\right) \tag{18}$$

$$w_1\left(x_1\right) = w_3\left(x_1\right) \tag{19}$$

$$\psi_1\left(x_1\right) = \psi_2\left(x_1\right) \tag{20}$$

$$\psi_1\left(x_1\right) = \psi_3\left(x_1\right) \tag{21}$$

$$w_2\left(x_2\right) = w_4\left(x_2\right) \tag{22}$$

$$w_3\left(x_2\right) = w_4\left(x_2\right) \tag{23}$$

$$\psi_2\left(x_2\right) = \psi_4\left(x_2\right) \tag{24}$$

$$\psi_3\left(x_2\right) = \psi_4\left(x_2\right) \tag{25}$$

$$u_2\left(x_1\right) - u_1\left(x_1\right) + \frac{h_3}{2}\psi_1\left(x_1\right) = 0 \tag{26}$$

$$u_3\left(x_1\right) - u_1\left(x_1\right) - \frac{h_2}{2}\psi_1\left(x_1\right) = 0 \tag{27}$$

$$u_2\left(x_2\right) - u_4\left(x_2\right) + \frac{h_3}{2}\psi_4\left(x_2\right) = 0 \tag{28}$$

$$u_3\left(x_2\right) - u_4\left(x_2\right) - \frac{h_2}{2}\psi_4\left(x_2\right) = 0 \tag{29}$$

**Continuity of moments and forces**

$$V_1\left(x_1\right) - V_2\left(x_1\right) - V_3\left(x_1\right) = 0 \tag{30}$$

$$N_1\left(x_1\right) - N_2\left(x_1\right) - N_3\left(x_1\right) = 0 \tag{31}$$

$$V_2\left(x_2\right) + V_3\left(x_2\right) - V_4\left(x_2\right) = 0 \tag{32}$$

$$N_2\left(x_2\right) + N_3\left(x_2\right) - N_4\left(x_2\right) = 0 \tag{33}$$

$$M_2\left(x_2\right) + M_3\left(x_2\right) - M_4\left(x_2\right) + \frac{h_3}{2}N_2\left(x_2\right) - \frac{h_2}{2}N_3\left(x_2\right) = 0 \tag{34}$$

$$M_2\left(x_2\right) + M_3\left(x_2\right) - M_4\left(x_2\right) + \frac{h_3}{2}N_2\left(x_2\right) - \frac{h_2}{2}N_3\left(x_2\right) = 0 \tag{35}$$

$$M_1\left(x_1\right) - M_2\left(x_1\right) - M_3\left(x_1\right) - \frac{h_3}{2}N_2\left(x_1\right) + \frac{h_2}{2}N_3\left(x_1\right) = 0 \tag{36}$$

The strains can be computed from the displacement field as in Eqns. (3-5). This analysis provides a capability of computing the strain field in the beam under a general loading, and for a specific length and location of the delamination damage. Note further, that the delamination damage is characterized in terms of three parameters - the x and z locations of the center of the delamination, and the length of the delamination.

## APPROXIMATE STRAIN RESPONSE USING ANN's

An important step in the design of sensor instrumented structures is to determine the appropriate placement of such sensors. A minimal number of such sensors is desirable from the standpoint of simplicity, but should be distributed so as to make use of available readings to correctly identify the location and intensity of damage. In the present problem, the sensors are assumed to record strain readings at discrete locations. To recover stiffness properties that yield these strains, and then correlate the stiffnesses to a state of the

**Table 1: Generalization error with missing input data**

| Number of sensors $N_s$ | BP - Network Average % Error/pattern | CP - Network Average % Error/pattern | Hybrid Network Average % Error/pattern |
|---|---|---|---|
| 7 | 126.49 | 10.04 | 112.3 |
| 9 | 113.67 | 9.79 | 14.9 |
| 12 | 81.65 | 4.62 | 7.0 |
| 15 | 5.81 | 1.36 | 5.79 |

