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New Sequential Partial Update Normalized Least Mean M-estimate Algorithms for Stereophonic Acoustic Echo Cancellation

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ABSTRACT

This paper proposes a family of new robust adaptive filtering algorithms for stereophonic acoustic echo cancellation in impulsive noise environment. The new algorithms employ sequential partial update scheme to reduce computational complexity, which is desirable in long echo path case. On the other hand, by employing robust M-estimate technique, the new algorithms become more robust to impulsive noises compared to their conventional least square-based counterparts. These two advantages enable the proposed algorithms to be good alternatives for stereophonic echo cancellation. Experiments are also conducted to verify their efficiency.

INTRODUCTION

In recent years, with the rapid development of digital communication, signal processing and VLSI techniques, there is an increasing demand for both the methods and quality of speech communication. Among the prevailing ones, hands-free communication systems, such as vedio/tele-conferencing, personal mobile phone, and home entertainment devices are finding numerous applications in this trend. Stereophonic speech communication system plays an important part in these systems. It consists of two microphones and two speakers, which enables the user to acquire an "immersive experience" [1] with higher sound reality and finer source localization ability. To enhance the speech communication quality, stereophonic acoustic echo cancellation (SAEC) (depicted in Fig. 1) [2] is one of the main tasks. Its kernel, the echo canceller, is known as a two channel adaptive filtering algorithm and basically much more difficult than the monophonic case [2]. This is because the near-end two speaker signals originate from a common source in the far-end room, which introduces a strong correlation between the transmitted stereo signals and thus prevents algorithm's normal convergence. In literature, a lot of efforts have been made to decorrelate the two channel input signals so as to mitigate the convergence problem [3]-[6]. In fact, there are a few other problems that are also very crucial to SAEC problem. In this paper, we focus on another important aspect of SAEC research which is less comprehensively explored: to derive a family of partial update robust algorithms with both computational simplicity and robustness to impulsive noises [7]. Accordingly, here we will employ two key techniques: (1) sequential partial update (PU) scheme, which is stable and easy to implement; (2) robust statistics technique, which fundamentally solves the fat tail signal distribution problem [8] and is proved effective in combating impulsive noises in a wide range of applications.

In acoustic echo cancellation, higher order adaptive filters are usually required to model the acoustic paths with long impulse responses. For SAEC, this computational load is especially heavy in considering of the two paths to be estimated. Among the various complexity reduction schemes, PU [9]–[14] is very efficient. It only updates a portion of the filter coefficients at each iteration. Due to its simplicity and stability, PU is very attractive for implementation in hardware, VLSI, and digital signal processors. There are generally two categories of PU adaptive filtering algorithms. The first one updates the coefficients using certain data-dependent selection criteria, such as [9]–[11]. They usually converge faster than those using fixed updating strategies, but with increased computational complexity due to coefficient selection. Moreover, it may also encounter convergence problem for non-stationary signals due to data-dependent updating [14]. On the other hand, the second category of algorithms uses pre-determined updating schemes to update the filter coefficients. Representative algorithm is the sequential LMS (S-LMS) algorithm [12], [13] which partitions the coefficients into non-overlapping groups and updates them sequentially at each iteration. These algorithms are simpler to implement and are found to be more stable for certain non-stationary signals than the first class of algorithms [14]. Recently, another variant of the S-LMS algorithm was developed: the stochastic partial update LMS

(SPU-LMS) algorithm [14], which randomly schedules coefficient updating, has a further improved stability in face of nonstationary input signal. Thanks to the structural similarity of these sequential PU algorithms, in this paper, we can work within the same framework to apply them into SAEC problem. Without losing generality, only their normalized versions are considered.

Nextly, as for the robustness issue, the above mentioned conventional sequential PU algorithms are not immune to impulsive noises which are widely encountered in speech communication systems. This is becauses they belong to the LMS algorithm family which is derived from Gaussian-distributed signal assumption. Their performances will deteriarite significantly in impulsive noise environment (non-Gaussian). To improve their robustness, we introduce the robust M-estimate technique [8] to develop a new set of sequential PU algorithms for SAEC which are more robust in combating impulsive noises than their LMS counterpart.

