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Existence and Iteration of Positive Solutions for Multipoint Boundary Value Problems Dependence on the First Order Derivative with One-Dimensional p-Laplacian

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Abstract

In this paper, we study the existence of monotone positive solutions for the following nonlinear m-point singular boundary value problem with p-Laplacian operator. The main tool is the monotone iterative technique. We obtain not only the existence of positive solutions for the problem, but also establish iterative schemes for approximating solution.

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1 Introduction

In this paper, we study the existence of positive solutions for the following nonlinear m-point singular boundary value problem with p-Laplacian operator

$$\begin{cases} (\phi_p(u'))' + a(t)f(t, u(t), u'(t)) = 0, \quad 0 < t < 1, \\ u'(0) = \sum_{i=1}^{m-2} a_i u'(\xi_i), \\ u(1) = \sum_{i=1}^{k} b_i u(\xi_i) - \sum_{i=k+1}^{s} b_i u(\xi_i) - \sum_{i=s+1}^{m-2} b_i u'(\xi_i), \end{cases} \tag{1.1}$$

where $\phi_p(s)$ is p-Laplacian operator, i.e. $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\phi_q = (\phi_p)^{-1}$, $\dfrac{1}{p} + \dfrac{1}{q} = 1$, $1 \le k \le s \le m-2$, a_i, $b_i \in (0, +\infty)$ with $0 <$

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$\sum_{i=1}^{k} b_i - \sum_{i=k+1}^{s} b_i < 1$, $0 < \sum_{i=1}^{m-2} a_i < 1$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $a(t) \in C((0,1),[0,\infty))$.

The study of multi-point boundary value problems for linear second-order ordinary differential equations was initiated by Il'in and Movisev [1,2]. Motivated by the study of [1,2], Gupta [3] studied certain three-point boundary value problems for nonlinear ordinary differential equations. Since then, more general nonlinear multi-point boundary value problems have been studied by several authors. We refer the reader to [4,5,6] for some references along this line. Multi-point boundary value problems describe many phenomena in the applied mathematical sciences. For example, the vibrations of a guy wire of a uniform cross-section and composed of N parts of different densities can be set up as a multi-point boundary value problems (see Moshinsky [7]); many problems in the theory of elastic stability can be handle by the method of multi-point boundary value problems(see Timoshenko [8])

In 2001, Ma [6] studied m-point boundary value problem (BVP)

$$\begin{cases} u''(t) + h(t)f(u) = 0, \qquad 0 \le t \le 1, \\ u(0) = 0, \qquad u(1) = \sum\limits_{i=1}^{m-2} \alpha_i u(\xi_i), \end{cases}$$

where $\alpha_i > 0 (i = 1, 2, \cdots)$, $0 < \sum\limits_{i=1}^{m-2} \alpha_i < 1$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, and $f \in C([0,+\infty),[0,+\infty))$, $h \in C([0,1],[0,+\infty))$. Author established the existence of positive solutions theorems under the condition that f is either superlinear or sublinear.

In[4], Xu studied the following m-point boundary value problem (BVP)

$$\begin{cases} (\phi_p(u'))' + a(t)f(u(t)) = 0, \quad 0 < t < 1, \\ u'(0) = \sum_{i=1}^{m-2} a_i u'(\xi_i), \\ u(1) = \sum_{i=1}^{k} b_i u(\xi_i) - \sum_{i=k+1}^{s} b_i u(\xi_i) - \sum_{i=s+1}^{m-2} b_i u'(\xi_i), \end{cases}$$

They show the existence of positive solutions if f is either superlinear or sublinear by applying the fixed point theorem in cones.