**Table 2: 800 test patterns - CPN (400K-N), Ns<7 Ep=11.29%**

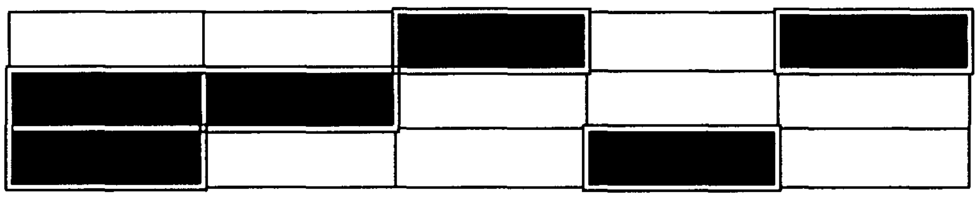


**Table 3: 800 test patterns - CPN (603K-N), Ns<7 Ep=11.19%**

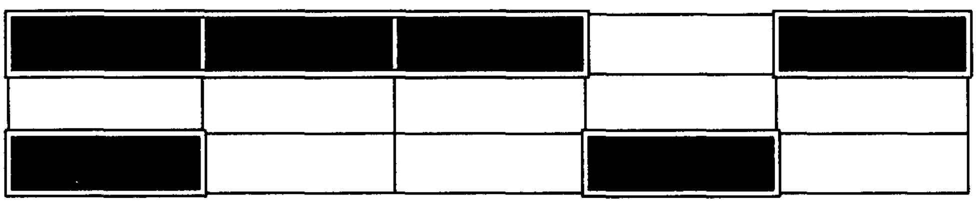


**Table 4: 800 test patterns - Hybrid (200K-N), Ns<7 Ep=21.18%**

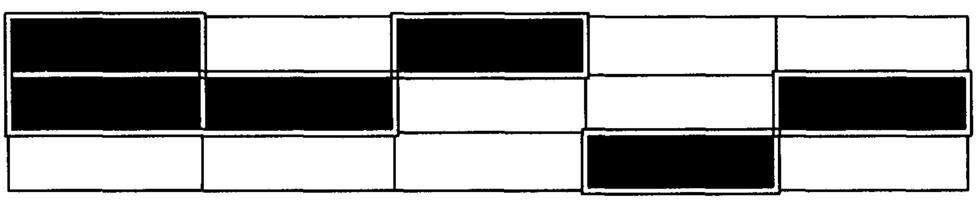


**Table 5: 800 test patterns - Hybrid (400K-N), Ns<7 Ep=12.82%**

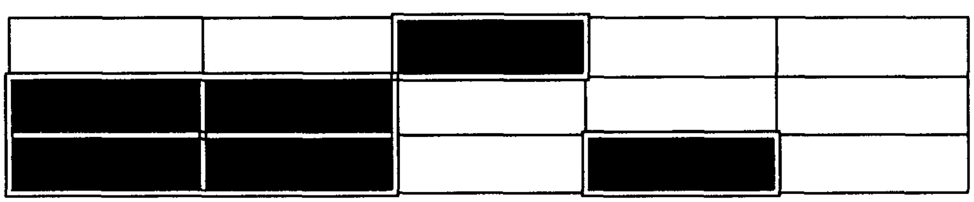
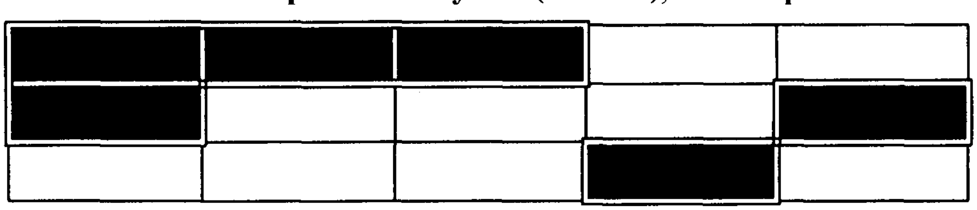


