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Quantitative Comparative Statics by Relative Derivatives on IS-LM with Five Production Factors Containing Multiple Energy Sources

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Abstract

This paper applies our established analytic technique of the relative derivative, (dy/dx)(a/b), to a quantitative comparative static analysis of a macroeconomy as based on the IS-LM framework coupled with a production function of five factors, capital, labor, oil, coal, and solar energy, resulting in twelve linear equations containing the general equilibrium growth rates of twelve endogenous variables, which are the six pairs of the (price, quantity) for the above output and five inputs. We conduct several simulations by substituting economically sensible values into all the parameters with some alterations for mathematical comparison, and finally we conclude with a summary remark.

Mathematics Subject Classification: 91B02, 26B10, 91B62, 91B64, 91B76

Keywords: Perturbation sensitivity, scale invariance, dimensionless elasticities, factor substitution, energy substitutability

1 Introduction

This paper serves as an extension of our previous publication [8] in this *Journal* which modeled the general equilibrium growth of a macroeconomy by an augmented IS-LM framework involving a simultaneous market clearance in goods, money, labor, and raw materials. Here we decompose raw materials into "oil," "coal," and "solar energy," to represent respectively "non-renewable but relatively clean," "relatively abundant but polluting," and "inexhaustible and clean" energies resources (for an extension of the IS-LM model to environmental economics, see, e.g., [6]). Again built upon our established analytic apparatus of the relative derivative $f_x := \left(\frac{df/b}{dx/a}\right)$, $a, b > 0$, $f \in C^1(\mathbb{R}, \mathbb{R})$, our expanded model here will comprise of a system of twelve simultaneous linear equations in twelve variables, namely: the general equilibrium growth rates of the general price level P, the "real" yearly gross domestic product y ("real" meaning adjusted for inflation), the real interest rate r (defined as $1 + interest/principal$), the real capital-stock employment K, the real wage rate w, the labor employment L, the real price of oil v_o, the employed quantity of oil O, the real price of coal v_c, the employed quantity of coal C, the real price of solar energy v_h, and the employed quantity of solar energy H. The prescribed parameters therein will consist of two kinds: (1) the partial relative derivatives as some of the coefficients of the above-said 12 variables, and (2) any exogenous percentage changes that can happen in the 12 equations (i.e., the right-hand-side constants in $\mathbf{Ax} = \mathbf{y}$, $\mathbf{A} \in GL(\mathbb{R}^{12}, \mathbb{R}^{12})$).

Our motivation here is to address the energy issue that confounds the world economy. Otherwise we seek to demonstrate once again the analytic facility of our proposed relative derivatives (for the advantages of dimensionless analysis, cf. [1, 2]); basically relative derivatives transform comparative statics from qualitative to quantitative analyses (for the wide-spread use of comparative static analyses using the regular derivatives $\partial\mathbf{y}/\partial\mathbf{x}$ in social sciences, cf., e.g., [3, 4, 5, 11]). In the next Section 2 we will first repeat all the pertinent essentials from our previous paper so as to make the present one self-contained; then we will simulate several comparative static results by varying certain interested parameters. Since our added contribution from this paper is a detailed breakdown of the previous production factor of raw materials, our simulations here below will all be about the "supply side" of an economy (for a general perturbation analysis of the system with a policy orientation, cf. our paper [7]). Finally in Section 3 we will conclude with a summary remark.

2 Analysis

2.1 The Model

Notation 1 *Assuming that all the functions below are C^1 and that the time period is one year, we denote respectively a proportional change in a variable u relative to a fixed constant u_o and the partial relative derivative of u with* *respect to a variable* x *evaluated at* $(x_{o,}u_o)$ *by*

$$\dot{u}^{\odot} := \frac{\frac{du}{dt}}{u_o},\ \textit{and}\ u_x := \frac{\partial u}{\partial x}\frac{x_o}{u_o}. \tag{1}$$

Remark 1 *We note that our relative derivative is a generalized elasticity in the sense that the above* $(x_{o,}u_o)$ *are fixed constants discretionarily chosen. As such, we break free from the rigid setup of the known elasticity* $u_x := \frac{\partial \ln u}{\partial \ln x}$ *(cf.* [10; 12*: p. 126*]*).*

Our first equation is to ensure an equal proportional change in the demand power withdrawn from the economy $s(y, r)$ and the demand power exerted onto the economy $i(r)$, i.e.,

$$\tilde{s}_o + s_r\dot{r}^{\odot} + s_y\dot{y}^{\odot} = \tilde{\imath}_o + i_r\dot{r}^{\odot}, \tag{2}$$

where $\tilde{s}_o$ represents an exogenous percentage change in the sum of the real private savings, government taxes, and the imports; analogously, $\tilde{\imath}_o$ represents an exogenous percentage change in the sum of the real private investments, government expenditures, and the exports.