The rest of this paper is organized as follows: in section 2, the SAEC problem and the sequential PU algorithms are introduced. In section 3, the proposed robust algorithms are derived. Simulation results are given in section 4 and conclusions are drawn in section 5.

Sequential PU NLMS algorithms

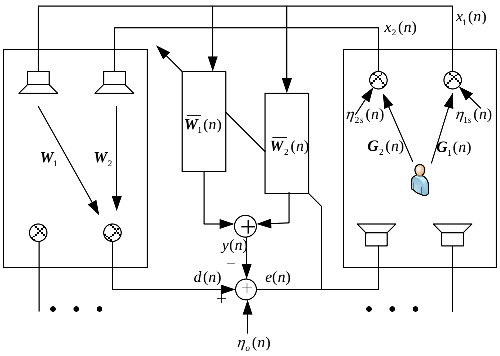


Figure 1. Stereophonic Adaptive Echo Cancellation (SAEC)

Fig. 1 depicts the SAEC problem. For simplicity, only one acoustic echo path is considered here. For telecommunications between the far-end and near-end rooms, the voice of the speaker in the far-end room passes through two acoustic paths $\boldsymbol{G_i}(n)$, $i=1,2$, and is then picked up by two microphones. $\eta_{is}(n)$, $i=1,2$, represents the measurement noise plus possible interference. The stereo signals $x_i(n)$ transmitted to the near-end room are thus mutually correlated. $x_i(n)$ are convolved with two echo paths $\boldsymbol{W_i}$, $i=1,2$ with order L and then picked up by the microphone as the echo $d(n)$. In the echo canceller, two adaptive filters $\overline{\boldsymbol{W}}_{\boldsymbol{i}}(n)$, $i=1,2$ with order L are employed to model $\boldsymbol{W_i}$ and generate an echo estimate. $\eta_o(n)$ represents the microphone measurement noise. To decorrelate the input signals $x_i(n)$, various schemes were proposed and among them the nonlinearity method [3] has gained wide application. Its main principle is to add a half-wave rectified signal to the original signal $x_i(n)$:

$$x_1'(n)=x_1(n)+\tfrac{1}{2}\alpha[x_1(n)+|x_1(n)|], \quad (1)$$

$$x_2'(n)=x_2(n)+\tfrac{1}{2}\alpha[x_2(n)-|x_2(n)|], \quad (2)$$

where α determines the amount of added distortion. Various practices suggest that satisfactory decorrelation can be achieved with no significant sound distortion when $\alpha\in(0.3,0.5)$ [3].

The two channel normalized least mean square (TCNLMS) algorithm with above input decorrelation scheme can be summarized as follows:

$$\boldsymbol{X}_i'(n)=[x_i'(n),x_i'(n-1),...,x_i'(n-L+1)],\ i=1,2, \quad (3)$$

$$d(n)=\boldsymbol{X}_1'^T(n)\boldsymbol{W}_1+\boldsymbol{X}_2'^T(n)\boldsymbol{W}_2+\eta_o(n), \quad (4)$$

$$y(n)=\boldsymbol{X}_1'^T(n)\overline{\boldsymbol{W}}_1(n)+\boldsymbol{X}_2'^T(n)\overline{\boldsymbol{W}}_2(n), \quad (5)$$

$$e(n)=d(n)-y(n), \quad (6)$$

$$\overline{\boldsymbol{W}}_i(n+1)=\overline{\boldsymbol{W}}_i(n)+\frac{\mu\boldsymbol{X}_i'(n)e(n)}{\boldsymbol{X}_1'^T(n)\boldsymbol{X}_1'(n)+\boldsymbol{X}_2'^T(n)\boldsymbol{X}_2'(n)+\varepsilon}, \quad (7)$$

where μ is the step size parameter and ε is a small positive value used to avoid division by zero.

