Throughput, Risk, and Economic Optimality of Runway Landing Operations

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Abstract

This paper analyzes the optimal level of operations on a single runway used only for arrivals. Two risks associated with landing procedures are the risk of a wake vortex encounter and the risk of simultaneous runway occupancy. We develop optimization models to maximize successful landing operations while mitigating these risk factors. The risks are mitigated by enforcing *go-around* procedures when separation distances are too small. In our capacity optimization, we assume that the go-around procedures are strictly enforced (making the operations risk-free) and their execution is absolutely safe. We develop two models as decision support tools which mimic the system dynamics and provide new insights into the landing process. One model maximizes the risk-free throughput (number of successful landings per unit of time) with and without wake-vortex effects. The second model accounts for dollar benefits and go-around costs in optimizing the system operations' level. This model maximizes expected net economic outcome (total dollar benefits minus total go-around costs) by adjusting the rate of landing attempts. Through these models, we calculate the maximum (risk-free) achievable throughput in the system. This provides a new definition of the *landing capacity* of the runway taking into account the probabilistic behavior of operations. Several numerical examples are given.

1 Introduction

There is increasingly high demand for runway slots (landing or departure) during peak periods at congested airports. The expense of runways and the high demand have made them very limiting and economically valuable resources of the air transportation network. The limitation of runway and airport capacity is a major cause of delays in the network. Thus, it is desirable to obtain the maximum possible output of the runway operations. The objective of this paper is to understand the probabilistic dynamics of landing operations and to estimate runway capacity based on these dynamics while also considering go-around procedures to mitigate risk.

There are extensive studies regarding capacity estimations and throughput maximizations of airports, see Gilbo [1], for example. Railsback *et al*. [2] provides an overview of methods and tools used to estimate runway capacities. Our study is focused on landing/approach operations only. Early studies of landing capacity go back to the 1940's when runways became congested and delays became a concern. Capacity is commonly considered a constant rate as the reciprocal of the minimum-allowed / safe-time spacing between aircraft, Bowen, *et al*. [3], and Bell [4]. In a classical capacity study, Blumstein [5] focused on landings under instrument flight rules (IFR) when an aircraft separation is required prior to the final approach and the velocity differences cause loss of capacity. In a more recent study, Lang *et al*. [6] studied the possibility of increasing throughput by using cross-wind information in sequencing the landings on closely spaced parallel runways. They studied the effect of safe reducing wake vortex separation minima. Some other works and tools are used to evaluate the capacity and delay benefits of different operating scenarios, for example Boesel [7].

This study is concerned with optimization of landing operations on a single runway for a given pair of follow-lead aircraft, large-large for example. In this paper, two major safety risks in aircraft landing procedures are considered. These are the risks of a wake vortex (WV) hazard and the risk of simultaneous runway occupancy (SRO). A wake-vortex hazard occurs when the following aircraft enters the wake vortex of its leading aircraft. When the wake is strong enough, the encounter may cause a loss of control, which may result in passenger injuries or even fatalities. SRO risk is the probability that a following aircraft reaches the runway threshold before the leading aircraft exits the runway. SRO is a precursor for a physical collision on the runway. These risks are to be avoided to assure a safe landing. Separation requirements to mitigate these risks are the major constraints on the capacity of the runway.

Existing probabilistic approaches to this problem study the relation between these risks (or

safety) and throughput rate, see for example Xie, et al. [8], Xie [9], Levy [10]. The maximum throughput occurs at the point of maximum allowable risk. For example, Jeddi *et al*. [11] illustrate a methodology for the case of a restricted SRO probability. In that paper, the probabilistic nature of aircraft separation is considered and the maximum throughput is set so that the probability of a SRO equals some pre-specified small value. However, the SRO risk was not completely eliminated from the operations. In this paper, we eliminate these risk factors from the operations by considering "go-around" (GA) or "missed approach" procedures, Nolan [12]. We assume that such procedures are strictly enforced and respected whenever the separation distance is below a specified threshold. The optimal level of operations is determined to maximize the number of successful landings, while also considering (and minimizing) the number of arriving aircraft that must take a go-around. Numerical examples here are based on probability distributions of landing time interval (*LTI*) and runway occupancy time (*ROT*) for Detroit Metropolitan airport estimated in Jeddi *et al*. [13]. These distributions are for the pairs with 3 nmi FAA minimum separation standard [14], [15].

The paper is organized as following. Section 3 explains *GA enforced* or *risk-free* landing operations. Section 3 formulates a model to maximize landing throughput. Section 4 formulates another model to find the optimal level of operations to maximize the economic output. Section 5 presents conclusions and proposals for future research.

Notation

The following notations are used throughout the paper. The input parameters are:

$LTI_{k,k+1}$ landing time interval between aircraft k and $k+1$ measured at the runway threshold in seconds and assumed to have the lower limit L

ROT_k runway occupancy time of aircraft k measured in seconds

R dollar benefit of one successful landing

C expected average cost of a go-around or unsuccessful landing

x_0 minimum wake vortex safe separation of successive aircraft given in seconds

DP1 decision point 1; nmi distance from threshold where pilot/controller decides whether to execute go-around procedure to avoid SRO. This is officially referred to as decision height.

