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Research Article

Delay-Dependent Stability Analysis for Uncertain Switched Time-Delay Systems Using Average Dwell Time

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We are concerned with the stability problem for linear discrete-time switched systems with time delays. The problem is solved by using multiple Lyapunov functions to develop constructive tools for the exponential stability analysis of the switched time-delay system. Furthermore, the uncertainties of the switched systems are also taken into consideration. Sufficient delay-dependent conditions are derived in terms of the average dwell time for the exponential stability based on linear matrix inequalities (LMIs). Finally, numerical examples are provided to illustrate the effectiveness of the proposed method.

1. Introduction

Switched systems represent dynamical systems described by a collection of differential equations with both continuous-time dynamics and discrete-time elements [1]. In recent years, hybrid and switched dynamic systems have attracted much attention because of their wide applications in control of mechanical systems, electrical systems [2, 3], networked systems [4–7], and many other fields. One of the important topics in the study of switched systems is stability analysis, and many results have been reported for linear switched system. By exploiting average dwell time, Hespanha and Morse derived some sufficient conditions for the uniform exponential stability of the switched linear systems [8]. A concise and timely survey on analysis and synthesis of switched linear system is presented in [9]. In [10], the stability of switched linear system is analyzed by using multiple Lyapunov functions and Lyapunov-Metzler inequalities. Note that these results can not be extended to switched time-delay systems due to the infinite dimensionality of time-delay systems.

Most existing results in switched systems are based on finite dimensional systems free of time delays. However, time-delay phenomena are very common in most practical industrial control systems [11]. As a matter of fact, switched time-delay systems have often appeared in the mathematical models of networked systems, hereditary systems, Lotka-Volterra systems, and so on. More importantly, the controller design of time-delay systems sometimes requires switching controller when one single controller cannot meet the design requirements. Thus, it is of great importance to investigate switched systems with time delays. To investigate the time-delay problem for switched systems, some important research efforts have been conducted. Sufficient conditions for exponential stability and weighted L_2-gain were developed for a class of switched systems with time-varying delays [12]. In [13], an average dwell time approach was used to analyze switched linear systems with time-varying delays. Furthermore, the literatures [14, 15] extended the average dwell time approach to switched singular time-delay systems. By using a Lyapunov functional and LMI approach, various delay-independent and delay-dependent stability results were provided for linear switched time-delay system in [16]. In [17, 18], the piecewise Lyapunov-Razumikhin functions were introduced for the stability analysis of the switched time-delay systems. It should be noticed that most of the aforementioned results do not consider the uncertainties of switched linear systems.

Due to the existence of model uncertainties in real applications, it is very desirable to consider the impact of uncertainties for the switched systems [19]. To the best of our knowledge, such problems for the switched systems with both uncertainties and time delay have rarely been studied till present. In [20], some sufficient conditions for the robust stabilization of a class of uncertain switched time-delay systems were developed based on average dwell time. Using a common Lyapunov function, several sufficient delay-independent conditions for the robust stability of the uncertain linear hybrid systems with time delay were given in [21]. Nevertheless, it may be hard to construct a common Lyapunov function for all the subsystems in most of the application cases. In addition, the research for the delay-dependent stability analysis is relatively a new topic. Generally speaking, the delay-dependent stability analysis is considered less conservative than the delay-independent case [22].

Motivated by the challenges discussed above, this paper considers generalized uncertain time-delay systems in a discrete domain where some sufficient delay-dependent conditions are derived by using multiple Lyapunov functions and average dwell time to guarantee global exponential stability of the closed loop systems. Compared with [21], stronger stability results are provided, that is, the exponential stability rather than the asymptotic stability.

The remainder of this paper is organized as follows. In Section 2, the mathematical model of the uncertain switched system with delay time is presented and some preliminaries are given. In Section 3, the stability of uncertain switched time-delay systems in the discrete-time domain is analyzed; some sufficient conditions with the dwell time for switching signal are given. In Section 4, some numerical examples are provided to illustrate the effectiveness of the results. Finally, some conclusions are drawn in Section 5.

The following notations will be used throughout this paper. Let $\mathbb{R} = (-\infty, +\infty)$ and $\mathbb{R}^+ = [0, +\infty)$, let $\mathbb{R}^n$ be the n-dimensional Euclidean space, and Z^+ denotes the set of all nonnegative integers and $\mathbb{R}^{n\times m}$ is the space of $n \times m$ real matrices. $P \in \mathbb{R}^{n\times n} \geq 0 (P \in \mathbb{R}^{n\times n} \leq 0)$ means that matrix P is symmetric and semipositive (seminegative) definite. $P \in \mathbb{R}^{n\times n} > 0 (P \in \mathbb{R}^{n\times n} < 0)$ means that matrix P is symmetric and positive (negative) definite. I_n denotes the $n \times n$ real identity matrix. A^T denotes the transpose of a square matrix A. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. For $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$, the norm of x is $\|x\| = (\sum_{i=1}^{n} x_i^2)^{1/2}$. For $A \in \mathbb{R}^{n\times m}$, the norm of A is $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$.