Our second equation relates proportional changes in i to that of K. Since i minus the real depreciation δ equals $\frac{dK}{dt}$ as an identity, i contributes to capital accumulation K through a modifying factor J as follows:

$$\dot{i}^{\odot} = \dot{K}^{\odot} + J, \text{ with} \tag{3}$$

$$J \equiv \frac{\frac{d}{dt}\left(\frac{dK}{dt} + \delta\right)}{\left(\frac{dK}{dt} + \delta\right)} - \frac{\frac{dK}{dt}}{K}, \tag{4}$$

where J will be simulated with a plausible value.

Our third equation is to ensure an equal proportional change in the real money supplied M/p and the real money demanded $l(y, r)$, i.e.,

$$\dot{M}^{\odot} - \dot{P}^{\odot} = l_o + l_y\dot{y}^{\odot} + l_r\dot{r}^{\odot}, \tag{5}$$

where l_o denotes an exogenous proportional change in the real money demand.

Our fourth equation is about the proportional change in $y = f(K, L, O, C, H)$, i.e.,

$$\dot{y}^{\odot} = \eta_K\dot{K}^{\odot} + \eta_L\dot{L}^{\odot} + \eta_O\dot{O}^{\odot} + \eta_C\dot{C}^{\odot} + \eta_H\dot{H}^{\odot} + \dot{A}^{\odot}, \tag{6}$$

where $\dot{A}^{\odot}$ denotes an overall technology growth rate.

Our fifth equation is to relate proportional changes in (K/L) to that in (w/r), (v_o/r), (v_c/r), and (v_h/r), observing the economic law of cost-minimizing input demands to be homogeneous of degree zero in the input prices,

$$\begin{aligned}\dot{K}^{\odot} &= \dot{L}^{\odot} + \sigma_{KLw} \cdot \left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KLvo} \cdot \left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KLvc} \cdot \left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KLvh} \cdot \left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right) + k_o, \end{aligned} \tag{7}$$

where k_o represents an exogenous proportional change in (K/L), reflecting a shift in the production technology, favoring K if $k_o > 0$ and favoring L if $k_o < 0$ (for a restrictive study involving the above two equations, (6), (7), cf. [9]).

Analogously we have the following equations six, seven, and eight in our linear system:

$$\begin{aligned}\dot{K}^{\odot} + o_o &= \dot{O}^{\odot} + \sigma_{KOw} \cdot \left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KOvo} \cdot \left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KOvc} \cdot \left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KOvh} \cdot \left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right), \end{aligned} \tag{8}$$

$$\begin{aligned}\dot{K}^{\odot} + c_o &= \dot{C}^{\odot} + \sigma_{KCw} \cdot \left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KCvo} \cdot \left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KCvc} \cdot \left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KCvh} \cdot \left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right), \text{ and} \end{aligned} \tag{9}$$

$$\begin{aligned}\dot{K}^{\odot} + h_o &= \dot{H}^{\odot} + \sigma_{KHw} \cdot \left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KHvo} \cdot \left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KHvc} \cdot \left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KHvh} \cdot \left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right). \end{aligned} \tag{10}$$

Our ninth equation is about the supply of L,

$$\dot{L}^{\odot} = L_o + L_w \cdot \left(\dot{w}^{\odot} + \alpha \dot{P}^{\odot}\right), \tag{11}$$

where L_o represents an exogenous proportional change in the quantity of the labor force, and $\alpha \in [0, 1]$ measures the degree of "money illusion" in labor supply; $\alpha = 0$ means that labor supply is a function of the real wage rate w, while $\alpha = 1$ means that labor supply is a function of the nominal wage rate wP.