Integrating sequential PU schemes in [12]-[14] into (1)–(7) yields a family of two channel sequential PU NLMS algorithms which we collectively call TCSNLMS algorithms:

$$\overline{\boldsymbol{W}}_i(n+1)=\overline{\boldsymbol{W}}_i(n)+\frac{\mu\boldsymbol{S_X}(n)\boldsymbol{X}_i'(n)e(n)}{\boldsymbol{X}_1'^T(n)\boldsymbol{X}_1'(n)+\boldsymbol{X}_2'^T(n)\boldsymbol{X}_2'(n)+\varepsilon} \quad (8)$$

where $\boldsymbol{S_X}(n)=\mathrm{diag}[s_1(n),....,s_L(n)]$; $s_i(n)\in\{0,1\}$, $i=1,2,\ldots,L$, is a selection matrix. In the S-LMS algorithms, $\boldsymbol{W}(n)$ is divided into C non-overlapping groups which are updated sequentially. The elements of $\boldsymbol{S_X}(n)$ are thus $P=L/C$ equally spaced or consecutive 1's (and 0's elsewhere) and they are shifted cyclically as time propagates. Consequently, only P coefficients are updated per iteration. At time instant *n*, when $s_i(n)$ is equal to one, the corresponding element $w_i(n)$ in $\boldsymbol{W}(n)$ will be updated. $\boldsymbol{S_X}(n)$ for the TCSNLMS, the TCSBNLMS (two channel sequential block NLMS), and the TCSPUNLMS algorithms are summarized in Tables 1.

Algorithms	$\boldsymbol{S_X}(n)$
TCSNLMS	$s_i(n)=\begin{cases}1, & (n+i)\bmod C=0\\ 0. & \text{otherwise}\end{cases}$
TCSBNLMS	$\boldsymbol{S_X}(n)=\mathrm{diag}([\boldsymbol{s}_1^T(n),\boldsymbol{s}_2^T(n),\ldots,\boldsymbol{s}_C^T(n)]^T)$, $\boldsymbol{s}_j(n)=[s_{(j-1)\frac{L}{C}+1},s_{(j-1)\frac{L}{C}+2},\ldots,s_{j\frac{L}{C}}]^T$, $j=1,2,\ldots,C$, $\boldsymbol{s}_j(n)=\begin{cases}[1,1,\ldots,1]^T, & n\bmod C+1=j\\ [0,0,\ldots,0]^T. & \text{otherwise}\end{cases}$
TCSPUNLMS	$r(n)$ is randomly selected from $[1,2,\ldots,C]$, $s_i(n)=\begin{cases}1, & (n+r(n)+i)\bmod C=0\\ 0. & \text{otherwise}\end{cases}$

Table 1. Structure of the TCSNLMS algorithms

The advantage of the TCSNLMS algorithms is their flexibility in compromising the convergence speed and computational complexity. In some situations, desired convergence speed can be achieved with just a mild computation load.

Given its efficiency and easiness for implementation, the TCSNLMS algorithms for SAEC has one outstanding disadvantage: since it is based on least square (LS) criteria like the conventional LMS algorithm, its performance will degrade considerably when the desired and/or input signals are corrupted by impulsive noise [7] which is commonly encountered in speech communication systems (i. e. in Fig. 1 $\eta_o(n)$ could mix with impulsive noise). Many techniques have been proposed to combat the adverse effects of impulsive noise on adaptive filters. Among them, the approaches based on robust statistics [8] have been proved very effective. A typical example is the least mean M-estimate (LMM) [15] algorithm. It is derived from robust M-estimate [8] and can be viewed as the robust counterpart of the LMS algorithm. Its improved robustness in impulsive noise environment has been thoroughly discussed in [15]. Here, we apply this technique to the TCSNLMS algorithms to obtain new algorithms with improved robustness to impulsive noise. In the next section, we shall introduce the M-estimate concept and then derive the new algorithms.