**Table 6: 800 test patterns - Hybrid (603K-N), Ns<7 Ep=12.31%**



structure, requires the solution of a very involved inverse problem, with no assurance of uniqueness. Furthermore, since online detection of damage is a key consideration, the strain measurements available at the sensor locations should be rapidly mapped into a physical state of the structure. This mapping can be developed by training a neural network with some predetermined training samples; in the present case, the training set consisted of strain measurements corresponding to a number of simulated delaminations, where both the location and size of the delamination were randomly varied between some prescribed range. The strain measurements were assumed to be available at a number of predetermined locations, and the locations themselves were distributed so as to adequately cover the regions of the structure where damage may occur. The underlying assumption is that once a neural network can be trained using such measurements, it would be capable of generalizing a set of new strain measurements into a given state of delamination. The present work explored the application of the backpropagation and the counterpropagation network to construct the mapping between the strains and the parameters characterizing the delamination damage.

The BP network can be described as a layer of input neurons (a neuron is the basic computing element which transforms a received input into an output through a preselected transfer/ activation function) connecting through one or more hidden layers of neurons to an output layer. A schematic sketch for such a network architecture is shown in Fig. 2, which also shows a widely used sigmoid activation function. Each neuron in the input and output layers corresponds to one input or output component, respectively. As shown in the figure, neurons in successive layers connect to one another through a set of interconnection weights $w_{ij}^{lm}$, where the subscripts and superscripts denote a connection between the i-th neuron of layer 'l' and the j-th neuron of layer 'm'. During the neural network training process, these strength of interconnections and the characteristics of the neuron activation function are adjusted so that for all samples in the training data (input-output pairs), the network learns to provide a good estimate to any input vector belonging to the domain of training. The training process is initiated by assigning random initial values to the weights. For the training patterns, presentation of the input to the neural network produces an output which may be considerably in error of the known output, and this error is backpropagated through the network to correct the interconnection weights - hence the name 'backpropagation network'. The BP network can be proven to approximate any function to an arbitrary degree of accuracy (Hornik et. al. 1989)]. Such networks have been used in recent engineering applications to construct function approximations (Hajela & Berke 1991, Rehak et. al. 1989 ). The principal concerns in training such a network reside in a proper selection (both number and distribution) of the training data, and in the specification of a network architecture that does not result in an under- or over-fitted response surface. In many instances, the time involved in training such networks is not trivial.

The CP network (Hecht-Nielsen 1988) is another approach of providing input-output mapping between given parameters, and is a combination of two specific architectures, namely the Kohonen and Grossberg models (Fig. 3). The data presented to the network is first classified as belonging to a specific category by the Kohonen layer of neurons. Hence, the interconnection weights to a particular neuron are nothing more than an averaged vector of all input patterns belonging to the particular category. The second layer of neurons (Grossberg layer) generates an output corresponding to each cluster identified by the Kohonen layer. The accuracy of generalization of this network is greatly dependent upon the radius of each cluster - the smaller the radius (larger number of Kohonen neurons), the better is the generalization effectiveness. This decrease in cluster radius is possible to the extent that the number of Kohonen neurons is equal to the number of training patterns. In this event, the network functions as a look-up table. A clear disadvantage to the approach of decreasing the cluster radius is the increase in storage requirements of the network.

An improvement to the basic approach was proposed by (Szewczyk and Hajela 1992). In this modified approach, the input vector to be generalized was categorized as belonging to more than one cluster, albeit to different degrees. A nonlinear averaging scheme was then used to construct the generalization for the input vector. This approach was shown to be effective in reducing the network size, and in providing improvements in the CP network generalization capability. An important feature of this network which is particularly relevant in the optimal distribution of sensors in the structure, is its pattern completion capability. As shown in Figure 3, the CP network allows for the generation of an identity map where the input X and output Y are mapped into their approximations X' and Y', respectively. If only a portion of the input, say a few components of the X or Y vector are input, the network generates an approximation to the complete output. The use of this capability is explained in greater detail in a subsequent section. Application of the CP network in function approximation has been examined in previous efforts (Hajela & Lee 1994, Szewczyk & Hajela 1993).