2. Problem Definition and Preliminaries

In this section, we introduce an uncertain linear discrete-time switched system with delays of the following general form:

$$\begin{aligned} x(k+1) = {} & [A_{\sigma(k)} + \Delta A_{\sigma(k)}(k)]\, x(k) \\ & + [B_{\sigma(k)} + \Delta B_{\sigma(k)}(k)]\, x(k-h) \end{aligned} \tag{1}$$

and the initial condition

$$x_{k_0}(\theta) = \phi(\theta), \quad \theta \in [-h, 0], \tag{2}$$

where $h \geq 0$, $k \in Z^+$, the state $x(k) \in \mathbb{R}^n$. Let $S = \{1, \ldots, N\}$, the switching signal $\sigma(k) = i \in S$, $x_{k_0}(\theta) = x(k_0 + \theta)$ and the set $\{k_q\}$ denotes switching sequence, which is assumed to be a closed discrete subset of Z^+ with $k_0 = 0 < k_1 < k_2 < \cdots < k_q < \cdots$, and $\lim_{q\to\infty} k_q = \infty$. For any $\sigma(k)$, $A_{\sigma(k)}, B_{\sigma(k)} \in \mathbb{R}^{n\times n}$ are given constant matrices, and $\Delta A_{\sigma(k)}(k), \Delta B_{\sigma(k)}(k) \in \mathbb{R}^{n\times n}$ are the parameter uncertainties which satisfy the following assumptions:

$$\begin{aligned} \Delta A_i(k) &= E_i F(k) F_i, \\ \Delta B_i(k) &= H_i F(k) G_i, \end{aligned} \tag{3}$$

where E_i, F_i, H_i, and G_i are given constant matrices of appropriate dimensions. The uncertain matrix $F(k)$ is assumed to satisfy the condition $F^T(k)F(k) \leq I$.

Definition 1. The discrete-time uncertain linear switched system with time delay (1) is robustly stable if there exist a positive definite scalar function $V(x(k))$ for all $x(k) \in \mathbb{R}^n$ and a switching signal $\sigma(k) \in S$ such that

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k)) < 0. \tag{4}$$

Definition 2. The induced norm of a matrix A is denoted by

$$\|A\| = \sup\left\{ \frac{\|Ax\|}{\|x\|} : x \in \mathbb{R}^n,\ \|x\| \neq 0 \right\}, \tag{5}$$

where $\|A\|$ and $\|x\|$ satisfy the inequality

$$\|Ax\| \leq \|A\| \, \|x\|. \tag{6}$$

Definition 3. A switching signal σ is said to have an average dwell time τ_a if there exist two positive numbers N_0 and τ_a such that

$$N(k_0, k) \leq N_0 + \frac{(k - k_0)}{\tau_a}, \quad \forall k \geq k_0 \geq 0, \tag{7}$$

where $N(k_0, k)$ is the number of switches in the interval $[k_0, k)$.

Definition 4 (see [11, 23]). The switched system (1)-(2) is said to be exponentially stable if its solutions satisfy

$$\|x(k)\| \leq c\lambda^{-(k-k_0)} \|\phi\|_L, \quad \forall k \geq k_0, \tag{8}$$

for any initial conditions (k_0, ϕ), where $\|\phi\|_L = \sup_{k_0-h\leq\theta\leq k_0} \|\phi(\theta)\|$, $c > 0$, and $\lambda > 0$ is the decay rate.

Lemma 5 (see [24]). *Let M, P, and Q be given matrices such that $Q > 0$, $Q = Q^T$. Then, the linear matrix inequality (LMI) $\begin{pmatrix} P & M \\ M^T & -Q \end{pmatrix} < 0$ holds if and only if $P + MQ^{-1}M^T < 0$.*

Lemma 6 (see [25, 26]). *Let $P > 0$, $F^T(k)F(k) \leq I$, and M, N are constant matrices. If there exists $\varepsilon > 0$ such that $\varepsilon I - M^T PM > 0$, then*

$$\begin{aligned}&[A + MF(k)N]^T P[A + MF(k)N]\\&\leq A^T R^{-1} A + \varepsilon N^T N,\end{aligned} \quad (9)$$

where $R = P^{-1} - (1/\varepsilon)MM^T$.

Lemma 7 (see [27]). *For a quadratic positive definite $V(x) = x^T Px$, there exist $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ such that*

$$\lambda_{\min}(P)\|x\|^2 \leq x^T Px \leq \lambda_{\max}(P)\|x\|^2. \quad (10)$$

3. Stability Analysis

In this section, we analyze the stability for uncertain switched time-delay systems, and some sufficient conditions are given. Firstly, let us consider switched time-delay systems in discrete-time domain instead.