DP2 decision point 2; nmi distance from threshold where pilot/controller decides whether to execute go-around procedure to avoid hazardous wake vortex encounter

The decision variables are

ω landing attempts per hour, i.e., flow rate through the glide slope, and $\omega = 3600/\text{mean}(LTI)$

λ arrival rate to TRACON or, equivalently, the runway throughput rate, landing per hour

p probability of go around P{GA}

2 Risk Free Landing (Go Around Enforced)

It is desired that the chance of a simultaneous runway occupancy or a WV hazard, i.e., moderate or severe encounter, be nearly or exactly zero. In conventional models in the literature, increasing the target separation between successive aircraft decreases these two risks. This relationship is studied by many authors, for example see [6]-[8]. The risk can also be reduced by implementing go-around procedures. For example, if two aircraft will be on the runway at the same time, the trailing aircraft can execute a go-around procedure to avoid a SRO. In reality, the go-around is not always taken.

In this paper, we assume that an aircraft is *always* enforced to execute a go-around whenever separation minima are not or will not be met. This will be discussed later in this article. In addition, we assume perfect information to make this decision. With these assumptions, the risk of a SRO or a wake-vortex hazard is exactly zero, though there is possibly an increase in the number of go-arounds.

Making the system risk-free by enforced GA creates a different dynamic and may change the optimal level of operations, i.e., the best number of attempts per hour. As we show in later sections, the optimal level of attempts per hour depends on the GA probability P{GA}. This section calculates this probability for two cases of with and without wake vortex effect.

In the approach / landing process, two different aircraft flows can be recognized: the flow through the glide slope ω measured in landing *attempts* per hour and the flow through the runway (or simply throughput) λ measured in *actual landings* per hour.
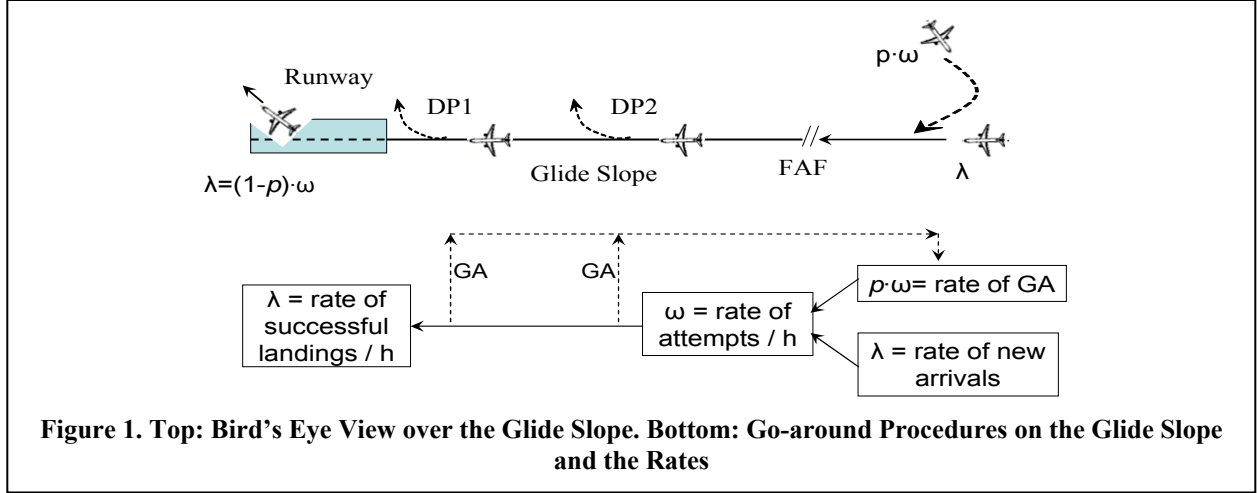
Figure 1. Top: Bird's Eye View over the Glide Slope. Bottom: Go-around Procedures on the Glide Slope and the Rates

Figure 1 demonstrates this dynamic with enforced GA procedures. When the following aircraft is at decision point 2 (DP2), e.g., 8 nmi from runway threshold, the controller/pilot decide(s) whether or not to take a GA procedure to avoid the risk of encountering a hazardous wake from the leading aircraft. We suppose that if the separation is less than a specific value, x_0, at this point, then the following aircraft must go-around (GA) to a holding position and return to the glide slope when cleared to attempt again. Such a minimum WV safe separation exists and can be estimated using wake vortex theories, [9], [16], [17], for example, and/or field observations, Shortle, *et al*. [18]. We name this operation as the "*wake vortex GA*" or "*wake vortex missed approach*" procedure in contrast with the well known GA procedure executed to avoid a SRO. We call the latter a "SRO GA" or "SRO missed approach". In this paper, for illustration purposes, we consider DP2 to be 8 nmi from the threshold, and $x_0 = 65$ s separation as the minimum WV safe separation.

If a safe separation is achieved at DP2, then the aircraft continues the approach. At DP1, which is called the *decision height*, the follower decides whether or not to execute a go-around to avoid simultaneous runway occupancy with the leading aircraft [12].

We define p to be the total GA probability. Note that p is a function of ω, the number of attempts per hour. The average GA rate (number of go-arounds per hour) is $p(\omega) \cdot \omega$ and the average successful landing rate is $\lambda(\omega) = [1-p(\omega)] \cdot \omega$. The rate of aircraft attempting to land is the arrival rate of aircraft $\lambda(\omega)$ plus the rate of aircraft executing a go-around $p(\omega) \cdot \omega$. As a check for consistency, the attempt rate is $[1-p(\omega)] \cdot \omega + p(\omega) \cdot \omega$ which equals ω.