Another network architecture that was explored in the present work is shown schematically in Figure 4, and is a combination of the BP and the counterpropagation (CP) networks. The motivation behind examining this architecture was seated in the generally accepted notion that higher quality approximations are generally available from the BP network; however, this network has no capacity for pattern completion. In those instances where generalization of an incomplete input vector is required, the front-end of the proposed hybrid combination (the CP network) can be used to complete the input vector, and which can then presented to the BP network for generalization.

## OPTIMAL SENSOR PLACEMENT

A moderately large number of sensors were first distributed uniformly in the beam structure. Strain readings at these locations under predefined loads, and for a number of

**Table 7: 800 test patterns - CPN (100K-N), Ns<=12 Ep=3.46%**

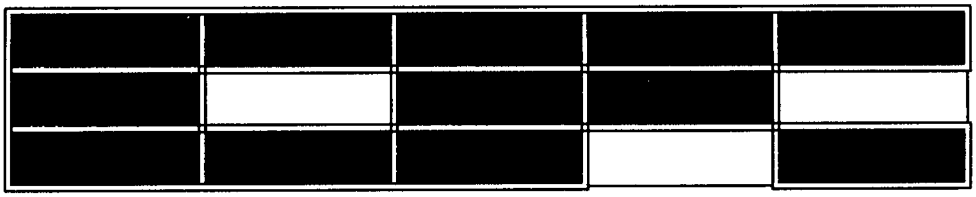


**Table 8: 800 test patterns - CPN (400K-N), Ns<=12 Ep=0.48%**

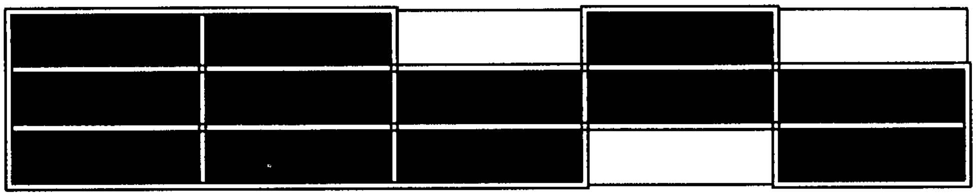
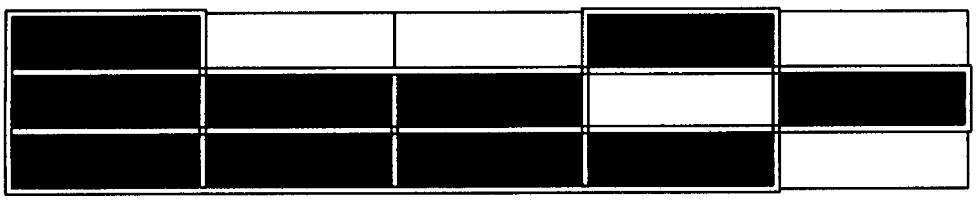