3.1. Switched Systems with Time Delay. Consider the discrete-time switched system with state delays given by

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}x(k-h) \quad (11)$$

and the initial condition

$$x_{k_0}(\theta) = \phi(\theta), \quad \theta \in [-h, 0]. \quad (12)$$

System (11)-(12) can be obtained from (1)-(2) as long as $\Delta A_{\sigma(k)}(k) = 0$ and $\Delta B_{\sigma(k)}(k) = 0$. $\sigma(k)$ is the switching signal, for each $k \in Z^+$; we get the following sets:

$$\begin{aligned}A_{\sigma(k)} &\in \{A_1, \ldots, A_N\},\\B_{\sigma(k)} &\in \{B_1, \ldots, B_N\}.\end{aligned} \quad (13)$$

Obviously, the switched linear system (11)-(12) has N different subsystems, that is,

$$x(k+1) = A_i x(k) + B_i x(k-h), \quad (14)$$

and the initial condition

$$x_{k_0}(\theta) = \phi(\theta), \quad \theta \in [-h, 0], \quad (15)$$

where $h > 0$, $i \in S = \{1, \ldots, N\}$, $k \in [k_{q-1}, k_q)$, and $q \in Z^+$. Clearly, there exists $j \in S$ such that A_i and B_i are constrained to jump to A_j and B_j among their own sets, respectively, where $j \neq i$.

Considering discrete-time switched system (11)-(12), we choose the following Lyapunov function given by

$$V_i(x(k)) = x^T(k)P_i x(k) + \sum_{s=k-h}^{k-1} \lambda^{2(s-k)} x^T(s) Q_i x(s), \quad (16)$$

where, $\forall P_i, Q_i > 0$ and $\lambda > 0$ is a given constant.

Proposition 8. *For a given scalar $\lambda > 1$ and the delay $h > 0$, there exist matrices $P_i > 0$, $Q_i > 0$, and $i \in S$ such that the following matrix inequality*

$$\Gamma = \begin{pmatrix}\lambda^2 A_i^T P_i A_i - P_i + Q_i & \lambda^{h+2} A_i^T P_i B_i\\ * & \lambda^{2(h+2)} B_i^T P_i B_i - Q_i\end{pmatrix} < 0 \quad (17)$$

holds; then the function $V_i(x(k))$ in (16) along any trajectory of system (14)-(15) guarantees the following growth estimation:

$$V_i(x(k)) \leq \lambda^{-2(k-k_0)} V_i(x(k_0)), \quad k \geq k_0. \quad (18)$$

Proof. Applying the transformation $x(k) = \lambda^{-(k-k_0)}\xi(k)$, we obtain the following system from (14)-(15):

$$\xi(k+1) = \lambda A_i \xi(k) + \lambda^{h+1} B_i \xi(k-h) \quad (19)$$

and the initial condition

$$\xi_{k_0}(\theta) = \xi(k_0 + \theta) = \lambda^\theta x_{k_0}(\theta), \quad \theta \in [-h, 0]. \quad (20)$$

Choose the following Lyapunov function for system (19)-(20):

$$W_i(\xi(k)) = \xi^T(k) P_i \xi(k) + \sum_{s=k-h}^{k-1} \xi^T(s) Q_i \xi(s). \quad (21)$$

The forward difference of the Lyapunov function $W_i(\xi(k))$ along the trajectory of system (19)-(20) is given by

$$\begin{aligned}\Delta W_i(\xi(k)) &= W_i(\xi(k+1)) - W_i(\xi(k))\\&= \left[\lambda A_i \xi(k) + \lambda^{h+1} B_i \xi(k-h)\right]^T\\&\quad \cdot P_i\left[\lambda A_i \xi(k) + \lambda^{h+1} B_i \xi(k-h)\right]\\&\quad - \xi^T(k) P_i \xi(k) + \xi^T(k) Q_i \xi(k)\\&\quad - \xi^T(k-h) Q_i \xi(k-h)\\&= y^T \Gamma y,\end{aligned} \quad (22)$$

where Γ is defined in (17) and $y = [\xi^T(k), \xi^T(k-h)]^T$. Using (17), we arrive at

$$\Delta W_i(\xi(k)) < 0 \quad (23)$$

which implies

$$W_i(\xi(k)) \leq W_i(\xi(k_0)). \quad (24)$$

From (16), we have

$$\begin{aligned}V_i(x(k)) &= \lambda^{-2(k-k_0)} \xi^T(k) P_i \xi(k)\\&\quad + \sum_{s=k-h}^{k-1} \lambda^{2(s-k)} \lambda^{-2(s-k_0)} x^T(k) Q_i x(k)\\&= \lambda^{-2(k-k_0)} W_i(\xi(k))\end{aligned} \quad (25)$$

and the fact that

$$W_i(\xi(k_0)) = V_i(x(k_0)). \quad (26)$$

It follows that

$$V_i(x(k)) \le \lambda^{-2(k-k_0)} V_i(x(k_0)), \quad k \ge k_0. \quad (27)$$

The proof is completed. □

Theorem 9. *For given scalars $\lambda > 1$ and $\mu \ge 1$ and the delay $h > 0$, assume that there exist $P_i > 0$ and $Q_i > 0$ such that equality (17) holds. Then, switched delay-time system (11)-(12) is exponentially stable if the following conditions hold:*