Recently, Ma etal.[5] proved the existence of at least one positive solutions for m-point boundary value problem (BVP)

$$\begin{cases} (\phi_p(u'))' + a(t)f(t, u(t)) = 0, \quad 0 < t < 1, \\ u'(0) = \sum_{i=1}^{m-2} a_i u'(\xi_i), \qquad u(1) = \sum_{i=1}^{m-2} b_i u(\xi_i), \end{cases}$$

where $0 < \sum_{i=1}^{m-2} b_i < 1$, $0 < \sum_{i=1}^{m-2} a_i < 1$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $a(t) \in L^1[0,1]$, $f \in C([0,1] \times [0,+\infty),[0,+\infty))$. They obtained the existence of monotone positive solutions by using the monotone iterative technique in cones. But the nonlinear term f does not depend on the first-order derivative.

Motivated by the results mentioned above, in this paper we study the existence of positive solutions of m-point boundary value problem (1.1). It is worth stating that the first term of our iterative scheme is a constant function or a simple function. Therefore, the iterative scheme is significant and feasible. To our knowledge, this is the first paper to use the technique of monotone iterative to deal with multipoint boundary value problem with one-dimensional p-Laplacian operator when nonlinear term f involves in the first-order derivative.

In the rest of the paper, we make the following assumptions:
(H_1) $a_i,\ b_i \in (0,+\infty),\ 0 < \sum_{i=1}^{k} b_i - \sum_{i=k+1}^{s} b_i < 1,\ 0 < \sum_{i=1}^{m-2} a_i < 1,\ 0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$;
(H_2) $f \in C([0,1]\times[0,+\infty)\times R, [0,+\infty))$, $a \in C([0,1],[0,+\infty))$ is not identically zero on any compact subinterval of $(0,1)$. In addition,

$$0 < \int_0^1 a(t)dt < 0.$$

We recall that a function u is said to be concave on [0,1], if

$$u(\lambda t_2 + (1-\lambda t_1) \geq \lambda u(t_2) + (1-\lambda)u(t_1), \qquad t_1, t_2, \lambda \in [0,1],$$

and a function is said to be monotone on [0,1], if $u(t)$ is nondecreasing or nonincreasing.

2 Preliminary Notes

In this section , we present some lemmas that are important to our main results.

Lemma 2.1. *Let (H_1) and (H_2) hold. Then for $x \in C^+[0,1]$, the problem*

$$\begin{cases} (\phi_p(u'))' + a(t)f(t,x(t),x'(t)) = 0, \quad 0<t<1, \\ u'(0) = \sum_{i=1}^{m-2} a_i u'(\xi_i), \\ u(1) = \sum_{i=1}^{k} b_i u(\xi_i) - \sum_{i=k+1}^{s} b_i u(\xi_i) - \sum_{i=s+1}^{m-2} b_i u'(\xi_i) \end{cases} \tag{2.1}$$

has a unique solution

$$u(t) = B_x - \int_t^1 \phi_p^{-1}\left(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr\right)ds,$$

where A_x, B_x satisfy

$$\phi_p^{-1}(A_x) = \sum_{i=1}^{m-2} a_i \phi_p^{-1}\left(A_x - \int_0^{\xi_i} a(s)f(s,x(s),x'(s))ds\right), \tag{2.2}$$

$$\begin{aligned} B_x &= -\frac{1}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}\Big(\sum_{i=1}^{k} b_i \int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr)ds \\ &- \textstyle\sum_{i=k+1}^{s} b_i \displaystyle\int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr)ds \\ &+ \textstyle\sum_{i=s+1}^{m-2} b_i \phi_p^{-1}(A_x - \displaystyle\int_0^{\xi_i} a(s)f(s,x(s),x'(s))ds)\Big). \end{aligned}$$

Define $l = \dfrac{\phi_p\left(\sum_{i=1}^{m-2} a_i\right)}{1-\phi_p\left(\sum_{i=1}^{m-2} a_i\right)}$, *then there exists a unique* $A_x \in \left[-l\int_0^1 a(s)f(s,x(s),x'(s))ds, 0\right]$ *satisfying (2.2).*

Proof. The method of the proof is similar to Lemma 2.1[5], we omit the details.

Lemma 2.2. *Let* (H_1) *and* (H_2) *hold. If* $x \in C^+[0,1]$, *the unique solution of the problem (2.1) satisfies* $u(t) \geq 0$.

