Introducing Probabilistic Coherence Spaces

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ABSTRACT

Coherence Spaces were defined by J. Y. Girard in [2] with the objective to study the semantics of Linear Logic.

Coherence Spaces are a special subcategory of Scott domains [4] having a strictly finitary structure. The objects are constructed over a set of tokens (basic elements) where a coherence (reflexive and symmetric) relation is defined. The order of information is the set inclusion relation.

In this work, we introduce the Probabilistic Coherence Spaces by associating probabilistic values with the objects of coherence spaces. As a result we get a notion of partial probability associated with the partial objects of the probabilistic coherence spaces.

It is possible to adopt a vector notation, introducing the Vector Coherence Spaces, so that Probabilistic Coherence Spaces can be used to represent state spaces of probabilistic processes. Since such states represent partial probabilities, computation with such states produces probabilistic approximation processes whose limits are the conventional probabilistic processes.

We also study linear functions on probabilistic coherence spaces to represent those probabilistic approximation processes and conventional probabilistic limits.

The aim to recast in terms of the special structure of Vector Coherence Spaces the fundamental notions of probabilistic and quantum computing [3], that were originally defined using complex vector spaces as the underlying semantic model.

One immediate application of the work is in the construction of a domain of Markov models [1] with partial probabilities.

References

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