(A1) *the positive definite matrices P_i and Q_i satisfy*

$$P_j \le \mu P_i, \quad Q_j \le \mu Q_i \quad \forall i, j \in S; \quad (28)$$

(A2) *there exists $1 < \nu \le \lambda$ such that the average dwell time τ_a satisfies*

$$\tau_a \ge \frac{\ln \mu}{(2 \ln \nu)}. \quad (29)$$

Proof. Choose the following Lyapunov function for system (11)-(12):

$$\begin{aligned} V_{\sigma(k)}(x(k)) &= x^T(k) P_{\sigma(k)} x(k) \\ &\quad + \sum_{s=k-h}^{k-1} \lambda^{2(s-k)} x^T(s) Q_{\sigma(k)} x(s). \end{aligned} \quad (30)$$

For $i \in S$, using Proposition 8, if equality (17) holds, we obtain

$$V_i(x(k)) = \lambda^{-2(k-k_{q-1})} V_i(x(k_{q-1})). \quad (31)$$

Using condition (A1), then we have

$$\begin{aligned} V_{\sigma(k_q)}(x(k_q)) &= x^T(k_q) P_{\sigma(k_q)} x(k_q) \\ &\quad + \sum_{s=k_q-h}^{k_q-1} \lambda^{2(s-k_q)} x^T(s) Q_{\sigma(k_q)} x(s) \\ &\le x^T(k_q) \mu P_{\sigma(k-1)} x(k_q) \\ &\quad + \sum_{s=k_q-h}^{k_q-1} \lambda^{2(s-k_q)} x^T(s) \mu Q_{\sigma(k_{q-1})} x(s) \\ &= \mu V_{\sigma(k_{q-1})}(x(k_q)). \end{aligned} \quad (32)$$

By virtue of (31) and (32), it follows that

$$\begin{aligned} V_{\sigma(k)}(x(k)) &\le \lambda^{-2(k-k_q)} V_{\sigma(k_q)}(x(k_q)) \\ &\le \lambda^{-2(k-k_q)} \mu V_{\sigma(k_{q-1})}(x(k_q)) \\ &\le \mu \lambda^{-2(k-k_q)} \lambda^{-2(k_q-k_{q-1})} \\ &\quad \cdot V_{\sigma(k_{q-1})}(x(k_{q-1})). \end{aligned} \quad (33)$$

Iterating k_q from $q-1$ to 0, we have

$$V_{\sigma(k)}(x(k)) \le \mu^N \lambda^{-2(k-k_0)} V_{\sigma(k_0)}(x(k_0)). \quad (34)$$

Applying (29), we have

$$V_N(x(k)) \le \mu^{N_0} e^{-2(\ln\lambda - \ln\nu)(k-k_0)} V_{\sigma(k_0)}(x(k_0)). \quad (35)$$

Thus, there exists $\ln \rho_1 = \ln\lambda - \ln\nu > 0$ and $K_1 = \mu^{N_0}$ such that

$$\begin{aligned} V_N(x(k)) &\le \mu^{N_0} e^{-2(k-k_0)\ln\rho_1} V_{\sigma(k_0)}(x(k_0)) \\ &= K_1 \rho_1^{-2(k-k_0)} V_{\sigma(k_0)}(x(k_0)). \end{aligned} \quad (36)$$

It follows that

$$\beta_1 \|x(k)\|^2 \le V_{\sigma(k)}(x(k)) \le K_1 \rho_1^{-2(k-k_0)} \beta_2 \|\phi\|_L^2 \quad (37)$$

which yields $\|x(k)\| \le \sqrt{K_1\beta_2/\beta_1} \rho_1^{-(k-k_0)} \|\phi\|_L$, where

$$\begin{aligned} \beta_1 &= \min_{i\in S} \lambda_{\min}(P_i), \\ \beta_2 &= \max_{i\in S} \lambda_{\max}(P_i) + h \max_{i\in S} \lambda_{\max}(Q_i). \end{aligned} \quad (38)$$

Hence, it is concluded from Definition 4 that switched delay-time system (11)-(12) is exponentially stable. The proof is completed. □

3.2. Uncertain Discrete-Time System with Time Delay. Consider the following subsystem of switched system (1)-(2):

$$\begin{aligned} x(k+1) &= [A_i + \Delta A_i(k)] x(k) \\ &\quad + [B_i + \Delta B_i(k)] x(k-h) \end{aligned} \quad (39)$$

and the initial condition

$$x_{k_0}(\theta) = \phi(\theta), \quad \theta \in [-h, 0], \quad (40)$$

$\forall i \in S$. Choose the following Lyapunov function for (39)-(40):

$$\begin{aligned} V_i(x(k)) &= x^T(k) P_i x(k) \\ &\quad + \sum_{s=k-h}^{k-1} \lambda^{2(s-k)} x^T(s) Q_i x(s), \end{aligned} \quad (41)$$

where $\forall P_i, Q_i > 0$ and $\lambda > 1$ is a given constant.