Proof. The method of the proof is similar to Lemma 2.2[4], we omit the details.

Let $E = C^1[0,1]$, then E is Banach space, with respect to the norm $\|u\| = \max\{\sup_{t\in[0,1]} |u(t)|, \ \ \sup_{t\in[0,1]} |u'(t)|\}$.

Now define an operator $T : P \to C[0,1]$ by setting

$$\begin{aligned} (Tx)(t) &= -\frac{1}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}\Big(\sum_{i=1}^{k} b_i \int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr)ds \\ &- \textstyle\sum_{i=k+1}^{s} b_i \displaystyle\int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr)ds \\ &+ \textstyle\sum_{i=s+1}^{m-2} b_i \phi_p^{-1}(A_x - \displaystyle\int_0^{\xi_i} a(s)f(s,x(s),x'(s))ds)\Big) \\ &- \int_t^1 \phi_p^{-1}\left(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr\right) ds. \end{aligned}$$

where $P = \{u \in E | \ \ u \geq 0, \text{ u is concave function}\}$ is a cone in E.

Obviously, BVP (1.1) has a solution $x = x(t)$ if and only if x is a fixed point of the operator T.

Lemma 2.3. *Let* (H_1) *and* (H_2) *hold. Then* $T : P \to P$ *is completely continuous.*

Proof. Clearly

$$\begin{aligned} u'(t) &= \phi_p^{-1}\left(A_x - \int_0^t a(s)f(s,x(s),x'(s))ds\right) \\ &= -\phi_p^{-1}\left(-A_x + \int_0^t a(s)f(s,x(s),x'(s))ds\right) \\ &\leq 0. \end{aligned}$$

It is easy to see that $u'(t_2) \leq u'(t_1)$ for any $t_1, t_2 \in [0,1]$ with $t_1 \leq t_2$. Hence $u'(t)$ is a nonincreasing function on [0,1]. This implies that

$$\sup_{t\in[0,1]} |u(t)| = u(0), \sup_{t\in[0,1]} |u'(t)| = |u'(1)|.$$

This means that the graph of $u(t)$ is concave down on (0,1). So we have $TP \subset P$. It is easy to see that T is continuous operator because f and a is continuous. Now, we prove T is compact. Let $\Omega \subset P$ be an bounded set. Then, there exists R, such that $\Omega \subset \{x \in P| \quad ||x|| \leq R\}$. For any $x \in \Omega$, we have $0 \leq \int_0^1 a(s)f(s, x(s), x'(s))ds \leq \max_{s\in[0,1],u\in[0,R],v\in[-R,R]} f(s,u,v) \int_0^1 a(s)ds = M$. Then, we have

$$|A_x| \leq lM,$$

$$|Tx| \leq \frac{(1 + \sum_{i=k+1}^{s} b_i + \sum_{i=s+1}^{m-2} b_i)\phi_p^{-1}[(l+1)M]}{1 - \sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i},$$

$$|(Tx)'| \leq \phi_p^{-1}[(l+1)M],$$

$$|(\phi_p(Tx)')'| \leq [(l+1)M].$$

The Arzela-Ascoli theorem guarantees that $T\Omega$ is relatively compact ,which means T is compact. The proof of Lemma 2.3 is completed.

3 Main Results

For natational convenience, let

$$A = \max\left\{ \frac{(1 + \sum_{i=k+1}^{s} b_i + \sum_{i=s+1}^{m-2} b_i)\phi_p^{-1}((l+1)\int_0^1 a(s)ds)}{1 - \sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}, \phi_p^{-1}((l+1)\int_0^1 a(s)ds) \right\},$$

Our main results are following theorems.