In addition, we make the following *assumptions*:

- $LTI_{k+1,k}$ and ROT_k are independent random variables
- The separation is minimized at DP2 and remains unchanged afterwards until the touchdown. In other words, the separation at DP2 equals *LTI*
- Shifting *LTI* to the right or left does not change the shape of its probability distribution
- Zero risk assumed for execution of both GA procedures
- GA are absolutely respected and enforced at both decision points
- The number of GA in an hour is not restricted
- Wake vortex GA and SRO GA conditions never simultaneously occur for a pair. That is, no simultaneous go around for aircrafts k and $k+1$ happens for all $k=1,2,\ldots$.

Where $LTI_{k,k+1}$ is the landing time interval between aircraft k and $k+1$ measured at the runway threshold (in seconds), and ROT_k is the runway occupancy time of aircraft k (measured in seconds).

2.1 Go-Around Probability Assuming no Wake Vortex Effect

In this section, we ignore the possible go-around at DP2. In other words, we only consider the risk of a SRO and not the risk of a hazardous wake vortex encounter. The probability of a SRO is

3

$$P\{SRO\} = P\{LTI < ROT \ \& \ \text{Follow aircraft lands}\}$$
$$= P\{\text{Follow aircraft lands} \mid LTI < ROT\}$$
$$* P\{LTI < ROT\}.$$

This probability can be reduced to zero by enforcing the go around procedure. In this case, $P\{\text{Follow aircraft lands} \mid LTI < ROT\} = 0$, and

$$p_1(\omega) = P\{GA\} = P\{LTI < ROT\}. \qquad (1)$$

Probability distribution functions of peak period *LTI* and *ROT* are estimated for DTW in [13] for the pairs of FAA 3 nmi minimum separation pairs. These follow-lead aircraft pairs include S-S, L-S, B757-S, H-S, L-L, B757-L, and H-L indicated in [17] and [18]. The estimations are used here for methodology illustrations. *LTI* is the peak period distribution calculated for arrival of aircraft to the glide slope (or the final approach fix) with the rate ω.

Figure 2 illustrates the *LTI* and *ROT* probability distributions obtained in [13] for 3 nmi pairs. $p(\omega)$ is estimated as 0.004, the mean of *LTI* is 106 s, and the average number of attempts per hour (during peak periods) is $\omega = 3600/106 = 34$ attempts/h. In this period, no go-around was observed, so $\omega = \lambda$, but $P\{SRO\} = 0.004$ instead of 0.0.
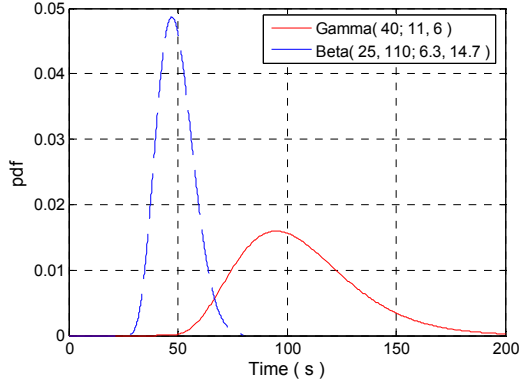
Figure 2. Distributions of *ROT* and *LTI*

In this paper, we assume that changes in arrival rates (that is, changes in λ or ω) can be modeled by shifting the *LTI* distribution to the left or right, i.e., by changing the location parameter. Shifting the *LTI* distribution also changes the probability that *LTI<ROT*, or equivalently, the probability of a go-around. $p(\omega) = P\{LTI < ROT\}$ is given by broken line in Figure 3.

Shifting *LTI* toward left or right, i.e., changing the location parameter, provides different values for ω and $p(\omega)$ assuming other parameters of the

distribution remain unchanged by shifting. $p(\omega) = P\{LTI < ROT\}$ is given by broken line in Figure 3.
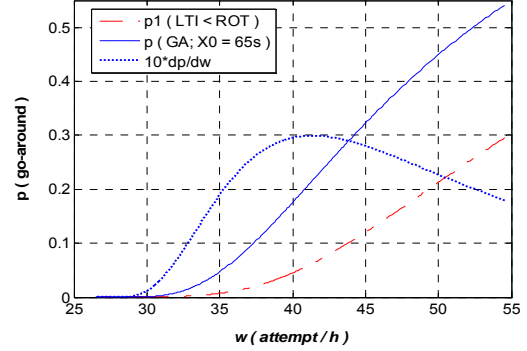
Figure 3. Total P(GA), P(*LTI<ROT*), and *dp/dω* for WV Safe Threshold of 65 s

2.2 Total go-around Probability with Wake Vortex Effect

When the wake vortex effect is taken into account and WV GA procedure is in place, an aircraft would possibly miss the approach for two reasons at two different points: WV safe threshold and runway safe threshold (decision height). For this situation, let x_0 to be the minimum wake vortex safe separation of successive aircraft given in seconds, and *L* to be the lower limit of *LTI* distribution; e.g., $L = 40$ s [13]. To calculate P(GA) two cases for *L* and x_0 shall be considered as follows:

Case 1: $L < x0$. For this case

$$P(GA) = \int_{x_0}^{\infty} F_{LTI}(y) dF_{ROT}(y) \qquad (2)$$
$$+ F_{LTI}(x_0) F_{ROT}(x_0),$$

where F_{LTI} and F_{ROT} are CDF of *LTI* and *ROT*, respectively. Detailed calculations are given in Appendix I.