To derive the exponential stability of switched time-delay system (1)-(2), we give the decay estimation of the Lyapunov function $V_i(x(k))$ along the trajectory (39)-(40) in the following proposition firstly.

Proposition 10. *For a given scalar $\lambda > 1$, $\varepsilon > 0$, and any delay $h > 0$, if there exist matrices $P_i > 0$, $Q_i > 0$, and $\varepsilon I - \begin{pmatrix} E_i^T P_i E_i & E_i^T P_i H_i \\ H_i^T P_i E_i & H_i^T P_i H_i \end{pmatrix} > 0$ such that the following inequalities,*

$$\begin{aligned} X_i + Y_i Z_i^{-1} \Omega_i &< 0, \\ Z_i &> 0, \\ R_i &> 0, \end{aligned} \quad (42)$$

hold, then function $V_i(x(k))$ in (41) along any trajectory of switched system (39)-(40) guarantees the decay estimation as follows:

$$V_i(x(k)) \le \lambda^{-2(k-k_0)} V_i(x(k_0)), \quad k \ge k_0, \tag{43}$$

where

$$\begin{aligned} R_i &= P_i^{-1} - \frac{1}{\varepsilon}\left(E_i^T E_i + H_i^T H_i\right), \\ X_i &= \lambda^2 \left(A_i^T R_i^{-1} A_i + \varepsilon F_i^T F_i\right) - P_i + Q_i, \\ Y_i &= \lambda^{h+2} A_i^T R_i^{-1} B_i, \\ \Omega_i &= \lambda^{h+2} B_i^T R_i^{-1} A_i, \\ Z_i &= Q_i - \lambda^{2(h+1)} \left(B_i^T R_i^{-1} B_i + \varepsilon G_i^T G_i\right). \end{aligned} \tag{44}$$

Proof. $\forall \lambda > 1$, by applying the transformation $x(k) = \lambda^{-(k-k_0)}\xi(k)$, we obtain the following system from (39)-(40):

$$\begin{aligned} x(k+1) &= \lambda\left[A_i + \Delta A_i(k)\right]\xi(k) \\ &\quad + \lambda^{h+1}\left[B_i + \Delta B_i(k)\right]\xi(k-h) \end{aligned} \tag{45}$$

and the initial condition

$$\xi_{k_0}(\theta) = \lambda^{\theta} x_{k_0}(\theta), \quad \theta \in [-h, 0]. \tag{46}$$

Choose the following Lyapunov function for switched system (45)-(46):

$$\begin{aligned} W_i(\xi(k)) &= \xi^T(k) P_{\sigma(k)} \xi(k) \\ &\quad + \sum_{s=k-h}^{k-1} \xi^T(k) Q_{\sigma(k)} \xi(k). \end{aligned} \tag{47}$$

Let $\overline{A}_i = \lambda[A_i + \Delta A_i(k)]$ and $\overline{B}_i = \lambda^{h+1}[B_i + \Delta B_i(k)]$. The forward difference of Lyapunov function $W_i(k)$ along any trajectory of system (45)-(46) is given by

$$\begin{aligned} \Delta W_i(\xi(k)) &= W_i(\xi(k+1)) - W_i(\xi(k)) \\ &= \left[\overline{A}_i \xi(k) + \overline{B}_i \xi(k-h)\right]^T \\ &\quad \cdot P_i \left[\overline{A}_i \xi(k) + \overline{B}_i \xi(k-h)\right] \\ &\quad - \xi^T(k) P_i \xi(k) + \xi^T(k) Q_i \xi(k) \\ &\quad - \xi^T(k-h) Q_i \xi(k-h) \\ &= y^T M_1 y, \end{aligned} \tag{48}$$

where

$$\begin{aligned} y &= \left[\xi^T(k), \xi^T(k-h)\right]^T, \\ M_1 &= \begin{pmatrix} \overline{A}_i^T P_i \overline{A}_i - P_i + Q_i & \overline{A}_i^T P_i \overline{B}_i \\ \overline{B}_i^T P_i \overline{A}_i & \overline{B}_i^T P_i \overline{B}_i - Q_i \end{pmatrix}. \end{aligned} \tag{49}$$

Note that

$$\begin{aligned} M_1 &= \begin{pmatrix} \overline{A}_i^T P_i \overline{A}_i - P_i + Q_i & \overline{A}_i^T P_i \overline{B}_i \\ \overline{B}_i^T P_i \overline{A}_i & \overline{B}_i^T P_i \overline{B}_i - Q_i \end{pmatrix} \\ &= \begin{pmatrix} -P_i + Q_i & 0 \\ 0 & -Q_i \end{pmatrix} \\ &\quad + \begin{pmatrix} \overline{A}_i^T \\ \overline{B}_i^T \end{pmatrix} P_i \begin{pmatrix} \overline{A}_i & \overline{B}_i \end{pmatrix}. \end{aligned} \tag{50}$$