**Table 9: 800 test patterns - Hybrid (100K-N), Ns<=12 Ep=6.16%**



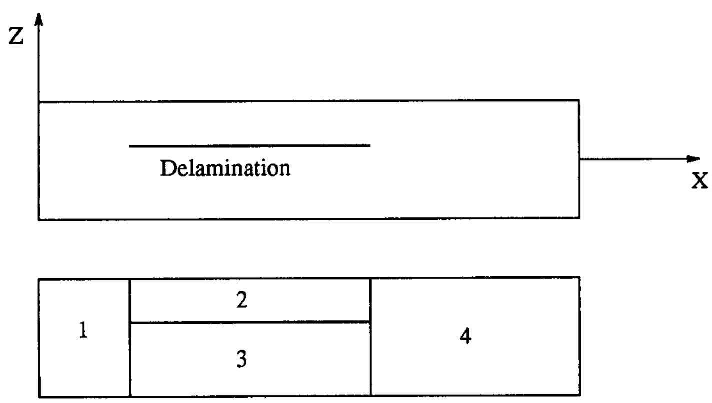



Fig. 1. Beam segmented into four regions due to delamination

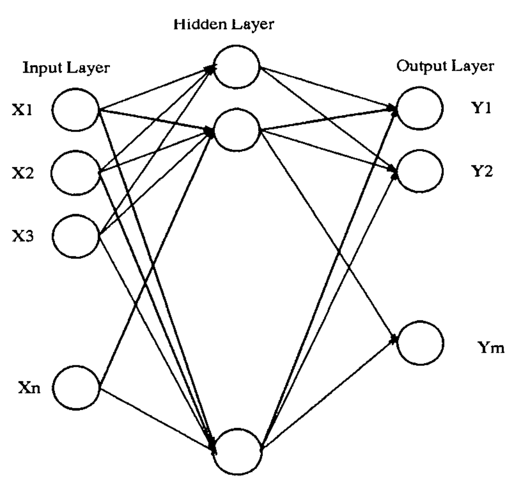



Fig. 2 Schematic sketch of a backpropagation neural network

random variations in characteristics defining the location and magnitude of damage, comprised the training set for the neural networks described in the previous section. The optimization problem in the present work was formulated to eliminate those sensors from the initial set that were deemed redundant. Such an approach would clearly depend upon the definition of redundancy, and in the present work, those retained sensors were defined as critical for which the error between the network predicted (based on limited strain inputs) and known values of location/extent of delamination corresponding to a set of test strain patterns, was a minimum. This can be mathematically stated as follows:

Given a set of possible sensor locations M by the set S, find $s_i \subset S$ as the positions at which sensors are actually placed so as to minimize

$$F(s) = \sqrt{\frac{1}{P}\sum_{P}\sum_{j=1}^{N}\left(\frac{Y_j^{actual} - Y_j^{predicted}}{Y_j^{actual}}\right)^2} \qquad (37)$$

where F(s) is the cumulative error in network prediction for P testing patterns, and N is the number of components used to describe the damage. In the case of delamination, N=3 and corresponds to x and z locations of delamination center, and the length of delamination. In this phase of the work, a number of training patterns were generated which assumed the existence of two strain components $\varepsilon_x$ and $\gamma_{xz}$ at each of the M locations of the sensors. Both the BP and the CP networks were trained to map these strains into the three parameters describing the delamination size and location.

Note that this optimization problem involves M binary variables, where a 1 or 0 value of the variable indicates whether a sensor is present or absent at a given location, respectively. A solution to this nonlinear integer programming problem is difficult to obtain with traditional mathematical programming algorithms. Consequently, a genetic algorithm based approach (Hajela, 1993) was used in the present work. A basic discussion of this approach is presented in a subsequent section. The nature of this stochastic search procedure is such that it requires a very significant number of functional evaluations to determine the optimal solution. In the absence of an approximation tool, a new analysis would have to be set up and executed for every trial arrangement of sensors proposed by the optimizer. With trained neural networks available to relate measured strains to the state of the structure, the function evaluations required by genetic search are very inexpensive. *The important issue was how to use a network trained for strain measurements at fixed locations, to describe the state of the structure on the basis of strain measurements available at only a subset of the previously defined locations, where this subset was specified by the optimizer.*

In general, the BP network can be trained to yield much higher quality approximations than the CP network (for the same number of training patterns), where the latter functions largely as a look-up table with somewhat lesser generalization capabilities. In the hybrid network approach used in this work, the strain measurements corresponding to the location selected by the optimizer were presented to the CP network; this network completed the pattern based on the concept of the nearest neighbor classification and yielded an approximation to the complete strain field. The completed pattern was then presented to the trained BP network for generalization purposes, and the output from this network used to compute the error function F(s). The pattern completion capability of the CP network would be compromised if the number of sensors dropped below some minimal levels. A second approach was to use the full feed-forward CP network output directly in computing the error function F(s).