Case 2: $L >= x_0$. This case means that *LTI* shifted to the right as much that its lower point *L* is above the wake vortex safety threshold of x_0. No wake vortex GA would ever occur in this case, and $P\{GA\}$ is obtained from equation (1).

Total $p(\omega) = P\{GA\}$ is shown in Figure 3 for peak period DTW IMC distributions, which are given in Figure 2, and for $x_0 = 65$ s. This figure also shows p_1 and derivative of $p(\omega)$ in terms of ω. The derivative $dp/d\omega$, which will be used in the following sections, is multiplied by 10 to make it more visible in this figure.

3 Maximizing Runway Throughput

We are interested to find the relation between glide-slope rate of ω attempts/h, runway throughput λ(ω), and go around probability p(ω). Then one can find out for what values of ω and p throughput λ is maximum while P{SRO} = 0 is maintained by enforced GA procedure. In other words, the objective is maximization of the runway throughput. That is,

$$\text{Maximize } \lambda(\omega) = [1\text{-}p(\omega)]\cdot\omega. \qquad (3)$$

This model is the same for both with and without WV effect assumptions. However, p(ω) differs depending on each of these cases, as discussed below.

3.1 Maximum Throughput without Wake Vortex Effect

In this case p(ω) is calculated from equation (1) and plugged into the problem (4) to maximize the throughput. Fig 4 provides λ(ω) in the left axis for the distributions in hand. λ(ω) is calculated for all pairs of (ω, p(ω)) in the right axis. By increasing the rate of attempts ω, the percentage of GA increases but the percentage of successful landings decreases. So that, after a point the decrease in the rate of successful landings dominates the increase in the rate of attempts. In other words, throughput λ(ω) has a unique maximum or optimal point. This can also be explained in mathematical terms. p(ω)>0 is increasing, and 1- p(ω) is decreasing in ω. So after a point, decrease of 1- p(ω) dominates increase of ω and [1-p(ω)]·ω would have a maximum.

For distributions in hand, the optimal (ω,λ,p) is (46.5, 39.6, 0.148), see Figure 4. To have a stable system, the arrival rate to the TRACON, λ, is adjusted so that ω is maintained in the optimal level of 46.5 attempts/h. 39.6 landings/h is the maximum and optimal throughput.

3.2 Maximum Throughput with Wake Vortex Effect

In previous section we analyzed the optimal level of landing attempts when the wake vortex constraint was relaxed and SRO was the only risk factor. In this section, wake vortex risk is also considered in maximization of runway throughput. Using P{GA} = p(ω) from equations (2) and (1) in problem (3) we obtain the solution.

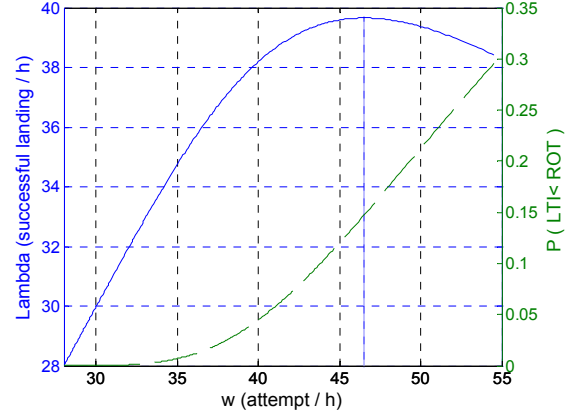**Figure 4. Landing/h (left) and P{GA} (right) vs. Attempts/h (ω)**

Throughput λ(ω) in terms of ω attempts / h is shown in Figure 5 with the solid line (indicated by r=0). Other plots of this figure are the subject of next section. The optimal (ω,λ,p) is about (37.0, 33.5, 0.08) and ω·p(ω) is 3 go-around per h.

We can obtain an interesting result here that WV is costing the system 39.6- 33.5 = 6 landings per peak hour. Thus WV is costing the system about 21,900 landings per year if there is 10 hours of peak period every day. This is for the cases of 3 nmi pairs, which majority of them are large aircrafts. Now we can multiply this value by R and obtain dollar value WV cost estimation. For R = $1000, we have about $22 m per year for a single landing runway under IMC.
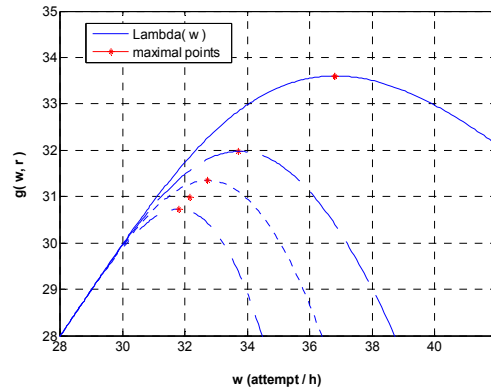

Figure 5. λ(ω) and g(ω;r) for r = 1, 2, and 4 from Top to the Bottom, Respectively

4 Maximizing Net Economic Gain

Maximizing the number of (successful) landings does not necessarily guarantee the overall economic optimality of the landing operations. This is because costs and gains from the operations are important optimality parameters and will be taken into account. For the landing operations when absolute safety is guaranteed by enforced GA procedure, the economic profit/benefit to the regional/whole economy (including airlines, airport, employees, etc) is the result of a successful landing. The overall cost to the regional/whole economy associates with the go-around procedure. The costs of successful landings are embedded in the landing profits, that is, total revenue to all stakeholders minus operational cost, except the cost of GA execution. The net economic gain or surplus, that is, total gain minus total cost of an hour of peak period operations, is desired to be maximized with respect to the number of attempts / h rate, ω. Since this net economic gain is a random variable, we consider maximizing its expected value, $ES = E\{\text{economic surplus}\}$.