Similarly we have the following supplies of O, C, and H as our equations ten, eleven, and lastly twelve in our linear system,

$$\dot{O}^{\odot} = Oil_o + oil_p \cdot \dot{v}_o^{\odot}, \tag{12}$$

$$\dot{C}^{\odot} = Coal_o + coal_p \cdot \dot{v}_c^{\odot}, \text{ and} \tag{13}$$

$$\dot{H}^{\odot} = Sun_o + sun_p \cdot \dot{v}_h^{\odot}, \tag{14}$$

where Oil_o, $Coal_o$, and Sun_o represent exogenous proportional changes in their supplies.

2.2 Simulation

We begin with the following "Basic Configuration" of the parameters, where economically sensible values are substituted into all the parameters:

$$\begin{aligned} \tilde{s}_o + s_r\dot{r}^{\odot} + s_y\dot{y}^{\odot} &= \tilde{\imath}_o + i_r\dot{r}^{\odot} = \dot{i}^{\odot} = \\ -1\% + 0.5\dot{r}^{\odot} + \dot{y}^{\odot} &= 1\% - \dot{r}^{\odot}, \end{aligned} \tag{15}$$

$$\dot{i}^{\odot} = \dot{K}^{\odot} + J = \dot{K}^{\odot} + 1\%, \tag{16}$$

$$\begin{aligned} \dot{M}^{\odot} - \dot{P}^{\odot} &= l_o + l_y\dot{y}^{\odot} + l_r\dot{r}^{\odot} = \\ 5\% - \dot{P}^{\odot} &= \dot{y}^{\odot} - 0.5\dot{r}^{\odot}, \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{y}^{\odot} &= \eta_K\dot{K}^{\odot} + \eta_L\dot{L}^{\odot} + \eta_O\dot{O}^{\odot} + \eta_C\dot{C}^{\odot} + \eta_H\dot{H}^{\odot} + \dot{A}^{\odot} \\ &= 0.4\dot{K}^{\odot} + 0.4\dot{L}^{\odot} + 0.09\dot{O}^{\odot} + 0.009\dot{C}^{\odot} + 0.001\dot{H}^{\odot} + 0.5\%, \end{aligned} \tag{18}$$

$$\begin{aligned} \dot{K}^{\odot} &= \dot{L}^{\odot} + \sigma_{KLw}\cdot\left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KLvo}\cdot\left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KLvc}\cdot\left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KLvh}\cdot\left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right) + k_o \\ &= \dot{L}^{\odot} + \left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + 2\%, \end{aligned} \tag{19}$$

$$\begin{aligned} \dot{K}^{\odot} + o_o &= \dot{O}^{\odot} + \sigma_{KOw}\cdot\left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KOvo}\cdot\left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KOvc}\cdot\left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KOvh}\cdot\left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right) = \\ \dot{K}^{\odot} - 0.5\% &= \dot{O}^{\odot} + 0.5\cdot\left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right), \end{aligned} \tag{20}$$

$$\begin{aligned} \dot{K}^{\odot} + c_o &= \dot{C}^{\odot} + \sigma_{KCw}\cdot\left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KCvo}\cdot\left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KCvc}\cdot\left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KCvh}\cdot\left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right) = \\ \dot{K}^{\odot} - 0.5\% &= \dot{C}^{\odot} + 0.5\cdot\left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right), \end{aligned} \tag{21}$$

$$\begin{aligned} \dot{K}^{\odot} + h_o &= \dot{H}^{\odot} + \sigma_{KHw}\cdot\left(\dot{w}^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KHvo}\cdot\left(\dot{v}_o^{\odot} - \dot{r}^{\odot}\right) \\ &\quad + \sigma_{KHvc}\cdot\left(\dot{v}_c^{\odot} - \dot{r}^{\odot}\right) + \sigma_{KHvh}\cdot\left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right) = \\ \dot{K}^{\odot} + 0.5\% &= \dot{H}^{\odot} + 0.5\cdot\left(\dot{v}_h^{\odot} - \dot{r}^{\odot}\right), \end{aligned} \tag{22}$$

$$\begin{aligned}\dot{L}^{\odot} &= L_o + L_w \cdot \left(\dot{w}^{\odot} + \alpha \dot{P}^{\odot}\right) \\ &= 0.5\% + \left(\dot{w}^{\odot} + 0.5\dot{P}^{\odot}\right),\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{O}^{\odot} &= Oil_o + oil_p \cdot \dot{v}_o^{\odot} \\ &= 0.5\% + 0.5\dot{v}_o^{\odot},\end{aligned} \tag{24}$$