Theorem 3.1. *Suppose conditions (H_1) and (H_2) hold. There exists a constant $a > 0$ such that*

(B_1) $f(t, x_1, y_1) \leq f(t, x_2, y_2)$ for any $t \in [0,1], 0 \leq x_1 \leq x_2 \leq a, 0 \leq |y_1| \leq |y_2| \leq a$;

(B_2) $\max_{t\in[0,1]} f(t, a, a) \leq \phi_p(\frac{a}{A})$;

(B_3) $f(t, 0, 0)$ is not identically zero on $[0,1]$.

Then the BVP (1.1) has two positive solutions ω^*, v^* such that

$$0 < \omega^* \leq a, 0 < |(\omega^*)'| \leq a,$$

and

$$\lim_{n\to\infty}\omega_n = \lim_{n\to\infty} T^n\omega_0 = \omega^*,$$

$$\lim_{n\to\infty}(\omega_n)' = \lim_{n\to\infty}(T^n\omega_0)' = (\omega^*)',$$

where $\omega_0(t) = a\left(\frac{(1+\sum_{i=k+1}^{s} b_i + \sum_{i=s+1}^{m-2} b_i)}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i} - t\right), t\in[0,1],$

$$0 < v^* \le a, 0 < |(v^*)'| \le a$$

and

$$\lim_{n\to\infty} v_n = \lim_{n\to\infty} T^n v_0 = v^*,$$

$$\lim_{n\to\infty}(v_n)' = \lim_{n\to\infty}(T^n v_0)' = (v^*)',$$

where $v_0(t) = 0$.

Proof. We denote $P_a = \{u \in P| \ ||u|| < a\}, \overline{P}_a = \{u \in P| \ ||u|| \le a\}$. In what follows, we first prove $T : \overline{P}_a \to \overline{P}_a$. If $u \in \overline{P}_a$, then $||u|| \le a$, we have

$$0 \le u(t) \le \max_{t\in[0,1]} u(t) \le ||u|| \le a,$$

$$|u'(t)| \le \max_{t\in[0,1]} u'(t) \le ||u|| \le a.$$

So by conditions (B_1) and (B_3) we have

$$0 \le f(t, u(t), u'(t)) \le f(t, a, a) \le \max_{t\in[0,1]} f(t, a, a) \le \phi_p(\frac{a}{A}), \quad \text{for} \quad 0 \le t \le 1.$$

In fact,

$$||Tu|| = \max\{Tu(0), -Tu'(1)\}.$$

So we have

$$\begin{aligned}
(Tx)(0) &= \frac{-1}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}\Big(\sum_{i=1}^{k} b_i \int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr)ds \\
&- \textstyle\sum_{i=k+1}^{s} b_i \displaystyle\int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr)ds \\
&+ \textstyle\sum_{i=s+1}^{m-2} b_i \displaystyle\int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr)ds\Big) \\
&- \int_0^1 \phi_p^{-1}\left(A_x - \int_0^s a(r)f(r,x(r),x'(r))dr\right)ds \\
&\le \frac{1}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}\Big(\sum_{i=1}^{k} b_i \int_0^1 \phi_p^{-1}((l+1)\int_0^1 a(r)f(r,x(r),x'(r))dr)ds \\
&+ \textstyle\sum_{i=s+1}^{m-2} b_i \displaystyle\int_0^1 \phi_p^{-1}((l+1)\int_0^1 a(r)f(r,x(r),x'(r))dr)ds\Big) \\
&+ \int_0^1 \phi_p^{-1}\left((l+1)\int_0^1 a(r)f(r,x(r),x'(r))dr\right)ds \\
&\le \frac{\left(1+\sum_{i=k+1}^{s} b_i + \sum_{i=s+1}^{m-2} b_i\right)\phi_p^{-1}\left[(l+1)\int_0^1 a(s)ds\right]}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}\frac{a}{A} \le a.
\end{aligned}$$

$$\begin{aligned}
-(Tu)'(1) &= -\phi_p^{-1}\left(A_x - \int_0^1 a(s)f(s,x(s),x'(s))ds\right) \\
&= \phi_p^{-1}\left(-A_x + \int_0^1 a(s)f(s,x(s),x'(s))ds\right) \\
&\le \phi_p^{-1}\left((l+1)\int_0^1 a(s)f(s,x(s),x'(s))ds\right) \\
&\le \phi_p^{-1}\left((l+1)\int_0^1 a(s)ds\right)\frac{a}{A} \le a.
\end{aligned}$$