The gain from one successful landing is R which occurs with probability $1-p(\omega)$ for every landing attempt. The loss of one landing attempt is the cost of go-around C which occurs with probability $p(\omega)$. Thus, since the number of attempts per hour is ω, then the expected value of the net gain from hourly landing attempts is $ES(\omega;R,C)$ given in (4) and the optimization objective is

$$\text{Maximize} \quad ES\left(\omega; R, C\right)$$
$$= \omega \cdot \left[\left(1 - p\left(\omega\right)\right) \cdot R - p\left(\omega\right) \cdot C\right] \quad (4)$$

We consider some dollar values for R and C to illustrate economic behavior of the system. For any given type of aircraft, C is the summation of cost components such as passenger delay cost, disturbed schedules cost of downstream flights, take off cost, aircraft operations cost, and airport cost. Any of these cost components depend on parameters such as aircraft load factors and the arrival rate at a given time, which are uncertain. Thus, C is a random variable. However, we consider its expected value as a suitable estimation of this parameter. Estimation of R and C is our ongoing research. However, at the time being and for the sake of illustration, we consider $C = \$4,000$ for a large aircraft in a peak hour. Three scenarios of $\$1,000$, $\$2,000$, and $\$4,000$ are considered for R here.

4.1 Economic Optimality without Wake Vortex Effect

For this case $p(\omega)$ is calculated from equation (1), and fed into problem (4). Figure 6 is a plot of $ES(\omega;R,C)$ in thousands of dollars, for assumed values of R and a fixed $C = \$4,000$.

To obtain a more general form, we write ES in terms of the ratio C/R. Factoring out R in equation (2) gives
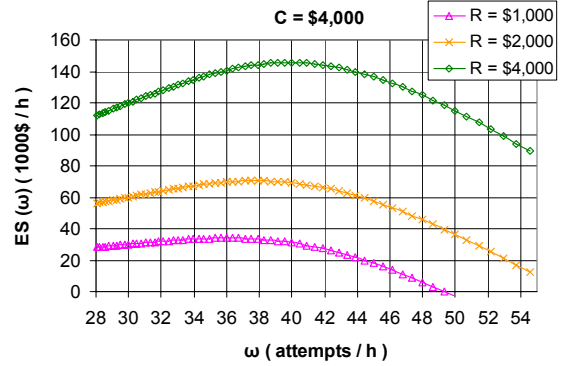

Figure 6. $ES(\omega;R,C)$ of One Hour Peak Period Landing Operations

$$ES\left(\omega; R, C\right) = R \cdot \omega \cdot \left[\left(1 - p\left(\omega\right)\right) - p\left(\omega\right) \cdot \frac{C}{R}\right]$$
$$= R \cdot \omega \cdot \left[1 - \left(1 + \frac{C}{R}\right)p\left(\omega\right)\right].$$

Thus, $ES(\omega;R,C)$ is a multiplication of constant dollar value R and a function of ω. Define the latter function to be $g(\cdot)$ as follows:

$$g\left(\omega; r\right) = \omega \cdot \left[1 - \left(1 + r\right)p\left(\omega\right)\right]$$

or

$$g\left(\omega; r\right) = \lambda\left(\omega\right) - r \cdot \omega \cdot p\left(\omega\right),$$

where $r = C/R$. Thus, maximizing $g(\omega;r)$ is equivalent to maximizing $ES(\omega;R,C)$. So the problem reduces to the more general form of

$$\text{Maximize } g\left(\omega; r\right) = \omega \cdot \left[1 - \left(1 + r\right)p\left(\omega\right)\right]$$
$$= \lambda\left(\omega\right) - r \cdot \omega \cdot p\left(\omega\right) \quad (5)$$

Figure 7 illustrates $g(\omega;r)$ for $r = 0, 1, 2, 4$ and ω in [28,55].
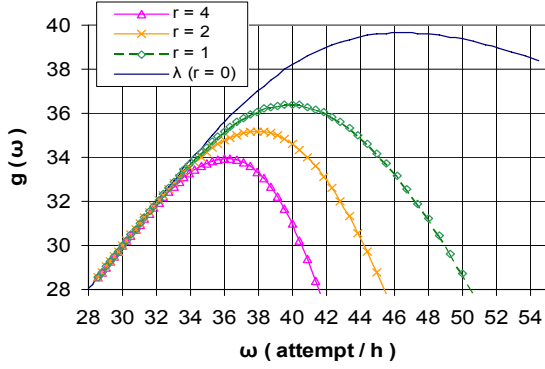
Figure 7. $g(\omega;r)$ for Different Values of Cost to Benefit Ratio. Optimal Throughput Increases as r Decreases