On the other hand, by virtue of Lemma 6, we have

$$\begin{aligned} &\begin{pmatrix} \overline{A}_i^T \\ \overline{B}_i^T \end{pmatrix} P_i \begin{pmatrix} \overline{A}_i & \overline{B}_i \end{pmatrix} \\ &= \left[\begin{pmatrix} \lambda A_i^T \\ \lambda^{h+1} B_i^T \end{pmatrix} + \begin{pmatrix} \lambda F_i^T & 0 \\ 0 & \lambda^{h+1} G_i^T \end{pmatrix} \right. \\ &\qquad \left. \times \begin{pmatrix} F^T(k) & 0 \\ 0 & F^T(k) \end{pmatrix} \begin{pmatrix} E_i^T \\ H_i^T \end{pmatrix}\right] P_i \\ &\quad \times \left[\begin{pmatrix} \lambda A_i & \lambda^{h+1} B_i \end{pmatrix} + \begin{pmatrix} E_i & H_i \end{pmatrix} \begin{pmatrix} F(k) & 0 \\ 0 & F(k) \end{pmatrix} \right. \\ &\qquad \left. \times \begin{pmatrix} \lambda F_i & 0 \\ 0 & \lambda^{h+1} G_i \end{pmatrix}\right] \\ &\le \begin{pmatrix} \lambda A_i^T \\ \lambda^{h+1} B_i^T \end{pmatrix} R_i^{-1} \left(\lambda A_i, \lambda^{h+1} B_i\right) \\ &\quad + \varepsilon \begin{pmatrix} \lambda F_i^T & 0 \\ 0 & \lambda^{h+1} G_i^T \end{pmatrix} \begin{pmatrix} \lambda F_i & 0 \\ 0 & \lambda^{h+1} G_i \end{pmatrix} \\ &= \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix}, \end{aligned} \tag{51}$$

where

$$\begin{aligned} R_i &= P_i^{-1} - \frac{1}{\varepsilon}\left(E_i^T E_i + H_i^T H_i\right), \\ (1,1) &= \lambda^2 \left(A_i^T R_i^{-1} A_i + \varepsilon F_i^T F_i\right), \\ (1,2) &= \lambda^{h+2} A_i^T R_i^{-1} B_i, \\ (2,1) &= \lambda^{h+2} B_i^T R_i^{-1} A_i, \\ (2,2) &= \lambda^{2(h+1)} \left(B_i^T R_i^{-1} B_i + \varepsilon G_i^T G_i\right). \end{aligned} \tag{52}$$

Then,

$$\begin{aligned} M_1 &= \begin{pmatrix} \overline{A}_i^T P_i \overline{A}_i - P_i + Q_i & \overline{A}_i^T P_i \overline{B}_i \\ \overline{B}_i^T P_i \overline{A}_i & \overline{B}_i^T P_i \overline{B}_i - Q_i \end{pmatrix} \\ &\le \begin{pmatrix} X_i & Y_i \\ \Omega_i & -Z_i \end{pmatrix}. \end{aligned} \tag{53}$$

Now taking into account (42) and using Lemma 5, we have

$$\Delta W_i(\xi(k)) < 0 \tag{54}$$

which implies that

$$W_i(\xi(k)) \le W_i(\xi(k_0)). \tag{55}$$

From (41), we have

$$\begin{aligned} V_i(x(k)) &= \lambda^{-2(k-k_0)}\xi^T(k)P_i\xi(k) \\ &\quad + \sum_{k-h}^{k-1}\lambda^{2(s-k)}\lambda^{-2(s-k_0)}\xi^T(k)Q_i\xi(k) \\ &= \lambda^{-2(k-k_0)}W_i(\xi(k)) \end{aligned} \tag{56}$$

and the fact that

$$W_i(\xi(k_0)) = V_i(x(k_0)). \tag{57}$$

It follows that

$$V_i(x(k)) \le \lambda^{-2(k-k_0)}V_i(x(k_0)). \tag{58}$$

The proof is completed. □

Theorem 11. *For given scalars $\lambda > 1$ and $\mu \ge 1$ and any delay $h > 0$, if there exist matrices $P_i > 0$ and $Q_i > 0$, such that inequalities (42) and the conditions*

$$P_j \le \mu P_i, \quad Q_j \le \mu Q_i, \quad \forall i, j \in S, \tag{59}$$

$$\tau_a > \frac{\ln\mu}{2\ln\lambda^*} \tag{60}$$

hold, then the uncertain switched time-delay system (1)-(2) is exponentially stable and guarantees a decay rate $\rho_2 = \lambda/\lambda^*$, where $1 < \lambda^* < \lambda$ and τ_a is the average dwell time.