Sequential PU NLMM algorithms

M-estimate

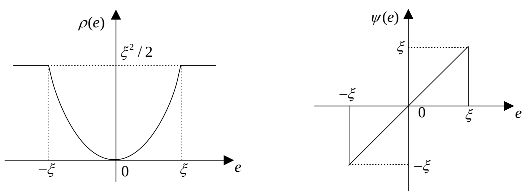


Figure 2. Modified Huber M-estimation

By minimizing the modified Huber (MH) M-estimate function $J_\rho = E[\rho(e(n))]$ [8] instead of the conventional least mean square criterion $J_{\text{LMS}} = E[e^2(n)]$, we get the score function $\psi(e) = \partial\rho(e)/\partial e$ and $q(e) = \psi(e)/e$ as illustrated in Fig. 2. $\psi(e) = q(e)e$ can thus be incorporated into various algorithms to generate the corresponding robust algorithms, e. g. the least mean M-estimate (LMM) algorithm [15] with respect to the LMS algorithm:

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \mu q(e(n))e(n)\boldsymbol{X}(n) . \quad (9)$$

Obviously, when $\eta_o(n)$ is impulsive, $e(n)$ will become larger than the threshold ξ, and $\psi(e)$ will become zero and prevent the weight vector from updating. Thus the resultant algorithms can effectively reduce the adverse effect of large estimation error due to impulsive noise. The threshold parameter ξ should be estimated adaptively using the following method [15]:

$$\xi = 2.576\hat{\sigma}_e(n) , \quad (10)$$

$$\hat{\sigma}_e^2(n) = \lambda_\sigma \hat{\sigma}_e^2(n-1) + c_1(1-\lambda_\sigma)\text{med}(A_e(n)) , \quad (11)$$

$$A_e(n) = \{e^2(n), \cdots, e^2(n-N_w+1)\} , \quad (12)$$

where med(·) is the median operator, $c_1 = 2.13$, $0 << \lambda_\sigma < 1$, $5 \le N_w \le 9$. The computational cost for performing the median filtering (additional cost compared to conventional algorithms) is $O(N_w \log N_w)$ operations per iteration which is relatively trivial.

We now analyze eq. (10)–(12) and discuss the optimal selection of c_1. Eq. (11) is in fact a combination of a moving median smoother and an exponential estimator. According to order statistics theory [16], in order to get rid of the impulsive interference in $e(n)$, a median filtering operation $\text{med}_{N'+1}(e^2(n))$ with window length $\frac{N'}{2}+1$ can be applied (It equals to an order statistics operation with order $\frac{N'}{2}+1$, where N' is even and $N_w = \frac{N'}{2}+1$). Compared to the original signal, the filtered signal has a slight variation in its probability density function (PDF) [16, 17]. Therefore, we can assume that eq. (11) can obtain an approximately accurate estimate of the power of the error signal which is free from contaminating impulsive noise. Moreover, this estimate is very close to the result of Tukey in [18] obtained through robust estimation using median filter on mass data. More specifically, from [19] we know for white input signal, the output of moving median filter has the following asymptotic mean value and variance, respectively:

$$\mu_{\text{med}} = t_{0.5} , \text{ and } \sigma_{\text{med}}^2 = 1/[4(N'+1) f_x^2(t_{0.5})] , \quad (13)$$

where $t_{0.5}$ is the median value of the PDF, and $f_x(t_{0.5})$ represents the value of the pdf at that point. From (13) it can be found that for the moving median filtering smoother, its estimate of the mean value of the signal PDF is consistent and unbiased (as $\lim_{N\to\infty} \sigma_{\text{med}}^2 = 0$). (13) also suggests the estimate is quite robust. This is because σ_{med}^2 is only proportional to $1/f_x^2(t_{0.5})$ but not to the variance of the signal it processes.