Note that the optimal solution $\omega^*(r)$ = Argmax$\{g(\omega;r)\}$ depends on the ratio $r = C/R$. The derivative of $g(\omega;r)$ with respect to ω is zero at ω^*. $ES(\omega;R,C)$ and $g(\omega;r)$ have the following interesting properties (at least for the *LTI* and *ROT* distributions in hand):

Property 1: For given R and C, $ES(\omega;R,C)$ and $g(\omega;r)$ are unimodal in a practical peak period rates of [28,55] attempts/h. That is, they have a unique maximum in this range. This is the necessary optimality condition. We justify this fact here by visual investigations of Figures 4 and 5 for given DTW peak period landing distributions. However, this fact remains to be proven mathematically. In Fig. 4, $ES(\omega;R,C)$ has a unique maximum for any given R and C. In Figure 5, $g(\omega;r)$ has a unique maximum for any given r.

Property 2: $g(\omega;r)$ decreases as r increases for any fixed ω. This is seen from equation (5). The only term that includes r is the term inside the brackets which is decreasing in r.

Property 3:

$$g(\omega;r) \rightarrow \lambda(\omega) = (1 - p(\omega))\cdot\omega \quad \text{when} \quad r \rightarrow 0^+,$$

which is obvious from equation (5). So, $g(\omega;r)$ is bounded by $\lambda(\omega) = [1- p(\omega)]\cdot\omega$ and is below it, see Fig. 7.

Property 4: $\omega^*(r)$ = Argmax$\{g(\omega;r)\}$ is decreasing in r for $28<\omega<$Argmax$\{dp/d\omega\}$.(This property is proven in Appendix A.)

These properties are seen in Fig. 6 and 7. They are unimodal (property 1). ω^* decreases as r increases (property 2). Argmax$\{dp/d\omega\}$ is calculated 49.7 attempts/h for the *LTI* and *ROT* in hand. $g(\omega;r)$ is below $\lambda(\omega)$. Peak of $g(\omega;r)$ moves down and left by

increasing the relative penalty of go-around to the landing benefit. Properties of $g(\omega;r)$ imply that the highest value for the optimal number of attempts per hour and the upper bound of the optimal throughput is the maximal point of $\lambda(\omega)$. This is achieved when C is much smaller than R.

In such a case, the problem reduces to maximizing $\lambda(\omega) = [1- p(\omega)]\cdot\omega$ scenario, which is discussed in Section 3. The highest throughput value is a good estimation of the average runway landing capacity. For our example, this capacity is 39.6 landings/h for the case that wake vortex effect is ignored, and GA procedure is enforced.

As examples, for $r = 1$, $(\omega, \lambda, p)^* = (40.0, 38.2, 0.045)$. For $r = 2$, $(\omega, \lambda, p)^* = (38, 37.1, 0.024)$. The latter means that to maximize the expected value of the net economic gain (surplus) from the landing operations, when go-around cost C is 2 times larger than landing profit R, the average glide slope throughput shall be adjusted at 38.2 attempts/h which gives 37.0 successful landings/h. Note that in this case, system throughput is 3 landings/h more than the current level of 34 landing/h with associated $P\{SRO\}= 0.004$, see Figure 4.

4.2 Economic Optimality with Wake Vortex Effect

In this case the problem is maximizing (6), or equivalently (4), where $P\{GA\} = p(\omega)$ is calculated from equations (2) and (1). Note that $\omega=3600/mean(LTI)$ where *LTI* is in seconds. All properties of $g(\omega;r)$ in Section 4.1 are still valid for this new $p(\omega)$. Justification procedures are the same that are provided in Section 4.1 and Appendix I. For DTW peak period IMC distributions of 3 nmi pairs derivative of $p(\omega)$ is maximized at 41.3 attempts/h when wake vortex safe threshold is 65 s or 2.2 nmi, see Figure 3. This is the condition for property 4.

The optimal results for DTW distributions are provided in Table 1 and Figure 5 for $x_0 = 65$, 70, and 75 seconds. Figure 8 is visualization of Table 1. The optimal solution $(\omega, \lambda, p)^*$ is (36.8, 33.6, 0.087) for $(x_0,r) = (65, 0)$. Since $r = 0$, that is, cost of go-around is negligible in comparison with the landing profit, then this is the optimal number of landings/h. So the average *landing capacity* of the system is 33.6 landings / h independent of the market condition. $(\omega, \lambda, p)^*$ is (32.7, 32.3, 0.014) for $(x_0,r) = (65,2)$, meaning that the optimal throughput is 32.3 landings / h if $C = 2R$ in the market, that is, the cost of go-around is two times bigger than the profit gained from a successful landing. Note that in Table 1

optimal throughput decreases as safe WV threshold increases; however, eventually the safe threshold is a certain number and once it is recognized other cases become irrelevant.

As a closing discussion some capacity estimations would be helpful. As we mentioned in the introduction of this article, the reciprocal of the minimum safe separation is sometimes considered as an estimation of the capacity. Using that method for the safe minimum separation of 65 s, one obtains the capacity of 55.4 landings per hour. One can intuitively recognize that this number is too high since in practice the capacity is generally between 30 and 40 landings / h. The problem with this method is that it ignores the probabilistic nature of the process. Achieving this level of throughput requires that the mean of *LTI* to be adjusted at 65 s which implies $P\{LTI<ROT\} = 0.30$ from Figure 4. In other words, with the enforced go-around procedure, there will be more than 30% loss of attempted landings at DP1. This would lead to the throughput level of less than 32 landing / h as can be seen in Figure 5. Further, in such an operation the system should tolerate the high cost of 13 go-around / h. So this reciprocal method is not suitable.