$$\begin{aligned}\dot{C}^{\odot} &= Coal_o + coal_p \cdot \dot{v}_c^{\odot} \\ &= 0.5\% + 0.5\dot{v}_c^{\odot},\end{aligned} \tag{25}$$

and

$$\begin{aligned}\dot{H}^{\odot} &= Sun_o + sun_p \cdot \dot{v}_h^{\odot} \\ &= 0.1\% + \dot{v}_h^{\odot}.\end{aligned} \tag{26}$$

Based on the above Basic Configuration of parameters in the 12 equations, we have the following solution:

$$\begin{aligned}\dot{P}^{\odot} &= 5.18\%,\ \dot{y}^{\odot} = 0.16\%; \\ \dot{r}^{\odot} &= 0.67\%,\ \dot{K}^{\odot} = -1.17\%; \\ \dot{w}^{\odot} &= -2.79\%,\ \dot{L}^{\odot} = 0.29\%; \\ \dot{v}_o^{\odot} &= -0.84\%,\ \dot{O}^{\odot} = 0.08\%; \\ \dot{v}_c^{\odot} &= -0.84\%,\ \dot{C}^{\odot} = 0.08\%; \\ \dot{v}_h^{\odot} &= -0.96\%,\ \dot{H}^{\odot} = -0.86\%.\end{aligned} \tag{27}$$

We now alter the above Basic Configuration by a singular variation in the exogenous supply of oil from $Oil_o = 0.5\%$ to -1%; then the new solution becomes:

$$\begin{aligned}\dot{P}^{\odot} &= 5.27\%,\ \dot{y}^{\odot} = 0.08\%; \\ \dot{r}^{\odot} &= 0.71\%,\ \dot{K}^{\odot} = -1.21\%; \\ \dot{w}^{\odot} &= -2.82\%,\ \dot{L}^{\odot} = 0.32\%; \\ \dot{v}_o^{\odot} &= 0.65\%,\ \dot{O}^{\odot} = -0.68\%; \\ \dot{v}_c^{\odot} &= -0.85\%,\ \dot{C}^{\odot} = 0.07\%; \\ \dot{v}_h^{\odot} &= -0.97\%,\ \dot{H}^{\odot} = -0.87\%.\end{aligned} \tag{28}$$

Consider now another singular variation in k_o from 2% in the Basic Configuration to 4%, indicating a technological shift to more capital using (or labor saving); then the new solution becomes:

$$\begin{aligned} \dot{P}^{\odot} &= 5.73\%,\ \dot{y}^{\odot} = -0.28\%; \\ \dot{r}^{\odot} &= 0.89\%,\ \dot{K}^{\odot} = -1.39\%; \\ \dot{w}^{\odot} &= -3.93\%,\ \dot{L}^{\odot} = -0.57\%; \\ \dot{v}_o^{\odot} &= -0.95\%,\ \dot{O}^{\odot} = 0.03\%; \\ \dot{v}_c^{\odot} &= -0.95\%,\ \dot{C}^{\odot} = 0.03\%; \\ \dot{v}_h^{\odot} &= -1.03\%,\ \dot{H}^{\odot} = -0.93\%. \end{aligned} \tag{29}$$

Our third simulation will be about an "oil crisis" through a composite variation from the Basic Configuration: to alter oil_p from 0.5 to 0 indicating that the oil supply does not respond to its price any more, to alter σ_{KOvo} from 0.5 to 0 indicating that capital machineries cannot substitute for oil any more, and lastly Oil_o from 0.5% to -1% indicating that there is an exogenous decline in the supply of oil; then the new solution becomes:

$$\begin{aligned} \dot{P}^{\odot} &= 5.31\%,\ \dot{y}^{\odot} = 0.05\%; \\ \dot{r}^{\odot} &= 0.72\%,\ \dot{K}^{\odot} = -1.22\%; \\ \dot{w}^{\odot} &= -2.83\%,\ \dot{L}^{\odot} = 0.33\%; \\ \dot{v}_o^{\odot} &= 27554,\ \dot{O}^{\odot} = -1\%; \\ \dot{v}_c^{\odot} &= -0.86\%,\ \dot{C}^{\odot} = 0.07\%; \\ \dot{v}_h^{\odot} &= -0.97\%,\ \dot{H}^{\odot} = -0.87\%. \end{aligned} \tag{30}$$