Since impulsive noise-free error signal is Gaussian-distributed with zero mean and variance $\sigma_e^2(n)$, we can assume $e^2(n)$ satisfies Chi-square distribution with dimension $k = 1$ (its signal power asymptotically converges to $\sigma_e^2(\infty)$). Hence, taking mathematical expectation on both sides of (11) yields:

$$E[\hat{\sigma}_e^2(n)] = \lambda_\sigma E[\hat{\sigma}_e^2(n-1)] + c_1(1-\lambda_\sigma)E[\text{med}(A_e(n))] , \quad (14)$$

Let $n \to \infty$, one gets

$$E[\hat{\sigma}_e^2(\infty)] = c_1\sigma_e^2(\infty)t_{0.5} , \quad (15)$$

where $t_{0.5}$ is the median value of the signal with Chi-square distritution，and its prototype signal satisfies N～(0,1) Gaussian distribution. Accordingly, employing the median value computation equation for Chi-square distributed signal one can get

$$t_{0.5} = k - \frac{2}{3} + \frac{4}{27k} - \frac{8}{729k^2} \Big|_{k=1} = 0.4705$$

and thus

$$c_1 = 1 / t_{0.5} = 2.13 \,. \quad (16)$$

The above derived value of c_1 differs from $[1.483(1+\frac{5}{N_w - 1})]^2$ in [15]. In the following experiment we can see $c_1 = 2.13$ is more accurate whereas the value in [15] will cause overestimate, which may wrongly ignore the impulsive interference with small amplitude.

TCSNLMM algorithms

Incorporating the techniques derived above into the TCSNLMS algorithms (8), we can get the new TCSNLMM algorithms as follows

$$\overline{\boldsymbol{W}}_i(n+1) = \overline{\boldsymbol{W}}_i(n) + \mu \frac{q(e(n))\boldsymbol{S}_X(n)\boldsymbol{X}_i'(n)e(n)}{\boldsymbol{X}_1'^T(n)\boldsymbol{X}_1'(n) + \boldsymbol{X}_2'^T(n)\boldsymbol{X}_2'(n) + \varepsilon} \,, \quad (17)$$

together with (1)-(6). Correspondingly, this family includes three members: the TCSNLMM, the TCSBNLMM, and the TCSPUNLMM algorithms.

Simulation results

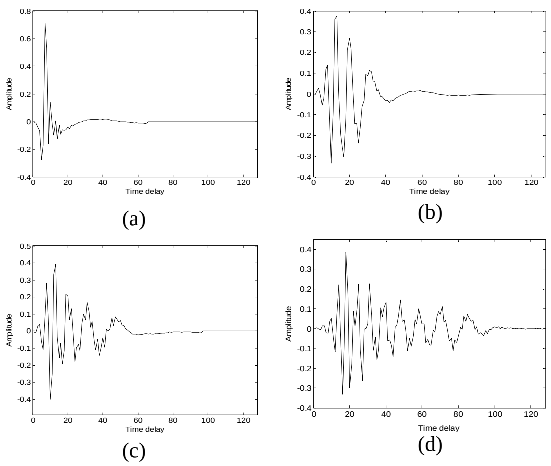


Figure 3. Acoustic and echo paths impulse responses (a) $\boldsymbol{G}_1$, (b) $\boldsymbol{G}_2$, (c) $\boldsymbol{W}_1$, (d) $\boldsymbol{W}_2$

We now verify the efficiency of the proposed TCSNLMM algorithms using computer simulations of SAEC problem. The input signal $x(n)$ is modeled as a speech signal using a first order AR process with the coefficient $a = 0.5$. The four acoustic and echo paths in both the near-end and far-end rooms $\boldsymbol{G}_1$, $\boldsymbol{G}_2$, $\boldsymbol{W}_1$, and $\boldsymbol{W}_2$ are the models given in the ITU-T recommendation G.168 [20] whose impulse responses all have 128 coefficients. They are respectively illustrated in Fig. 3 (a)-(d). The step sizes of the TCSNLMS and TCSNLMM algorithms are all set to be 0.5. All curves are the averaged results of 200 independent runs.

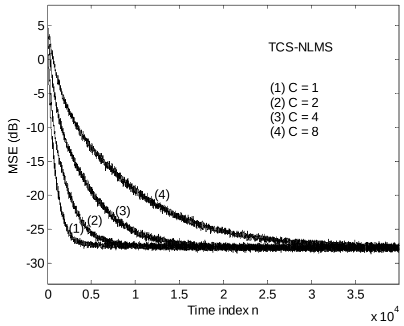
Firstly, we examine the role of the decimation factor C in the TCSNLMS algorithms. The measurement noise $\eta_o(n)$ is now a Gaussian white noise with SNR=35dB. $C = 1,2,4,8$ are tested and the results are shown in Fig. 4. The TCSNLMS, the TCSBNLMS, and the TCSPUNLMS algorithms all exhibit the similar performance with regard to various C value. The convergence speed decreases with the less weight coefficients being updated.