Table 1 Optimal Values for Different safe WV threshold and *r*

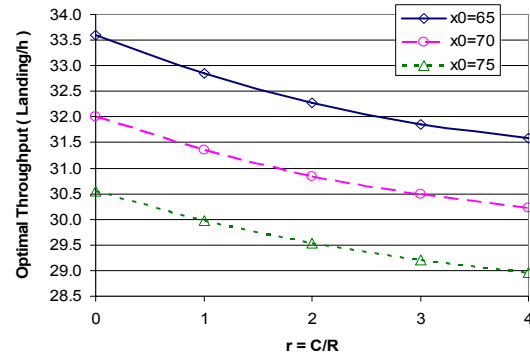| WV threshold	*r*	ω^*	λ^*	*P**
$x_0 = 65$ s	0	36.8	33.6	0.087
	1	33.7	32.8	0.026
	2	32.7	32.3	0.014
	3	32.1	31.9	0.009
	4	31.8	31.6	0.007
$x_0 = 70$ s	0	34.7	32.0	0.079
	1	32.1	31.4	0.024
	2	31.3	30.8	0.013
	3	30.8	30.5	0.009
	4	30.4	30.2	0.006
$x_0 = 75$ s	0	33.0	30.5	0.073
	1	30.7	30.0	0.022
	2	29.9	29.5	0.012
	3	29.5	29.2	0.008
	4	29.1	29.0	0.006

Figure 8. Optimal Throughput in *r* for Safe WV Thresholds of 65, 70, and 75 s

Our methodology, which considers the probabilistic behavior of the system, estimated the average capacity of 33.6 landings per hour for 65 s safe WV threshold and risk free landings. Economic considerations may reduce the optimal throughput to about 32 landings / h with maximized net economic gain, 1% go-around, and risk free (safe) operations.

Conclusion

We studied the landing / approach process to a single independent runway which is used for landing operations only. The goal is to take the most advantage of this scarce and increasingly demanded resource of the air transportation network. However, increasing utilization trade-off with risking safety and human lives. Through this research, we initiated four steps to manage this trade-off and achieve the goal:

1) We proposed a go-around procedure to avoid wake vortex incidents and to assure the landing safety. We suggested that enforced go-around can be economically rational.

2) We suggested that increasing go-around rate (by shortening the average separation spacing) might be helpful to increase the landing throughput. One optimization model developed to mimic this dynamic of the system. The maximization model is solved for the peak period landing distributions of Detroit airport, and the results supported our hypothesis of increased throughput.

3) The aforementioned optimization model estimates *landing capacity* of the runway, with or without the presence of wake vortex effect. These estimates are 39.6 and 33.6 landings per hour, respectively. This provides a logical framework to estimate the economic effect of wake vortex

phenomenon in the system. Using these figures we roughly estimated the cost of WV phenomenon (for Detroit airport case) as about 22,000 landings of large aircraft per year. This translates to WV cost of tens of millions of dollars per year in a moderately busy airport.

4) We hypothesized that maximizing the throughput (by adjusting the average separation spacing) does not necessarily assure the most economic use of the runway. Another model developed to mimic these economic dynamics of the approach operations accounting for the go-around cost (to all beneficiaries) and the befit of every successful landing (to all beneficiaries). System beneficiaries include airlines, passengers, airports, employees, etc. We showed that economically optimal level of operations depends on cost to benefit ratio rather than depending on specific values of go-around cost and successful landing profit. The results validated our hypothesis.

We illustrated the methodologies for specific pairs of follow-lead aircraft without loss of generality. An immediate extension to our models is to consider a varied fleet mix that includes a mix of varied separation standards. The economic model presented in this article should be extended to account for different types of aircrafts. Some general properties of the developed optimization models are discussed for specific probability distributions. Property 1 needs to be investigated with more rigorous mathematical expressions. It might be true for more general classes of landing time interval distributions, Gamma class distributions, for example. Some real world estimations of the cost of go-around and the successful landing profit are necessary to obtain. Also, the proposed wake vortex go-around procedure needs to be further explored and investigated for real world implementation purposes. These subjects are practical research topics to consider.

Appendix I

Proof of Property 4: The proof is by contradiction. $\omega^*(r)$ is the point where $dg/d\omega = 0$ or $p(\varpi^*) + \varpi^* \cdot p'(\varpi^*) = 1/(1+r)$. Increasing r decreases the right hand side and consequently the left hand side of this equation. On the other hand, ω, $p(\omega)$, and $dp/d\omega$ are all increasing in ω for $\omega < \text{Argmax}\{dp/d\omega\}$. Thus if ω does not decrease, the LHS will not decrease. This is a contradiction, and completes the proof. ◄

Derivation of equation (2). For the case $L < x_0$, note that

P(GA) = P{separation at DP2 $< x_0$ or, exclusively, separation at DP1 $< ROT$}

$\qquad$ = P{$LTI < x_0$ or $LTI < ROT$}.