Thus we have

$$||Tu|| \le a.$$

We have shown that

$$T : \overline{P}_a \to \overline{P}_a.$$

Let

$$\omega_0(t) = a\left(\frac{(1+\sum_{i=k+1}^{s} b_i + \sum_{i=s+1}^{m-2} b_i)}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i} - t\right), \quad \text{for} \quad 0 \le t \le 1.$$

Let $\omega_1(t) = T\omega_0(t)$, $\omega_2(t) = T\omega_1(t) = T^2\omega_0(t)$, then denote $\omega_{n+1}(t) = T\omega_n = T^n\omega_0(t)$, $n = 0,1,2,\cdots$. Since $T : \overline{P}_a \to \overline{P}_a$, we have $\omega_n \in T\overline{P}_a \subset \overline{P}_a$, $n = 0,1,2,\cdots$. Since T is completely continuous, $\{\omega_n\}_{n=1}^{\infty}$ is a sequentially compact set. Since

$$\begin{aligned}
\omega_1(t) &= (T\omega_0)(t) \\
&= \frac{-1}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}\Big(\sum_{i=1}^{k} b_i \int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r) f(r,\omega_0(r),\omega_0^{'}(r))dr)ds \\
&- \textstyle\sum_{i=k+1}^{s} b_i \displaystyle\int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r) f(r,\omega_0(r),\omega_0^{'}(r))dr)ds \\
&+ \textstyle\sum_{i=s+1}^{m-2} b_i \displaystyle\int_{\xi_i}^{1} \phi_p^{-1}(A_x - \int_0^s a(r) f(r,\omega_0(r),\omega_0^{'}(r))dr)ds\Big) \\
&- \int_t^1 \phi_p^{-1}\left(A_x - \int_0^s a(r) f(r,\omega_0(r),\omega_0^{'}(r))\right)ds \\
&\le \frac{1}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i}\Big(\sum_{i=1}^{k} b_i \int_0^1 \phi_p^{-1}((l+1)\int_0^1 a(r) f(r,\omega_0(r),\omega_0^{'}(r))dr)ds \\
&+ \textstyle\sum_{i=s+1}^{m-2} b_i \displaystyle\int_0^1 \phi_p^{-1}((l+1)\int_0^1 a(r) f(r,\omega_0(r),\omega_0^{'}(r))dr)ds\Big) \\
&+(1-t)\phi_p^{-1}\left((l+1)\int_0^1 a(r) f(r,\omega_0(r),\omega_0^{'}(r))dr\right)ds \\
&\le \left(\frac{(1+\sum_{i=k+1}^{s} b_i + \sum_{i=s+1}^{m-2} b_i)}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i} - t\right)\phi_p^{-1}((l+1)\int_0^1 a(s)ds)\frac{a}{A} \le \omega_0(t).
\end{aligned}$$

$$\begin{aligned}
|\omega_1^{'}(t)| &= |T\omega_0^{'}(t)| \\
&= -\phi_p^{-1}\left(A_x - \int_0^1 a(s) f(s,\omega_0(s),\omega_0^{'}(s))ds\right) \\
&= \phi_p^{-1}\left(-A_x + \int_0^1 a(s) f(s,\omega_0(s),\omega_0^{'}(s))ds\right) \\
&\le \phi_p^{-1}\left((l+1)\int_0^1 a(s) f(s,\omega_0(s),\omega_0^{'}(s))ds\right) \\
&\le \phi_p^{-1}\left((l+1)\int_0^1 a(s)ds\right)\tfrac{a}{A} \le a = |\omega_0^{'}(t)|.
\end{aligned}$$