Next, we verify the robustness of the TCSNLMM algorihtms. The measurement noise $\eta_o(n)$ is now impulsive and modeled as the contaminated Gaussian (CG) type impulsive noise [7]. More precisely, it is given by:

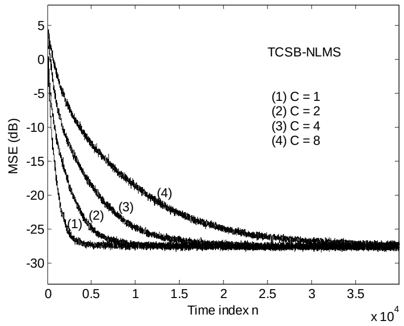
$$\eta_o(n) = \eta_g(n) + \eta_{im}(n) = \eta_g(n) + b(n)\eta_w(n) \,, \quad (18)$$

where $\eta_g(n)$ and $\eta_w(n)$ are both independent and identically distributed (i.i.d.) zero mean Gaussian sequences with respective variance σ_g^2 and σ_w^2. $b(n)$ is an i.i.d. Bernoulli random sequence whose value at any time instant is either zero or one, with occurrence probabilities $P_r(b(n)=1) = p_r$ and $P_r(b(n)=0) = 1 - p_r$. The variances of the random processes $\eta_{im}(n)$ and $\eta_o(n)$ are then given by $\sigma_{im}^2 = p_r \sigma_w^2$ and $\sigma_{\eta_o}^2 = \sigma_g^2 + \sigma_{im}^2 = \sigma_g^2 + p_r \sigma_w^2$. The ratio $r_{im} = \sigma_{im}^2 / \sigma_g^2 = p_r \sigma_w^2 / \sigma_g^2$ is a measure of the impulsive characteristic of the CG noise. In this experiment, we set $p_r = 0.005$, $r_{im} = 100$ .Other parameters include $\lambda_\sigma = 0.99$, $N_w = 7$, SNR=35dB. For clear view of the impact of the impulsive noise on the desired signal, the locations of the impulses are fixed and only those at four time instants *n*=5174, 11685, 16463, and 20882 are shown. This is realized by generating one fixed Bernoulli sequence $b(n)$ with $p_r = 0.005$ and using it in all of the independent runs. Fig. 5 depicts the case of *C*=4 (others are omitted due to space limitation) we can clearly see the performances of the TCSNLMS algorithms are significantly degraded by the impulsive noise. In contrast, the TCSNLMM algorithms show improved robustness and quickly assume normal convergence after the instant disturbance of the impulsive noise.

Finally, we study the effects of various value of c_1 on algorithm performance. Here we employ the TCSPUNLMM algorithm with echo path model $L = 24$. All coefficients are randomly generated and normalized to unit energy. Similar impulsive noise as in previous experiments is applied to the desired signal. *C*=4 and the program is only run once. Three values of c_1, 1, $[1.483(1+\frac{5}{N_w - 1})]^2$ (in [15]), and 2.13 obtained in previous sectin are tested. From Fig. 6 it can be observed that when $c_1 = 1$, the error signal power is underestimated, and for $c_1 = [1.483(1+\frac{5}{N_w - 1})]^2$, is overestimated. In contrast, when $c_1 = 2.13$, the estimate is relatively accurate. In fact, under- and overestimated error signal power will both result in the wrong judgement of the presence of impulsive noise (i. e. wrong presence decision and ignoring, respectively) and thus critically deteriariate the algorithm performance.