## GENETIC SEARCH

GA's can be described as belonging to a general category of stochastic search techniques that have a philosophical basis in Darwin's theory of survival of the fittest. A set of design alternatives which are analogous to a population of a species in natural evolution, are subjected to transformation operators that allow favorable characteristics of the more fit members of the population to be combined in the progeny population. If the measure which indicates the fitness of the population is also the desired goal of the design process, successive generations will result in better objective function values. The method does not require the use of gradients of objective or constraint functions, and is therefore effective in problems where this information is unavailable. It works with discrete representation of design variables rather than the variables themselves, and is therefore ideally suited to handling of the 0/1 type variables of the present problem.

Central to this approach is the step by which a design is coded into a bit string of finite length, and a commonly used approach is to represent each variable by a fixed length binary number representation of the variable value. In the present problem, however, this problem was considerably simplified. The design of placing sensors at M possible locations can be represented by an M-digit binary string, where each digit on the string corresponds to a particular sensor location; the presence of a 0 or a 1 indicates the absence or presence of a sensor at that location, respectively. Several such chromosomal strings are defined to constitute a population of designs. This population of designs is then subjected to three basic genetic transformations including reproduction, crossover, and mutation.

The reproduction process is nothing more than biasing the population of designs by introducing multiple copies of better designs (measured in terms of objective function value) and simultaneously eliminating designs with poorer values of the fitness function. Reproduction represents an elitist selection process which retains only the most fit members of a population for mating; it does not in any way improve or create new designs. It is the crossover transformation that allows the characteristics of the designs in the population pool to be altered, thereby exploring new designs. A simple crossover operation (two-point) consists of randomly selecting two mating parents from the pool, randomly choosing two sites on the genetic strings, and swapping strings of 0's and 1's between two chosen sites among the mating pair. A