Our last simulation will be a variation of the 4×4 matrix of input mutual substitutability as embedded in the four equations, (19), (20), (21), and (22), for the cost-minimizing input demands, from the Basic Configuration

$$\begin{array}{ccccc} \sigma & w & vo & vc & vh \\ K/L & 1 & 0 & 0 & 0 \\ K/O & 0 & 0.5 & 0 & 0 \\ K/C & 0 & 0 & 0.5 & 0 \\ K/H & 0 & 0 & 0 & 0.5 \end{array} \tag{31}$$

to the following more sensible matrix entries,

$$\begin{array}{ccccc} \sigma & w & vo & vc & vh \\ K/L & 1 & 0.2 & 0.2 & 0.1 \\ K/O & 0.2 & 0.5 & -0.2 & -0.2 \\ K/C & 0.2 & -0.2 & 0.5 & -0.2 \\ K/H & 0.2 & -0.2 & -0.2 & 0.5 \end{array} \tag{32}$$

where (input i/ input j) is more responsive to (price j/ price i) as shown by the dominant diagonal entries. Otherwise, since the raw materials of oil, coal, and solar energy substitute more for capital machineries than for labor, as their prices increase, the demand for capital relative to labor increases, accounting for the first row in the matrix (with less substitutability between capital and solar energy, shown as $\sigma_{KLvh} = 0.1$). For the second row, labor substitutes more for capital machineries than for oil, thus $\sigma_{KOw} = 0.2$; however, coal and the solar energy substitute more for oil than for capital machineries, thus, $\sigma_{KOvc} = \sigma_{KOvh} = -0.2$. The third and fourth rows follow the same argument. Then the new solution becomes:

$$\begin{aligned} \dot{P}^{\odot} &= 4.99\%,\ \dot{y}^{\odot} = 0.31\%; \\ \dot{r}^{\odot} &= 0.59\%,\ \dot{K}^{\odot} = -1.09\%; \\ \dot{w}^{\odot} &= -2.40\%,\ \dot{L}^{\odot} = 0.59\%; \\ \dot{v}_o^{\odot} &= -0.77\%,\ \dot{O}^{\odot} = 0.12\%; \\ \dot{v}_c^{\odot} &= -0.77\%,\ \dot{C}^{\odot} = 0.12\%; \\ \dot{v}_h^{\odot} &= -0.90\%,\ \dot{H}^{\odot} = -0.80\%. \end{aligned} \tag{33}$$

2.3 Discussion

Remark 2 *The feature of "non-renewable but relatively clean" for our generically termed "oil" was reflected by setting $o_o = -0.5\%$ to show an exogenous shift in the production technology to substitute capital machineries for oil ("making the machines more energy efficient") and by setting $Oil_o = 0.5\%$ to show a continuing increase in the exogenous supply of oil.*

Remark 3 *The feature of "relatively abundant but polluting" for our generically termed "coal" was reflected by setting $c_o = -0.5\%$ to show an exogenous shift in the production technology to substitute capital machineries for coal to reduce its pollution to the environment and by setting $Coal_o = 0.5\%$ to show a continuing increase in the exogenous supply of oil due to its relative abundance.*

Remark 4 *The feature of "inexhaustible and clean" for our generically termed "solar energy" was reflected by setting $h_o = 0.5\%$ to show an exogenous shift in the production technology to substitute solar energy for capital machineries (such as alternative architectural designs of buildings) and by setting $Sun_o = 0.1\%$ to show a continuing increase in the exogenous supply of solar energy independent of its price v_h.*

Remark 5 *Our above simulation of an "oil crisis" showed, rather surprisingly, that other than a skyrocketing increase in the oil price,* $\dot{v}_o^\odot = 27554$, *the economy continued to function smoothly. In fact, even mathematically our linear system of equations suffered only a removable discontinuity, of the form* $\frac{0}{0}$ *with a limiting value of* $\dot{v}_o^\odot$ *equal to* 27554, *i.e.,*

$$\dot{v}_o^\odot \to 27554, \text{ as } oil_p \to 0 \text{ and } \sigma_{KOvo} \to 0 \text{ at } Oil_o = -1\%. \tag{34}$$