(a)



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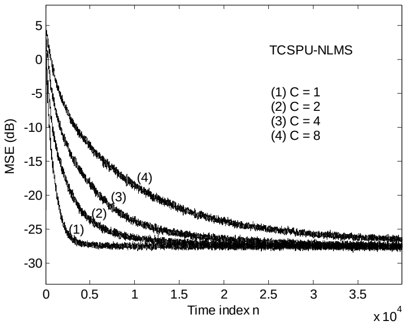
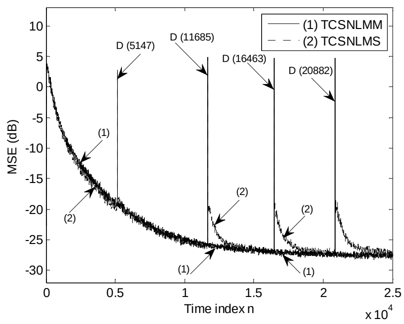
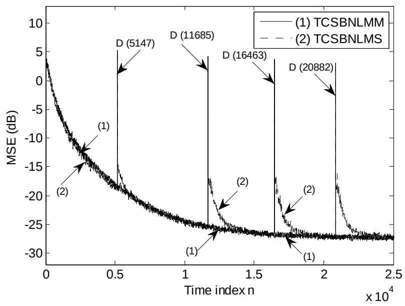


(c)

Figure 4. Convergence performance of TCSNLMS algorithms with various *C* value: (a) TCSNLMS, (b) TCSBNLMS, (c) TCSPUNLMS.



(a)



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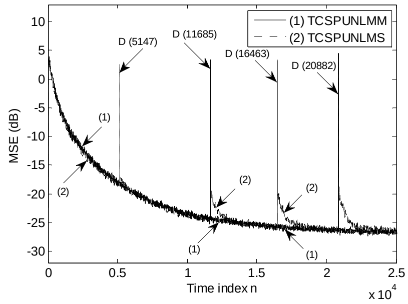
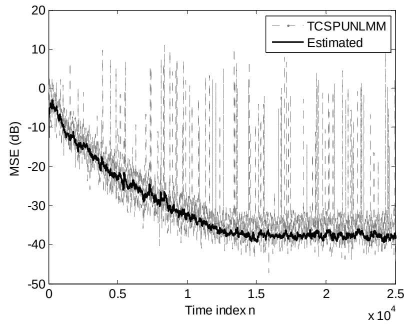
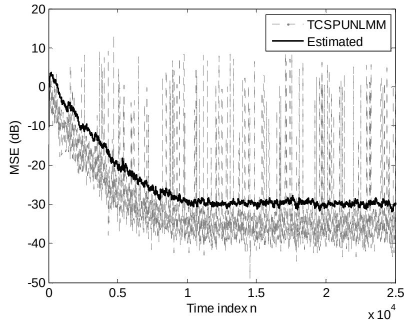


(c)

Figure 5. Convergence performance of TCSNLMS and TCSNLMM algorithms with CG noise: (a) TC-SNLMS/NLMM, (b) TC-SBNLMS/NLMM, (c) TC-SPUNLMS/NLMM.



(a)



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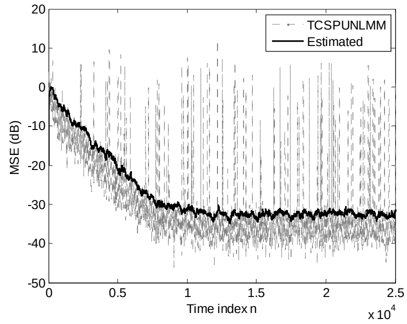

(c)

Figure 6. The effect of c_1 on the estimate of the error signal power: (a) $c_1 = 1$, (b) $c_1 = [1.483(1+\frac{5}{N_w - 1})]^2$, (c) $c_1 = 2.13$.

Conclusion

In this paper, we develop a new family of two channel sequential PU normalized least mean M-estimate algorithms for stereophonic acoustic echo cancellation in impulsive noise. These algorithms are obtained by applying the robust statistics technique into the conventional two channel sequential PU NLMS algorithms.

The resultant algorithms show improved robustness over their NLMS-based counterparts in combating impulsive noise corrupting the desired signal. The corresponding issue on key parameter selection is also discussed.

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