Proof. Considering system (1)-(2), choose the following Lyapunov function given by

$$\begin{aligned} V_{\sigma(k)}(x(k)) &= x^T(k)P_{\sigma(k)}x(k) \\ &\quad + \sum_{s=k-h}^{k-1}\lambda^{2(s-k)}x^T(s)Q_{\sigma(k)}x(s). \end{aligned} \tag{61}$$

By virtue of (59), we have

$$V_{\sigma(k_q)}(x(k_q)) \le \mu V_{\sigma(k_{q-1})}(x(k_q)). \tag{62}$$

On the other hand, function $V_i(k)$ ensures the decay estimation (43) under condition (42). Hence, we have

$$\begin{aligned} V_{\sigma(k)}(x(k)) &\le \lambda^{-2(k-k_q)}V_{\sigma(k_q)}(x(k_q)) \\ &\le \lambda^{-2(k-k_q)}\mu V_{\sigma(k_{q-1})}(x(k_q)) \\ &\le \mu\lambda^{-2(k-k_q)}\lambda^{-2(k_q-k_{q-1})}V_{\sigma(k_{q-1})}(x(k_{q-1})) \\ &\le \mu^{N(k,k_0)}\lambda^{-2(k-k_0)}V_{\sigma(k_0)}(x(k_0)) \\ &\le \mu^{N_0}e^{((k-k_0)/\tau_a)\ln\mu}\lambda^{-2(k-k_0)}V_{\sigma(k_0)}(x(k_0)). \end{aligned} \tag{63}$$

Let $K_2 = \mu^{N_0}$; by virtue of (60), we have

$$\begin{aligned} V_{\sigma(k)}(x(k)) &\le K_2(\lambda^*)^{2(k-k_0)}\lambda^{-2(k-k_0)}V_{\sigma(k_0)}(x(k_0)) \\ &\le K_2\rho_2^{-2(k-k_0)}V_{\sigma(k_0)}(x(k_0)), \end{aligned} \tag{64}$$

where $\rho_2 = \lambda/\lambda^* > 1$. It follows that

$$\begin{aligned} \alpha_1\|x(k)\|^2 &\le V_{\sigma(k)}(x(k)) \\ &\le K_2\rho_2^{-2(k-k_0)}\alpha_2\|\phi\|_L^2 \end{aligned} \tag{65}$$

which yields $\|x(k)\| \le \sqrt{K_2\alpha_2/\alpha_1}\rho_2^{-(k-k_0)}\|\phi\|_L$, where

$$\begin{aligned} \alpha_1 &= \min_{i\in S}\lambda_{\min}(P_i), \\ \alpha_2 &= \max_{i\in S}\lambda_{\max}(P_i) + h\max_{i\in S}\lambda_{\max}(Q_i). \end{aligned} \tag{66}$$

Hence, by virtue of Definition 4, the switched system (1)-(2) is exponentially stable. The proof is completed. □

Remark 12. In literature [21], the common Lyapunov function was employed to derive the delay-independent conditions. From condition (59), it can be seen that the common Lyapunov function approach can be treated as a special case of Theorem 11 if and only if μ satisfies $\mu = 1$. In this sense, we get away from the common Lyapunov conditions as μ increases from 1, which indicates the conservativeness of the common Lyapunov function approach. In contrast, this paper presents the delay-dependent exponential stability conditions by constructing multiple Lyapunov functions.

Remark 13. It should be noted that [11] only considers the switched time-delay systems without the parameter uncertainties. The present paper extends the results in [11] to the uncertain switched time-delay system in discrete-time domain by constructing different Lyapunov functions and employing the concept of the average dwell time.

4. Numerical Example

In this section, we use an example to illustrate the results in Section 3.

Example 1. Consider the delay-time switched system (11)-(12) given by

$$\begin{aligned} A_1 &= \begin{pmatrix} 0 & 0.3 \\ -0.2 & 0.1 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} -0.1 & 0 \\ 0.1 & 0 \end{pmatrix}, \\ A_2 &= \begin{pmatrix} 0.2 & -0.5 \\ 0 & 0.8 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 0 & 0.1 \\ 0 & 0.3 \end{pmatrix}, \end{aligned} \tag{67}$$

where $S = \{1, 2\}$. Let $h = 1$. Assuming that the average dwell time is $\tau_a = 2$ and $\mu = 1.5$, then we have $\nu \ge 1.1067$. If we take $\nu = 1.2$, then we can choose $\lambda = 1.8$. From equalities (17) and (28), by using LMIs toolbox, we can obtain the feasible solutions for P_1, Q_1 and P_2, Q_2 given by

$$P_1 = \begin{pmatrix} 1.7733 & 0.1339 \\ 0.1339 & 1.5144 \end{pmatrix}, \qquad Q_1 = \begin{pmatrix} 1.3174 & 0.1059 \\ 0.1059 & 0.5855 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 0.0929 & 0.0247 \\ 0.0247 & 0.0137 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 0.5327 & 0.0549 \\ 0.0549 & 0.2748 \end{pmatrix}. \tag{68}$$

From Theorem 9, it is concluded that the delay-time switched system (11)-(12) is exponentially stable.