Remark 6 *The equation in our simulation of the production function of the economy*

$$\begin{aligned} \dot{y}^\odot &= \eta_K \dot{K}^\odot + \eta_L \dot{L}^\odot + \eta_O \dot{O}^\odot + \eta_C \dot{C}^\odot + \eta_H \dot{H}^\odot + \dot{A}^\odot \\ &= 0.4\dot{K}^\odot + 0.4\dot{L}^\odot + 0.09\dot{O}^\odot + 0.009\dot{C}^\odot + 0.001\dot{H}^\odot + 0.5\% \end{aligned}$$

had

$$\eta_K + \eta_L + \eta_O + \eta_C + \eta_H = 0.9 < 1, \tag{35}$$

showing a decreasing return to scales, with $1 - 0.9 = 10\%$ *of the gross domestic product* y *appearing as the excess profits due to the existence of some non-replicable factors in the production process, i.e., "quasi-rent."*

Remark 7 *From standard economics one has, in equilibrium,*

$$\eta_K \quad : \quad = \frac{\partial y}{\partial K}\frac{K}{y} = \frac{rK}{y}, \text{ so that} \tag{36}$$

$$\frac{\frac{d\eta_K}{dt}}{\eta_K} \equiv \dot{\eta}_K^\odot = \dot{r}^\odot + \dot{K}^\odot - \dot{y}^\odot, \text{ i.e.,} \tag{37}$$

$$\frac{d\eta_K}{dt} = F_1(\eta_K, \eta_L, \eta_O, \eta_C, \eta_H). \tag{38}$$

Similarly one has

$$\frac{d\eta_L}{dt} = F_2(\eta_K, \eta_L, \eta_O, \eta_C, \eta_H), \tag{39}$$

$$\frac{d\eta_O}{dt} = F_3(\eta_K, \eta_L, \eta_O, \eta_C, \eta_H), \tag{40}$$

$$\frac{d\eta_C}{dt} = F_4(\eta_K, \eta_L, \eta_O, \eta_C, \eta_H), \text{ and} \tag{41}$$

$$\frac{d\eta_H}{dt} = F_5(\eta_K, \eta_L, \eta_O, \eta_C, \eta_H). \tag{42}$$

In our above Basic Configuration we had

$$\begin{aligned} \dot{\eta}_K^\odot &= \dot{r}^\odot + \dot{K}^\odot - \dot{y}^\odot = 0.67\% - 1.17\% - 0.16\% = -0.66\%, \\ \dot{\eta}_L^\odot &= \dot{w}^\odot + \dot{L}^\odot - \dot{y}^\odot = -2.79\% + 0.29\% - 0.16\% = -2.66\%, \\ \dot{\eta}_O^\odot &= \dot{v}_o^\odot + \dot{O}^\odot - \dot{y}^\odot = -0.84\% + 0.08\% - 0.16\% = -0.91\%, \\ \dot{\eta}_C^\odot &= \dot{v}_c^\odot + \dot{C}^\odot - \dot{y}^\odot = -0.84\% + 0.08\% - 0.16\% = -0.91\%, \text{ and} \\ \dot{\eta}_H^\odot &= \dot{v}_h^\odot + \dot{H}^\odot - \dot{y}^\odot = -0.96\% - 0.86\% - 0.16\% = -1.97\%, \end{aligned}$$

so that the income distribution over the five production factors was not in equilibrium (suggesting incidentally a deterministic cause for the phenomenon of business cycles). Whether the above $F_j(\eta_K, \eta_L, \eta_O, \eta_C, \eta_H) = 0$, $j = 1, 2, .., 5$, *in Equations* (38), (39), (40), (41), *and* (42), *have positive solutions for* η_K, η_L, η_O, η_C, *and* η_H *with an asymptotically stable general equilibrium depends clearly on the parameters.*

3 Summary Remark

From our above simulation, we see that the key to a global economic prosperity lies in the existence of abundant supplies of environment-friendly energies along with an overall technological growth, which falls in the domain of physical sciences, in particular, chemistry. Otherwise, we have once again demonstrated the utility of our proposed analytical tool of the relative derivative. To be sure, one can easily extend the production inputs beyond the five factors as included in the model of this paper.

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