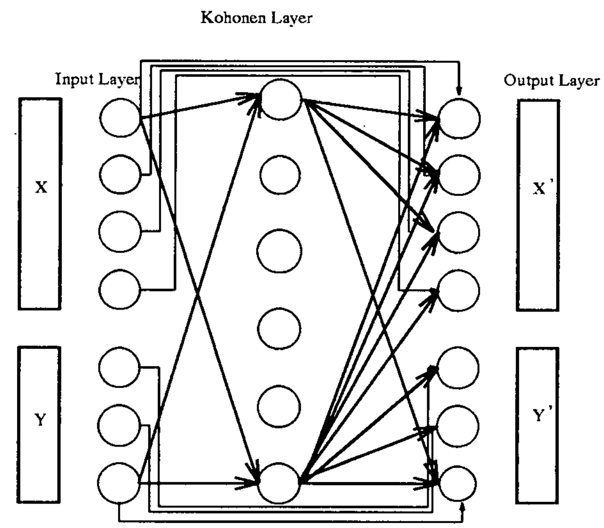



Fig. 3. Schematic sketch of a counterpropagation neural network

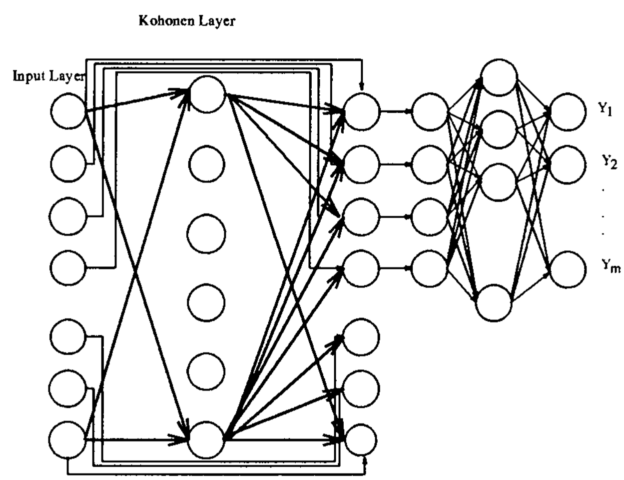



Fig. 4. Schematic sketch of a hybrid (CP-BP) neural network

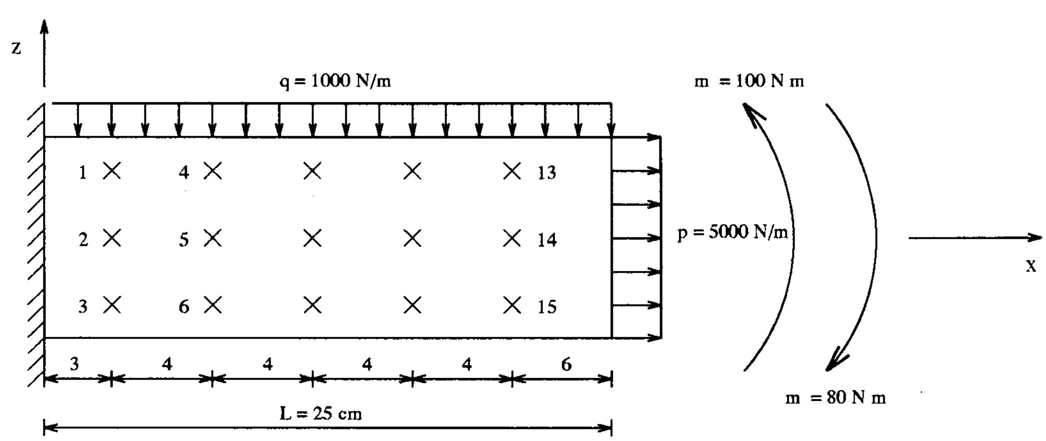



Fig. 5. Loading and sensor arrangement on beam

probability of crossover $p_C$ is defined to determine if crossover should be implemented. The third transformation operator, mutation, safeguards the genetic search process from a premature loss of 'genetic' information due to reproduction. The process of mutation simply involves selecting few members from the population pool, and to switch a 0 to 1 or vice versa at a randomly selected mutation site on the chosen string. This operation is done with a low probability of mutation $p_m$. The process of reproduction, crossover and mutation are repeated over a number of generations so that all designs in the population converge to a design with the best fitness value. More details on the use of the GA approach are available in Goldberg (1989).

## NUMERICAL EXPERIMENTS

A hygrothermally curvature stable, unsymmetric beam made of Graphite/Epoxy composite laminae of thickness 0.15 mm, was used in this study. The layup sequence was as follows:

$$\left[30/-60_2/\pm30/60_2/\mp30/-60_2/\pm30/-60_2/-30\right]$$

The beam was subjected to 4 distinct load conditions as shown in Figure 5 - a distributed transverse load of 1000N/m, a tensile end-load of 5000 N/m, and positive and negative bending moments of 100 N-m and 80 N-m, respectively. Strains corresponding to each of these load conditions were included in constructing the mapping between strains and the delamination characteristics; use of multiple load conditions was important to make the identification space as unique as possible. Assuming that sensors were placed at each of the 15 indicated sites on the beam, 800 sets of training data (strains) were generated in which the x and z location of the delamination center varied between 4-20 cms and 0.15-2.25 mm, respectively. Another set of 15 testing patterns were generated for the range of training data, but were not used for network training. At each sensor location, five strain components were considered to be critical; $\varepsilon_x$, $\gamma_{xz}$, due to transverse loading, and $\varepsilon_x$ for the tensile load and the two bending moments. This resulted in a total of 75 input strains used to relate to the state of the structure. The following numerical experiments were performed with this data:

Case A: The first phase of these experiments was directed towards validating the use of neural network based modeling of the relationship between strain measurements and parameters describing the location/magnitude of the damage. A BP network was trained to within 0.01% training accuracy with 200 training patterns. Additionally, the larger training sample with 800 sets of training data were used to train the CP network to comparable levels of resolution. These trained networks were also used to construct the hybrid BP-CP network described in the earlier section. Each of these networks was tested for its generalization capacity by using the 15 test patterns (strains) not used in training to predict the state of damage in the structure. Strain readings from a number of sensors were eliminated (all strain components from a particular sensor) to simulate the situation of incomplete data, and to assess the generalization capability of each network.