Then we get

$$\omega_1(t) \le \omega_0(t), |\omega_1^{'}(t)| \le |\omega_0^{'}(t)|, \quad 0 \le t \le 1.$$

So,

$$\omega_2(t) = T\omega_1(t) \le T\omega_0(t) = \omega_1(t) \quad 0 \le t \le 1,$$

$$|\omega_2'(t)| = |(T\omega_1)'(t)| \le |(T\omega_0)'(t)| = |\omega_1'(t)| \quad 0 \le t \le 1.$$

Hence by induction we have

$$\omega_{n+1}(t) \le \omega_n(t), |\omega_{n+1}'(t)| \le |\omega_n'(t)| \;\; 0 \le t \le 1, n = 1, 2, \cdots.$$

Thus, there exists $\omega^* \in \overline{P}_a$ such that $\omega_n \to \omega^*$. Applying the continuity of T and $\omega_{n+1} = T\omega_n$, we get $T\omega^* = \omega^*$.

Let $v_0(t) = 0, 0 \leq t \leq 1$, then $v_0(t) \in \overline{P}_a$. Set $v_1 = Tv_0, v_2 = Tv_1 = T^t v_0$, then we denote $v_{n+1} = Tv_n = T^n v_0, n = 1, 2, \cdots$. Since we have $\omega_n \in T\overline{P}_a \subset \overline{P}_a, \quad n = 0, 1, 2, \cdots$. Since $T : \overline{P}_a \to \overline{P}_a$, we have $v_n \in T\overline{P}_a \subset \overline{P}_a, \quad n = 0, 1, 2, \cdots$. Since T is completely continuous, we see that $\{v_n\}_{n=1}^{\infty}$ is a sequentially compact set.

Since $v_1 = Tv_0 = T0 \in \overline{P}_a$, we have

$$v_1(t) = Tv_0(t) = (T0)(t) \geq 0, \ 0 \leq t \leq 1,$$

$$v_1'(t) = (Tv)_0'(t) = (T0)'(t) \geq 0, \ 0 \leq t \leq 1.$$

So,

$$v_2(t) = Tv_1(t) \geq (T0)(t) = v_(t), \ 0 \leq t \leq 1,$$

$$v_2'(t) = (Tv)_1'(t) \geq |(T0)'(t)| = |v_1'(t)|, \ 0 \leq t \leq 1.$$

By an induction argument similar to the above we obtain

$$v_{n+1} \geq v_n, |v_{n+1}'| \geq |v_n'|, \ 0 \leq t \leq 1, n = 1, 2, \cdots.$$

Hence, there exists $v^* \in \overline{P}_a$ such that $v_n \to v^*$. Applying the continuity of T and $v_{n+1} = Tv_n$, we get $Tv^* = v^*$.

If $f(t, 0, 0)$ is not identically zero on $[0, 1]$, then the zero function is not the solution of BVP(1.1). Thus, $\max_{0 \leq t \leq 1} |v^*(t)| > 0$, we have

$$v^* \geq \min\{t, 1-t\} \max_{0 \leq t \leq 1} |v^*(t)| > 0, 0 < t < 1.$$

It is well known that each fixed point of T in P is a solution of BVP(1.1). Hence we have shown that ω^* and v^* are two positive, concave solutions of BVP(1.1). The proof of Theorem 3.1 is completed.

The following corollaries follow easily.

Corollary 3.1. *Suppose conditions (H_1), (H_2), (B_1), (B_3) hold. There exists a constant $a > 0$ such that*

(B_4) $\underline{\lim}_{l \to \infty} \max_{t \in [0,1]} f(t, l, a) \leq \phi_p(\frac{1}{A})$, *(particularly* $\underline{\lim}_{l \to \infty} \max_{t \in [0,1]} f(t, l, a) = 0$*).*

Then the BVP (1.1) has two positive solutions ω^*, v^* such that the conclusion of Theorem3.1 hold.