Example 2. Consider the delay-time switched system (1)-(2), where $i \in \{1, 2\}$, and

$$\begin{aligned} A_1 &= \begin{pmatrix} -0.6 & 0 \\ 0 & 0.1 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} -0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \\ A_2 &= \begin{pmatrix} 0.2 & 0 \\ 0 & -0.8 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} -0.1 & 0 \\ 0 & 0.3 \end{pmatrix}, \\ E_i &= \begin{pmatrix} e_i^1 & 0 \\ 0 & e_i^2 \end{pmatrix}, \qquad H_i = \begin{pmatrix} h_i^1 & 0 \\ 0 & h_i^2 \end{pmatrix}, \\ F_i &= \begin{pmatrix} f_i^1 & 0 \\ 0 & f_i^2 \end{pmatrix}, \qquad G_i = \begin{pmatrix} g_i^1 & 0 \\ 0 & g_i^2 \end{pmatrix}, \end{aligned} \tag{69}$$

where

$$\begin{aligned} &e_1^1 = 0.02, \quad e_1^2 = 0.04, \quad e_2^1 = 0.01, \quad e_2^2 = 0.03, \\ &h_1^1 = 0.01, \quad h_1^2 = 0.03, \quad h_2^1 = 0.02, \quad h_2^2 = 0.04, \\ &f_1^1 = 0.01, \quad f_1^2 = 0.02, \quad f_2^1 = 0.01, \quad f_2^2 = 0.03, \\ &g_1^1 = 0.04, \quad g_1^2 = 0.05, \quad g_2^1 = 0.04, \quad g_2^2 = 0.09. \end{aligned} \tag{70}$$

Let $h = 1$. Assuming that the average dwell time is $\tau_a = 1$ and $\mu = 1.4$, then we have $\lambda^* \geq 1.1832$. If we take $\lambda^* = 1.2$, then we can choose $\lambda = 1.32$. By virtue of Theorem 11, the decay rate is $\rho_2 = 1.1$. If we choose $\varepsilon = 0.5$ and the matrices

$$P_1 = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.8 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}, \tag{71}$$

we can verify the conditions

$$\begin{aligned} &\varepsilon I - \begin{pmatrix} E_1^T P_1 E_1 & E_1^T P_1 H_1 \\ H_1^T P_1 E_1 & H_1^T P_1 H_1 \end{pmatrix} > 0, \\ &\varepsilon I - \begin{pmatrix} E_2^T P_2 E_2 & E_2^T P_2 H_2 \\ H_2^T P_2 E_2 & H_2^T P_2 H_2 \end{pmatrix} > 0, \\ &Z_1 = \begin{pmatrix} 0.1590 & 0 \\ 0 & 0.5874 \end{pmatrix} > 0, \\ &Z_2 = \begin{pmatrix} 0.1493 & 0 \\ 0 & 0.2016 \end{pmatrix} > 0. \end{aligned} \tag{72}$$

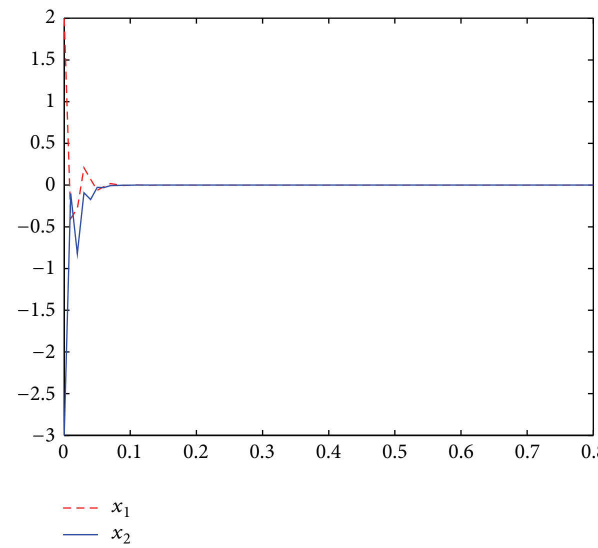

Figure 1: State trajectories of switched systems.

From equalities (42) and (59), by using LMIs toolbox, we can obtain the feasible solutions for Q_1 and Q_2 given by

$$Q_1 = \begin{pmatrix} 0.2056 & 0 \\ 0 & 0.6365 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 0.1609 & 0 \\ 0 & 0.3183 \end{pmatrix}, \tag{73}$$

where

$$R_1 = \begin{pmatrix} 1.2490 & 0 \\ 0 & 1.2450 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 4.9990 & 0 \\ 0 & 4.9950 \end{pmatrix}. \tag{74}$$

The initial states are given by $x_0 = (2, -3)^T$. According to Theorem 11, the state curves can be obtained by simulation (as shown in Figure 1).

5. Conclusions

In this paper, we have investigated the stability for switched time-delay systems in discrete-time domain. Several sufficient conditions have been proposed by utilizing multiple Lyapunov functions with the average dwell time. On one hand, the exponential stability conditions have been derived for the switched systems in the presence of time delays. On the other hand, the exponential stability conditions have been developed for the uncertain switched linear system with time delays based on LMIs conditions. Finally, the illustrative examples have been given to verify the main theoretical results.

We are currently working on the switched systems with interval time delays as well as controller synthesis methods for such systems, in the hope that switching controller design can in the long run offer a new look of synthesis of systems with large and/or time-varying delays when a single controller cannot suffice the design requirements.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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