Then,

$$P(GA) = P(LTI < x_0) + P\{LTI < ROT \text{ and } LTI \geq x_0\}. \quad (I.1)$$

On the other hand,

$$P\{LTI < ROT \text{ and } LTI \geq x_0\} = \int_{x_0}^{\infty} \int_{x_0}^{y} dF_{LTI,ROT}(x,y)$$

$$= \int_{x_0}^{\infty} \left[\int_{x_0}^{y} dF_{LTI}(x) \right] dF_{ROT}(y)$$

$$= \int_{x_0}^{\infty} [F_{LTI}(y) - F_{ROT}(x_0)] dF_{ROT}(y) \quad (I.2)$$

$$= \int_{x_0}^{\infty} F_{LTI}(y) dF_{ROT}(y) - F_{LTI}(x_0) \cdot \int_{x_0}^{\infty} dF_{ROT}(y)$$

$$= \int_{x_0}^{\infty} F_{LTI}(y) dF_{ROT}(y) - F_{LTI}(x_0) \cdot (1 - F_{ROT}(x_0))$$

$$= \int_{x_0}^{\infty} F_{LTI}(y) dF_{ROT}(y) - F_{LTI}(x_0) + F_{LTI}(x_0) F_{ROT}(x_0),$$

where F_{LTI} and F_{ROT} are CDF of LTI and ROT, respectively. Joint distribution of LTI and ROT is broken into multiplication of their marginal distributions because of their independence. Plugging in (I.2) in (I.1) gives equation (2).

References

[1] Gilbo, E.P., Sept. 1993, *Airport Capacity: representation, Estimation, Optimization*, IEEE Transactions on Control Systems Technology, Vol. 1, No. 3, pp. 144-154.

[2] Railsback, P., L. Sherry, 2006, A Survey of Rationales and Methods for Determining Declared Airport Capacity, Presented at Annual Meeting of the Transportation Research Board.

[3] Bowen, E.G., T. Pearcey, 1948, Delays in the Flow Air Traffic, Journal of Royal Aeronautical Society, Vol. 52, pp 251-258

[4] Bell, G.E., 1949, Operational Research into Air Traffic Control, Journal of Royal Aeronautical Society, Vol. 53, pp 965-976.

[5] Blumstein, A., 1959, The Landing Capacity of a Runway, Operations Research, Vol. 7, No. 6., pp 752-763.

[6] Lang, S., A. Mundra, W. Cooper, B.S. Levy, C.R. Lunsford, A.P. Smith, J.A. Tittsworth, 2003, A Phased Approach to Increase Airport Capacity Through Safe Reduction of Existing Wake

Turbulence Constraints, Air Traffic Control Quarterly, Vol. 11, No. 4, pp. 331-356.

[7] Boesel, J., D. Bodoh, 2004, Simulating Airspace Redesign for Arrivals to Detroit Wayne County Airport, Proceedings of Winter Simulation Conference, pp. 1318-1325.

[8] Xie, Y., J. Shortle, G. Donohue, 2003, Runway landing safety analysis: a case of Atlanta Hartsfield airport, presented at the Digital Avionics Systems Conf., Indianapolis, IN.

[9] Xie, Y., 2005, Quantitative Analysis of Airport Runway Capacity and Arrival Safety Using Stochastic Methods, Ph.D. dissertation, Dept. of Sys. Eng. and Oper. Res., George Mason Univ., Fairfax, VA.

[10] Levy, B., J. Legg, and M. Romano, 2004, Opportunities for Improvements in Simple Models for Estimating Runway Capacity, presented at the 23rd Digital Avionics Systems Conference, Salt Lake City, UT.

[11] Jeddi, B.G., J.F. Shortle, L. Sherry, 2006, Statistical separation Standards for the Aircraft – Approach Process, Proceedings of 25th Digital Avionics System Conference, Portland, OR, pp. 2A1-1 to 2A1-13.

[12] Nolan, M.S., 2003, Fundamentals of Air Traffic Control, 4th Ed., Brooks/Cole Publishing Company.

[13] Jeddi, B.G., J.F. Shortle, L. Sherry, 2006, Statistics of the Approach Process at Detroit Metropolitan Wayne County Airport, Presented in 2nd ICRAT conference, Serbia and Montenegro.

[14] FAA Order 7110.65, Air Traffic Control, Federal Aviation Administration, Sept. 1993, Wake Turbulence, Para 2,1,19.

[15] FAA Order 7110.65, Same Runway Separation, Federal Aviation Administration, Sept. 1993, Wake Turbulence, Para 3,9,6.

[16] Holzäpfel, F., March-April 2003, Probabilistic Two-Phase Wake Vortex Decay and Transport Model, Journal of Aircraft, Vol. 40, No. 2.

[17] Robins, R.E., and D.P. Delisi, June 2002, NWRA AVOSS wake vortex Prediction Algorithm Version 3.1.1, NASDA/CR-2002-211746.

[18] Shortle, J.F., B.G. Jeddi, 2007, Probabilistic Analysis of Wake Vortex Hazards for Landing Aircraft Using Multilateration Data, Presented at 86th Transportation Research Board Annual Meeting, Washington DC.

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Disclaimer

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Probability, Approach/Landing, Risk, Safety, Maximization, Runway capacity, Throughput, Simultaneous runway occupancy, Wake Vortex, Go around. Economic gain

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