Case B: This set of numerical experiments were designed to study the optimal placement of sensors on the structure. The objective was to limit the number of sensors below a specified number, by selecting those locations for removing sensors which had the least adverse effect on the generalization capability of the network trained with the original 15 sensors. To implement this example, an additional set of 20 test patterns were developed in the range of network training, and used in the optimal sensor location problem; the sum of the squares of network generalization error for each of these 20 patterns was minimized. The optimization problem was solved using a genetic algorithm, where the location and number of sensors were design variables. A population size of 30 was selected for these simulations and probabilities of crossover and mutation were selected as 0.8 and 0.02, respectively.

## DISCUSSION OF RESULTS

Numerical results corresponding to network testing are summarized in Table 1, which shows the average error per pattern from each of the three network architectures considered in this work. A total of 15 test patterns were generated randomly in the range of variable variation for which the networks were trained. For the case when all 15 sensor readings are provided at the input, all three architectures produce minimal errors ranging from 1.36% to 5.81%, with the CP network doing better due to the fact that a larger number of training patterns were used. Strain measurements from the outboard sensors were systematically eliminated from the input vector to assess the quality of approximations available from the network. Quite clearly, the BP network performed the worst of the three architectures, and showed no pattern completion capabilities. This poor performance continues to exist even as the number of training patterns are increased.

The optimal placement problem was investigated using both the CP network and the hybrid network to compute the objective function for the optimization problem. In both of these networks, the effect of cluster size was investigated by changing the number of Kohonen layer neurons in the network. It is important to emphasize the smaller number of Kohonen neurons implies lower data storage requirements, and further, faster look-up capabilities during the network generalization phase. Smaller number of Kohonen neurons also implies a coarser generalization result. Three different cases of cluster size were considered, including 100, 400 and 603 Kohonen neurons. For the number of sensors restricted to less than 7, the results of optimal placements using a CP network are shown in Tables 2-4. Comparable results of optimal placement using a hybrid network are summarized in Tables 5-7. In each of these tables, a filled-in area in a particular column or row represents the presence of a sensor as determined by the optimizer. It is obvious from these tables that as the number of Kohonen neurons increases, the generalization capability of both the CP and the hybrid networks improves; in this particular case, increasing the number of Kohonen neurons to above 400 shows only a marginal improvement in the generalization capability. Although results of optimal placements are qualitatively similar for the different cases, there are minor differences which may be attributed to the nature of the stochastic search process.

A similar set of results and conclusions can be drawn with respect to another simulation, where a total of upto 12 sensors were permitted on the structure. The average error per test pattern is considerably lower in this case, with a maximum of 3.46% error for a case with only 100 Kohonen neurons. Increasing the number of Kohonen neurons to 400 drops the average error to less than 0.5%. When using the hybrid network, with 100 Kohonen neurons, the average generalization error was 6.16%. Note however, that the upper bound of 12 sensors was not selected by the optimization routine, indicating premature convergence of the optimization process. Results of these simulations are summarized in Tables 8-10.

## CLOSING REMARKS

The paper describes an approach in which neural networks are trained to relate strain measurements taken during operation to damage in composite beams. For the network to be most effective in identifying the extent and location of damage under prescribed loading using a minimum number of sensors, the latter were required to be optimally located on the structure. The optimal placement problem requires function information corresponding to a varying analysis model, and a neural network based approximation to this analysis was used with success in the present work. A CP network and a hybrid CP-BP network were explored as alternatives for generating function approximations. The pattern completion capability of the CP network is of significance in this class of problems, and can also be used to enhance the quality of function approximations available from the BP network (hybrid network). The approach can be extended to include an integrated design of instrumented composite structures, where the base structure is simultaneously designed to enhance the performance of damage detection and mitigation strategies.

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