Corollary 3.2. *Suppose conditions (H_1), (H_2), (B_1), (B_3) hold. There exist constants $0 < a_1 < a_2 < \cdots < a_n$ such that*

(B_5) $\max_{t \in [0,1]} f(t, a_k, a_k) \leq \phi_p(\frac{a_k}{A})$, *(particularly* $\underline{\lim}_{l \to \infty} \max_{t \in [0,1]} f(t, l, a_k) = 0, \ k = 1, 2, \cdots$*).*

Then the BVP (1.1) has $2n$ *positive solutions* ω_k^*, v_k^* *such that*

$$0 < \omega_k^* \le a_k, 0 < |(\omega_k^*)'| \le a_k,$$

and

$$\lim_{n\to\infty} \omega_{k_n} = \lim_{n\to\infty} T^n \omega_{k_0} = \omega_k^*,$$

$$\lim_{n\to\infty} (\omega_{k_n})' = \lim_{n\to\infty} (T^n \omega_{k_0})' = (\omega_k^*)',$$

where $\omega_{k_0}(t) = a_k \left(\frac{(1+\sum_{i=k+1}^{s} b_i + \sum_{i=s+1}^{m-2} b_i)}{1-\sum_{i=1}^{k} b_i + \sum_{i=k+1}^{s} b_i} - t \right)$, *for* $0 \le t \le 1$,

$$0 < v_k^* \le a, 0 < |(v_k^*)'| \le a,$$

and

$$\lim_{n\to\infty} v_{k_n} = \lim_{n\to\infty} T^n v_{k_0} = v_k^*,$$

$$\lim_{n\to\infty} (v_{k_n})' = \lim_{n\to\infty} (T^n v_{k_0})' = (v_k^*)',$$

where $v_{k_0}(t) = 0,\ t \in [0,1]$.

Corollary 3.3. *Suppose conditions* (H_1), (H_2), (B_1), (B_3) *hold. There exist constants* $0 < a_1 < a_2 < \cdots < a_n$ *such that*
(B_6) $\underline{\lim}_{l\to\infty} \max_{t\in[0,1]} f(t,l,a_k) \le \phi_p(\frac{1}{A})$, *(particularly* $\underline{\lim}_{l\to\infty} \max_{t\in[0,1]} f(t,l,a_k) = 0,\ k = 1, 2, \cdots$*).*
Then the BVP (1.1) has $2n$ *positive solutions* ω_k^*, v_k^* *such that the conclusion of Corollary 3.2.*

References

[1] V.A.Il'in, E.I.Moviseev, Nonlocal boundary value problem of the second kind for a Sturm-Liouville operator, Differ .Eqn.23 (1987) 979-987.

[2] V.A.Il'in, E.I.Moviseev, Nonlocal boundary value problem of the first kind for a Sturm-Liouville operator in its differential and finite difference aspects, Differ .Eqn.23 (1987) 803-810.

[3] C.P.Gupta, Solvability of a three-point nonlinear boundary value problem for a second order ordinary differential equation, J.Math.Anal.Appl. 168 (1992) 540-551.

[4] F.Xu, Positive solutions for multipoint boundary value problems with one-dimensional p-Laplacian operator, Appl.Math.Comput. (2007) (in press).

[5] D. Ma, Z. Du, W.Ge, Existence and iteration of monotone positive solutions for multipoint boundary value problems with p-Laplacian operator, Comput.Math.Appl. 50 (2005) 729-739.

[6] R. Ma, Positive solutions for a nonlinear m-point boundary value problems, Comput.Math.Appl.42(2001) 775-765.

[7] M. Moshinsky, Sobre los problems de condiciones a la frontiera en una dimension de caracteristicas discontinuas, Bol.Soc.Mat.Mexicana. 7(1950)1-25.

[8] S.Timoshenko, Theory of Elastic Stability.Mc.Graw, New York,1961.

[9] M. A. Krasnoselskii, Positive solution of operator equations, Noordhoof,Gronignen,1964.

[10] D. Guo and V. Lakshmikantham, Nonlinear Problems in Abstract Cone. Academic Press, Sandiego, 1988.

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